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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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STRAIGHT LINE PLOTTING OF SKEW FREQUENCY DATA

By R. D. Goodrich,* M. Am. Soc. C. E.

To Be Presented September 1, 1926.

Synopsis

Valuable and suggestive papers have been presented to the Society in recent years on methods for the investigation of the frequency of rainfall and run-off of various intensities, for use in engineering design.

This paper deals with the application of new equations for both the "Integral" or "Duration" Curve and the "Frequency" or "Differential" Curve; and for expressing the variations in the occurrence of phenomena which follow "Skew Frequency Laws" as to their distribution. These equations are particularly applicable to the plotting of hydraulic data (rainfall and run-off) having a limited number of observations in the series.

The equation of the Duration Curve is more easily obtained and is of simpler form than that of the Frequency Curve; it is important to note, however, that with either equation given the other can be written at once. Fractional exponents are to be expected, but they do not affect the fixed relations between the two equations. The equations of the Duration Curve are such that a curve can always be plotted as a straight line function of data that follow various skew frequency laws. One general equation is given, which can be simplified for the condition of finite or infinite limits.

Numerous examples of the use of these equations in the plotting of curves on a new type of skew frequency paper are shown. Ordinary and logarithmic co-ordinate paper may also be used. These curves cover a greater variety of conditions than have heretofore been presented. Curves are plotted with equal facility when either or both the coefficient of variation and coefficient of skew are negative as well as when the mode is greater than the mean. The same plotting paper is used when either one or both the limits are finite. The equations can also be applied to the representation of certain population curves.

The equations and methods can be used by any engineer, as the work is largely graphical and most of the difficulties and objections to the use of curves and equations based on statistical methods of analysis are obviated.

Note.—This paper is issued before the date set for presentation and discussion. Correspondence is invited and may be sent by mail to the Secretary. Discussion on the paper will be closed in November, 1926, and, when finally closed, the paper, with discussion in full, will be published in *Transactions*.

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Introduction

For several years the Chihli River Commission has been engaged in studying flood conditions in the Province of Chihli, North China, but great difficulty has been experienced in securing data as to rainfall and run-off, especially regarding flood flows.

In order to make the fullest use of the meager data available, all the recent papers and discussions which have appeared in the publications of the Society have been studied with great care and with valuable results. While more records covering longer periods of observation are greatly to be desired, the methods of utilizing the existing short-term records are being constantly improved by engineers throughout the world. It is with the hope of making another step forward in this direction and of adding something to the value and usefulness of the shorter series of observations which are becoming more generally available that the following method is presented for treating skew frequency data so that they can be plotted according to a "straight line" law of variation.

The difficulties and objections to the use of methods developed for frequency studies of statistical data suggested a new angle for attacking the problem of applying frequency curves to engineering. The following entirely empirical equations and the methods developed for their use are the result of several years' study consisting of a search for equations that could be used as guides in estimating the magnitude and frequency of floods, and in developing and extending the equations when found. A skew frequency paper having logarithmic co-ordinates was devised. On this paper original data may be plotted and then replotted as a straight line function for several different skew frequency laws of distribution, irrespective of the signs of the "coefficient of variation" and "coefficient of skew", and also for any value of the "skewness" whether it be positive or negative. The methods are here presented in the hope that they will prove as useful to other engineers as they have to the writer. It is also hoped that a thorough study and discussion by other members of the profession may lead to the development of other uses and applications for these equations.

Mathematics of Skew Frequency Equations

Nomenclature.—

Let t = serial number of records when arranged in order of magnitude, or the percentage of the time.
n = number of records in the series, or 100 per cent.
R = value of any record in the series.
a = lower finite limit of the curve representing the records (not the lowest record).
b = upper finite limit of the curve representing the records (not the highest record).
c, d = exponents of $f(R)$, values of which are found from the curve.
h, k = coefficient of $f(R)$, values of which are found from the curve; but $h = k M$.

$$M = \log_{10} e = 0.43429\,; \quad \frac{1}{M} = 2.30259.$$

$\Gamma\left(\frac{1}{c}+1\right)$ = gamma function of $\left(\frac{1}{c}+1\right)$.

r = radius of gyration of the area under the frequency curve about the t' axis.

sv = standard variation.

cv = coefficient of variation.

cs = coefficient of skew.

$y = \text{colog}\left(1 - \frac{t}{n}\right)$.

The equations are distinguished by letters indicating the application of the particular form. For example, the general formula is labeled Equation (G), and the special form, Equation (F), etc. The designation, Equation (A), signifies that this form has the a-limit; Equation (B) has both the a-limit and the b-limit. Equation (A') is the differential of Equation (A) and Equation (A''), etc. The subscripts indicate variations in the base equations, as, for example, Equation (A_1) is a special form of Equation (A).

General Equation.—The general equation* of the integral or duration curve is:

$$t = n - n\,(10)^{-h\frac{(R-a)^c}{(b-R)^d}}$$

or,

$$t = n - n\,e^{-k\frac{(R-a)^c}{(b-R)^d}} \qquad (G)$$

In this equation it is evident that when $R = a$, $t = 0$; also when $R = b$, $t = n$ and, hence, n is the length of the ordinate at the end of the curve where the tangent (or asymptote) is parallel to the R-axis.

Special Forms of the Equation.—In some cases the equation may be conveniently used in the following forms:

$$t = n\,(10)^{-h\frac{(R-a)^c}{(b-R)^d}}$$

or,

$$t = n\,e^{-k\frac{(R-a)^c}{(b-R)^d}} \qquad (F)$$

When both c and d are positive, the curve for this equation is the reverse of that for Equation (G). That is, the curve begins above the t-axis at the height, n, and the R-axis becomes the tangent or asymptote at the right-hand end of this curve. If c and d are negative, the curve begins on the R-axis as in Equation (G).

In practice, Equation (F) may be used with c and d positive, if t is defined as the percentage of time for which any given value of R may be exceeded. Numerical computations are reduced by using this equation when plotting curves of rainfall or run-off on ordinary or logarithmic co-ordinate paper.

* This equation was derived by an entirely empirical process and subsequently compared with those of Pearson, Kapteyn, and others which were not known to the writer at the time of its development.

However, the ordinates to the differential or frequency curve derived from this equation would be negative; this might seem incorrect although in reality it matters but little. If the frequency curve is derived from Equation (G), the ordinates will be positive, and, hence, for convenience in the mathematical investigations, the second form (Equation (G)) will be discussed. When $d = 0$, Equation (G) becomes:

$$t = n - n\,(10)^{-h\,(R-a)^c}$$

or,

$$t = n - n\,e^{-k\,(R-a)^c} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A)$$

In Equation (A) the lower limit of R is a, which may be either positive, negative, or zero, while the upper limit is ∞, and it may be used, therefore, in place of Types III, V, and VI of the Pearson curves.*

When $a = 0$, it becomes a special case and is in the simplest form, that is,

$$t = n - n\,(10)^{-h\,R^c}$$

or,

$$t = n - n\,e^{-k\,R^c} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (S)$$

If Equation (S) had been derived from Equation (F) it would be in the form of Equation No. XV, in "Empirical Equations", by Professor Running, a study of which stimulated the investigation that finally led to the development of the general form.

When d and c are unequal, Equation (G) is too complicated for general use;† but when $d = c$, while a and b are both finite, it takes the following form which may be used in place of Type I of the Pearson curves:

$$t = n - n\,(10)^{-h\left[\frac{R-a}{b-R}\right]^c}$$

or,

$$t = n - n\,e^{-k\left[\frac{R-a}{b-R}\right]^c} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

When $c = 0$, Equation (G) becomes:

$$t = n - n\,(10)^{\frac{-h}{(b-R)^d}}$$

or,

$$t = n - n\,e^{\frac{-k}{(b-R)^d}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C)$$

The curve for Equation (C) has $-\infty$ as its lower limit and b as its upper limit. This type of curve does not appear to be included in any of Professor Pearson's types of skew frequency equations while the writer's equations do not include his Types II, IV, and VII. In Equation (F), when $d = 0$ and c is negative, the lower limit is a and the upper limit is $+\infty$. If $d = c$ (both negative) the limits are finite and it takes the form of Equation (B), but with $\frac{R-a}{b-R}$ inverted. It may be stated, therefore, that the equations thus

* The several types of curves here referred to are shown in *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), pp. 179, 182.

† The form with $d = 2c$ is useful in studies of certain population statistics.

far given cover the entire practical range of skew frequency curves as fully as the Pearson equations. The general equation, (G), contains too many constants for ordinary use and a limited experience shows that Equations (A), (B), and (S) are sufficient for frequency investigations of most rainfall and run-off data.

Although Equation (C) can be discussed in a manner similar to that given subsequently for Equations (S) and (A), it does not apply to the conditions met in ordinary hydraulic problems and, therefore, it will not be considered further.

Frequency Curves.—The general equation of the differential or frequency curve in this system of representation is obtained from Equation (G) by differentiation, thus,

$$t' = \frac{d\,t}{d\,R} = n\,k\,\frac{(R-a)^{c-1}}{(b-R)^{d+1}}\,[c\,(b-R)+d\,(R-a)]\,e^{-k\,\frac{(R-a)^c}{(b-R)^d}} \ldots (G')$$

From this, the equations of the frequency curves corresponding to the integral Equations (S), (A), and (B) are obtained by substituting the proper values of d and a as before

$$t' = n\,c\,k\,R^{(c-1)}\,e^{-k\,R^c} \ldots (S')$$

$$t' = n\,c\,k\,(R-a)^{c-1}\,e^{-k\,(R-a)^c} \ldots (A')$$

$$t' = n\,c\,k\,(b-a)\,\frac{(R-a)^{c-1}}{(b-R)^{c+1}}\,e^{-k\left[\frac{R-a}{b-R}\right]^c} \ldots (B')$$

Equation (S') is strikingly similar to Equation (8)* presented by J. Alden Foster, Assoc. M. Am. Soc. C. E., with R^c in place of z in the exponent of e. Since Equation (G') was derived by differentiation from Equation (G), and is entirely general, the total area under the curve of Equation (G') will always equal the maximum ordinate, n, of Equation (G'') as for other systems of frequency curves. These equations, therefore, evidently represent forms of skew frequency curves; the particular form of curve depending on the character of $f(R)$ and the values of the constants, k and c.

To continue the examination further, differentiate Equation (G'). Then:

$$t'' = n\,k\,\frac{(R-a)^{c-1}}{(b-R)^{d+1}}\,[c\,(b-R)+d\,(R-a)]\,e^{-k\,\frac{(R-a)^c}{(b-R)^d}}$$
$$\left\{\frac{d-c}{c\,(b-R)-d\,(R-a)} - \left[\frac{c-1}{R-a}+\frac{d-1}{b-R}\right] - k\,\frac{(R-a)^{c-1}}{(b-R)^{d+1}}\right.$$
$$\left.\left[c\,(b-R)+d\,(R-a)\right]\right\} \ldots (G'')$$

This equation may similarly be simplified to the following three forms:

$$t'' = n\,c\,k\,R^{c-2}\,[(c-1)-c\,k\,R^c]\,e^{-k\,R^c} \ldots (S'')$$

$$t'' = n\,c\,k\,(R-a)^{c-2}\,[(c-1)-c\,k\,(R-a)^c]\,e^{-k\,(R-a)^c} \ldots (A'')$$

$$t'' = n\,c\,k\,\frac{(R-a)^{c-2}}{(b-R)^{c+2}}\left[(c-1)\,\frac{2\,(R-a)}{b-a} - c\,k\left(\frac{R-a}{b-R}\right)^c\right]$$
$$(b-a)^2\,e^{-k\left(\frac{R-a}{b-R}\right)^c} \ldots (B'')$$

* "Theoretical Frequency Curves and Their Application to Engineering Problems," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 151.

In these equations it is evident that when $f(R) = \infty$, $t'' = 0$ and hence the frequency curves become tangent to the R-axis at the right-hand end.

When $f(R) = 0$, there are several different conditions which depend upon the value of c:

1.—If $c > 2$, $t'' = 0$ and the frequency curve is tangent to the R-axis at the left. (See Figs. 1 and 2.)

2.—If $1 < c < 2$, $t'' = \infty$ and the frequency curve is perpendicular to the R-axis at the left; but for all three equations the curve begins on the R-axis. (See Figs. 3 and 7.)

3.—If $c = 2$, the frequency curves will leave the R-axis at a definite angle.

4.—If $c = 1$, all the frequency curves will start at a definite height above the R-axis, equal to $n\,k$ and at an angle the tangent of which is $-n\,k^2$. The duration curve, therefore, will leave the R-axis at a definite angle. Furthermore, in this case all values of t'' will be negative and there will be no hump in the frequency curve.

5.—If $c < 1$, $t'' = \infty$ and the frequency curve begins at the left with the ordinate through a as tangent or asymptote, and as all values of t'' will be negative, there will be no hump in the frequency curve. The duration curve will begin perpendicular to the R-axis. (See Figs. 6, 8, and 10.)

6.—In Equation (F), if $d = 0$ or c, and c is negative, the frequency curve begins and ends on the R-axis. In this case the first and second derivatives are identical with those for Equation (9). (See Fig. 11.)

L. Standish Hall, Assoc. M. Am. Soc. C. E., has called attention to these types of frequency curves.* In Mr. Hall's paper, however, only the first type is considered, while the writer's equations and methods of plotting apply to all these types as well as to those in which the maximum point is to the right of the center of gravity of the area under the curve. (See Figs. 4 and 5.) All these types are illustrated in the preceding diagrams. The type of frequency curve is usually determined by the value of c which is the slope of the auxiliary line obtained by this method of plotting. (Fig. 9 is an exception to this rule since the mode is less than the mean while $c > 3.3$.)

Location of Mode and Mean.—When c is greater than unity, the value of R for the maximum point on the frequency curve, that is, the mode, is obtained by placing the expression within the brackets in the equations for t'' equal to zero. This results in the following:

$$\text{Mode}_S = \left(\frac{c-1}{c\,k}\right)^{\frac{1}{c}};\quad \text{Mode}_A = a + \left(\frac{c-1}{c\,k}\right)^{\frac{1}{c}} \qquad \ldots\ldots\ldots\ldots (1)$$

The mode for Equation (B') is not so easily obtained. It is that value of R which satisfies the following expression:

$$\frac{R-a}{b-R} = \left[\frac{c-1}{c\,k}\,\frac{2\,(R-a)}{c\,k\,(b-a)}\right]^{\frac{1}{c}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

* See Fig. 12 of his paper entitled "The Probable Variations in Yearly Run-off as Determined from a Study of California Streams," *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 240.

This expression can be solved only approximately and the following is the best the writer has been able to obtain. In Equation (2) substitute R' for $R - a$. The equation may then be written:

$$\frac{\frac{R'}{b-a}}{1-\frac{R'}{b-a}} = \left(\frac{c-1}{c\,k}\right)^{\frac{1}{c}} \left\{1 + \frac{2}{c\,(c-1)}\,\frac{R'}{(b-a)}\right\}^{\frac{1}{c}}$$

Expand the left-hand member by simple division, divide both sides of the equation by the first factor of the right-hand member and then expand the second factor by the binomial theorem. The result is:

$$\left(\frac{c\,k}{c-1}\right)^{\frac{1}{c}} \left[\frac{R'}{b-a} + \left(\frac{R'}{b-a}\right)^2 + \left(\frac{R'}{b-a}\right)^3 + \left(\frac{R'}{b-a}\right)^4 + \ldots\ldots\ldots\ldots\right]$$
$$= 1 + \frac{2}{c\,(c-1)}\left(\frac{R'}{b-a}\right) - \frac{2}{c^2\,(c-1)}\left(\frac{R'}{b-a}\right)^2 + \frac{4\,(2\,c-1)}{c^3\,(c-1)^2}\left(\frac{R'}{b-a}\right)^3$$
$$- \frac{2\,(2\,c-1)\,(3\,c-1)}{3\,c^4\,(c-1)^3}\left(\frac{R'}{b-a}\right)^4 + \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

By collecting the coefficients of corresponding powers of R' and discarding terms with the third and higher powers, a quadratic is obtained which can be solved for R'. The value of R as obtained from this will be approximately equal to the mode. The result will probably be 2 or 3% too small. By computing the coefficients for one or more of the higher powers of R' and solving by methods for incommensurate roots, the results could be made as accurate as desired.

The mean for a frequency curve has been defined as the abscissa of the center of gravity of the area under the curve. Hence:

$$\text{Mean} = \frac{\int R\,t'\,d\,R}{\int t'\,d\,R}$$

Although the writer has not been able to solve this integral for the case when both limits of the frequency curve are finite, it can be solved for Equations (S') and (A'), in which R varies from zero or a to ∞.

The method will be shown for Equation (S'). Substitute the value of t' from Equation (S') in the integral; then:

$$\text{Mean}_S = \frac{\int_0^\infty n\,c\,k\,R^c\,e^{-k\,R^c}\,d\,R}{\int_0^\infty n\,c\,k\,R^{c-1}\,e^{-k\,R^c}\,d\,R} = c\,k\int_0^\infty R^c\,e^{-k\,R^c}\,d\,R$$

This follows since the integral of the denominator is the total area under the frequency curve, which is n. Now, let $R^c = x$, then,

$$d\,R = \frac{1}{c}\,x^{\frac{1}{c}-1}\,d\,x$$

Hence:

$$\text{Mean}_S = k\int_0^\infty x^{\frac{1}{c}}\,e^{-k\,x}\,d\,x$$

This is a standard form of definite integral which can be evaluated in terms of the gamma function.*

$$\text{Mean}_S = k^{\frac{-1}{c}}\,\Gamma\left(\frac{1}{c}+1\right) \quad \ldots\ldots(4)$$

Similarly, the mean for Equation (A') is:

$$\text{Mean}_A = k^{\frac{-1}{c}}\,\Gamma\left(\frac{1}{c}+1\right)+a \quad \ldots\ldots(5)$$

Standard Deviation and Skewness.—If the square of the "standard variation" of a series is defined as equal to the square of the radius of gyration of the area under the frequency curve for the given series, about the center of gravity of that area, it can be determined for Equation (S') by integrating in the same manner as was done for the mean. In integral form, the square of the radius of gyration of the area under the curve about the t'-axis is:

$$r^2 = \frac{\int R^2\,t'\,d\,R}{\int t'\,d\,R} = c\,k\int_0^\infty R^{c-1}\,e^{-k\,R^c}\,d\,R$$

This becomes:

$$r^2 = k^{\frac{-2}{c}}\,\Gamma\left(\frac{2}{c}+1\right)$$

The square of the standard variation is found by subtracting the square of the mean, that is, the distance from the t'-axis to the center of gravity of the area under the curve. Hence:

$$(s\,v)^2 = k^{\frac{-2}{c}}\left[\Gamma\left(\frac{2}{c}+1\right)-\Gamma\left(\frac{1}{c}+1\right)^2\right] \quad \ldots\ldots(6)$$

This formula applies also to Equation (A') as the abscissa of the point of beginning of the curve is eliminated in the process.

The coefficient of variation is the standard variation divided by the mean, and, therefore, for Equations (S') and (A'), it is:

$$(c\,v)_S = \frac{\left[\Gamma\left(\frac{2}{c}+1\right)-\Gamma\left(\frac{1}{c}+1\right)^2\right]^{\frac{1}{2}}}{\Gamma\left(\frac{1}{c}+1\right)} \quad \ldots\ldots(7)$$

$$(c\,v)_A = \frac{\left[\Gamma\left(\frac{2}{c}+1\right)-\Gamma\left(\frac{1}{c}+1\right)^2\right]^{\frac{1}{2}}}{a\,k^{\frac{1}{c}}+\Gamma\left(\frac{1}{c}+1\right)} \quad \ldots\ldots(8)$$

With Equations (6), (7), and (8), it is now possible to compute the "skewness" for the same equations, by the relation:

$$\text{Skewness} = \frac{\text{Mean} - \text{mode}}{\text{Standard variation}} \quad \ldots\ldots(9)$$

While it is possible thus to solve the integral for r^2 for the "third moment" about the t'-axis, the writer has not been able to do so for the "gravity axis"

* An excellent table of log $\Gamma(p)$ is given by Mr. Foster, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), Table 1, pp. 152–153.

and hence the "coefficient of skew", as used by Allen Hazen, M. Am. Soc. C. E., Mr. Foster and others, has not been determined.*

Equations (6), (7), and (8) show that the "skewness" of curves fitted by Equation (*A*), depends only on the value of the exponent, *c*. It varies inversely as this exponent and is a maximum when the mode is equal to *a*, that is, when $c = 1$. If *c* is less than unity the mode is imaginary, but practically it remains the same as for $c = 1$. The skewness becomes zero when,

$$\Gamma\left(\frac{1}{c}+1\right) = \left(1-\frac{1}{c}\right)^{\frac{1}{c}}$$

from which, $c = 3.3121$.

As previously stated, the value of *c* is determined from the slope of the straight line drawn through the data when plotted as described for the auxiliary curve. Hence, for all data fitted by Equations (*S*) and (*A*) the slope of the line indicates at once whether the skewness of the frequency curve is positive or negative.

It does not seem to be possible to develop suitable formulas for the equation for curves with both limits finite, corresponding to the foregoing coefficients and functions used in connection with systems of skew frequency curves developed by statistical methods. However, the mode, mean, standard variation, coefficient of variation, and doubtless other functions of these curves, can all be found by graphical methods for any given case. For example, after having completed the straight line plotting of the data, the curve may be replotted with ordinary rectangular co-ordinates. From this curve, by plotting the first and second differential curves, the intersection of the second curve with the *R*-axis will of course give the mode.

The duration curve is the first integral of the frequency curve and, therefore, its ordinates give areas under the frequency curve. By plotting a second integral curve, moments are obtained from which the position of the center of gravity of the area under the frequency curve can be determined, giving the mean. In the same way, the radius of gyration can be obtained after plotting a third integral curve. Sufficient data, therefore, can be obtained by graphical means for calculating the same functions of the curve with finite limits, as were obtained mathematically for the other curves.

In addition to the difficulty of expressing the functions of a new system of skew frequency curves in terms of an entirely different system derived by radically different methods, and designed for different uses, it seems to the writer that the coefficients taken from these older systems have little, if any, physical significance when applied to ordinary hydraulic problems.

One or two functions of these curves will be suggested, which can be easily calculated with data obtained from the duration curve. These functions will give a ready means of comparing curves for different streams or localities

* While the term, "coefficient of skew," will occasionally be used in the following text, it should be understood to denote the "skewness" as measured in Equation (9), except where the context clearly indicates otherwise. It should be noted that Equation (9) gives a positive or negative value according to whether the mean is greater or less than the mode; it is, therefore, more indicative of the true form of the curve than is the "coefficient or skew" derived from the sum of the squares of the variations from the mean.

and will also call attention to conditions which should be given consideration in designing engineering works.

Factors for Classifying Frequency Curves.—When the foregoing equations are used for the investigations of skew frequency data especially as applied to hydraulic problems, the following functions or quantities may be used for the purpose of comparing different curves as well as for obtaining general information about an individual curve. The first ones proposed may be called the "average range" and the "average frequency" of the curves or data.

The average range is obtained by taking from the duration curve as plotted by any of the foregoing methods, the values of R (expressed as a percentage of the mean when desired for comparison with other curves) for 25% and for 75% of the time. The difference of these values may be called the "range of average run-off" (rainfall, floods, etc.), or briefly, the "average range" for the given stream or locality. This range is such that 50% of the records will lie within its limits. That the record for the next succeeding year will come within this range is just as probable as that it will fall outside. It is evident that the area under the frequency curve between the ordinates corresponding to these two points on the duration curve, is one-half the total area under the curve, and hence if the data followed the "normal law of error" this quantity would be equal to twice the "probable error of the series". In general, the average range varies directly as the coefficient of variation for the series. One-half the average range is given as one of the measures of dispersion by G. Udny Yule,* who designates it the "quartile deviation" or "semi-interquartile range". Attention is called to the fact that this "is not a measure of the deviation from any particular average". Yule states that "for distributions approaching the ideal forms [of normal or moderately skew curves] the semi-interquartile range is usually about two-thirds of the standard deviation", and, therefore, that nine times the semi-interquartile range, or four and one-half times the average range, will approximately cover 99% of the total frequency. He also states that this measure of dispersion has the clearest and simplest meaning of any of the three which he describes, and that it is calculated with the greatest ease.

The average frequency of the average range of the data, or briefly the average frequency, may be obtained by dividing one-half the total area under the frequency curve by the percentage covered by the average range. This gives the average height of the frequency curve between the limits of this range. If the range is small, the average frequency will be high, that is, greater than unity, and *vice versa*. This is illustrated in Table 1, where these factors are given for several streams, together with the coefficients of variation as published for the same stream.

In addition to the formula for the skewness of a frequency curve as used by Pearson (the mean minus the mode divided by the standard deviation) Yule* gives another measure of skewness, and one which can be easily com-

* "An Introduction to the Theory of Statistics," by G. Udny Yule, Fourth Edition, Chapter VIII, p. 147.

puted with quantities taken from the duration curve. This might be called the "quartile ratio" to distinguish it. It is obtained from values of the variable corresponding to 25%, 50%, and 75% of the time. If these quantities are, respectively, $Q_{0.25}$, $Q_{0.50}$, $Q_{0.75}$, then:

$$\text{Quartile ratio} = \frac{(Q_{0.75} - Q_{0.50}) - (Q_{0.50} - Q_{0.25})}{\frac{1}{2}(Q_{0.75} - Q_{0.25})} = \frac{Q_{0.25} + Q_{0.75} - 2\,Q_{0.50}}{\frac{1}{2}(Q_{0.75} - Q_{0.25})}$$

This ratio becomes zero for a symmetrical or nearly symmetrical distribution, "and while its highest possible value is 2, it would rarely in practice attain higher numerical values than 1." When this quantity is positive, the mode will be less than the mean, while for negative values the reverse will be true. In Table 1, values of this ratio are given for the streams in Column (5), and these may be compared with the coefficients of skew given in Column (6).

TABLE 1.—Comparison of Coefficients and Factors.

Name of stream. (1)	Average range, percentage. (2)	Average frequency. (3)	Coefficient of variation. (4)	Normal* ratio. (5)	Skewness. (6)
Richelieu River, New York	21	2.38	−0.16	0.87	−0.02
West Canada Creek, New York	35	1.43	0.21	1.78	0.60
Sacramento River, California	38	....	0.29	0.89	0.14
Rock Creek, California	44	1.15	0.21	1.16	....
Merrimac River, Massachusetts	49	1.02	0.24	2.56	−0.01
Kings River, California	59	0.85	0.41	1.30	0.41
Mackenzie River, Australia	62	0.81	−0.44	0.83	−0.22
Yuba River, Colorado	65	0.77	0.40	0.81	....
Kern River, California	68	0.74	0.56	5.55	1.23
Donner Creek, California	76	0 66	0.50	1.00	....
Sweetwater River, California	97	0.52	1.87	25.00	3.66

* Normal ratio $= \frac{\text{Abnormal range}}{\text{Subnormal range}}$, discussed later.

Preparation of Skew Frequency Paper

In order to use the foregoing equations for straight line plotting of data in the form of integral or duration curves, they must be put into logarithmic form. Equation (S) will be used as an illustration, but the transformations are equally applicable to the more general forms. By transposing to the left-hand member of the equation the terms containing n, and then dividing both sides by $-n$, it becomes;

$$1 - \frac{t}{n} = (10)^{-h\,R^c}$$

This can now be transformed into the logarithmic form:

$$\log_{10}\left(1 - \frac{t}{n}\right) = -h\,R^c, \text{ or } \operatorname{colog}_{10}\left(1 - \frac{t}{n}\right) = h\,R^c$$

Since n is always greater than t, $\frac{t}{n}$ will always be less than unity and $1 - \frac{t}{n}$ will also be less than unity. Hence, $-\log\left(1 - \frac{t}{n}\right)$, or $\operatorname{colog}\left(1 - \frac{t}{n}\right)$, will

be a positive quantity. Let this be represented by y. The equation now becomes;

$$y = h R^c \ldots\ldots\ldots\

When log y is plotted to log R, the curves for Equations (A_2) and (B_2) will have the same forms as when Equations (A_1) and (B_1) are plotted on logarithmic paper, or when Equations (A) and (B) are plotted on skew frequency paper.

To make a skew frequency paper on which corresponding values of the percentage of the time, t, and rainfall or run-off, R, or $f(R)$, will plot as linear functions, it is only necessary to take the ordinary logarithmic ruling in one direction for the R and $f(R)$-scales, and then compute a suitable series of values of log colog $\left(1 - \frac{t}{n}\right)$ to give the required spacing for the t-scale.

An example of such ruling is given in Fig. 12. All the other diagrams have been plotted on this kind of paper. The origin for the spacing of the t-scale is the 90% line. This becomes evident since $\left(1 - \frac{90}{100}\right) = 0.1$. The colog of 0.1 is unity and the log of unity is zero. Hence, for percentages greater than 90, the ordinates laid off to the right as the log colog will be positive, while for percentages less than 90, the values of log colog will be negative and the ordinates must be laid off to the left of that line.

Although the computation of suitable spacings is not at all difficult if a table of cologarithms is available, Table 2 may be convenient in laying out this skew frequency paper and its use will avoid uncertainties and errors. Table 2 is easily modified or extended as desired.

TABLE 2.—SPACING OF ORDINATES FOR SKEW FREQUENCY PAPER.

t = percentage of time.
x = distance from origin for skew frequency ruling.

t.	$+x$.	t.	$-x$.	t.	$-x$.	t.	$-x$.	t.	$-x$.
99.99	0.602	90	0.000	50	0.521	10	1.339	0.6	2.583
99.98	0.568	88	0.036	48	0.547	9	1.387	0.5	2.662
99.95	0.519	86	0.068	46	0.573	8	1.441	0.45	2.708
99.9	0.477	84	0.099	44	0.599	7	1.501	0.4	2.759
99.8	0.431	82	0.128	42	0.626	6	1.571	0.35	2.818
99.6	0.380	80	0.156	40	0.654	5	1.652	0.3	2.886
99.4	0.347	78	0.182	38	0.683	4.5	1.699	0.25	2.963
99.2	0.322	76	0.208	36	0.713	4	1.751	0.2	3.060
99	0.301	74	0.233	34	0.744	3.5	1.811	0.15	3.186
98	0.230	72	0.251	32	0.776	3	1.878	0.1	3.363
97	0.183	70	0.282	30	0.810	2.5	1.959	0.05	3.664
96	0.146	68	0.305	28	0.846	2.0	2.057	0.04	3.759
95	0.114	66	0.329	26	0.883	1.8	2.103	0.03	3.886
94	0.087	64	0.353	24	0.924	1.6	2.155	0.02	4.061
93	0.063	62	0.376	22	0.967	1.4	2.213	0.01	4.363
92	0.040	60	0.400	20	1.014	1.2	2.281	.....	.....
91	0.020	58	0.424	18	1.064	1.0	2.361	.....	.....
90	0.000	56	0.448	16	1.121	0.9	2.406	.....	.....
		54	0.472	14	1.184	0.8	2.457	.....	.....
		52	0.496	12	1.256	0.7	2.516	.....	.....

APPLICATION OF FORMULAS TO SPECIFIC DATA

Hydraulic Data of the Usual Types.—It has been possible to plot satisfactory frequency curves for nearly all the hydraulic data which have come

to the writer's notice by the use of Equations (A), (B), and (S). Equation (C) and forms involving other functions of R have not been investigated thus far.

Since it is possible to use simple graphical methods for nearly all the investigations necessary in the solution of engineering problems, they will be explained first, while the mathematical investigations of these equations will be taken up later.

Several diagrams and tables are given to illustrate the great variety of frequency curves included, and to enable one who is not familiar with such curves to become so by making use of the data given. The data have been selected to illustrate as many types of skew frequency distributions as possible, in most part from hydraulic records. Some data from other sources have been included to illustrate particular forms. A possible application of the equations to population curves will be indicated subsequently as this does not seem to be strictly a problem in frequencies.

The work of plotting diagrams is greatly simplified by the use of the skew frequency paper as it will accommodate curves having any coefficient of variation and any degree of skewness, either or both of which may be positive or negative.

Fig. 1 shows the curves for the frequency distribution of the annual run-off of the Sacramento River, at Red Bluff, Calif., as given in Table 3, Column (1). These and other data were taken from Mr. Hall's paper previously mentioned. They were first arranged in the order of increasing magnitude. The percentages in Column (2) opposite each item in Column (1), were computed by taking the series of odd numbers 1, 3, 5 . . . . . $(2n - 3)$, $(2n - 1)$, and dividing each by $2n$, n being the number of records in the series.* Corresponding quantities from Columns (1) and (2) were then plotted as shown in Fig. 1, and a straight line drawn so as to average the points as nearly as possible. It is seen that the points follow this line, and the line therefore may be taken as the integral or duration curve for the skew frequency distribution of the total annual discharge of this river.

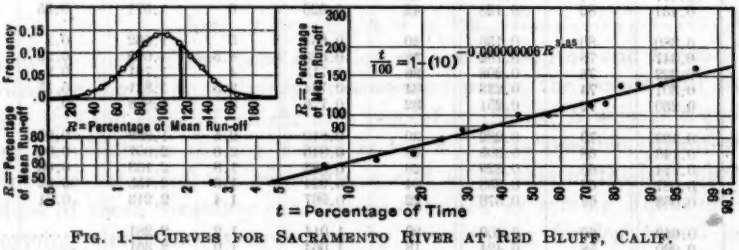

FIG. 1.—CURVES FOR SACRAMENTO RIVER AT RED BLUFF, CALIF.

Where such a short series of records lie so close to a straight line, it is not necessary to plot any other curves or make any but the simplest computations to arrive at conclusions which may be adopted as guides in engineering reports

* This method using the series of odd numbers is equivalent to finding the percentage at the center of each block of a histogram, whereas the simple finding of the ratio in percentage which each item bears to the whole would give the corresponding percentage of the larger valued side of each block. Furthermore, by the latter method the last value would correspond with 100% which cannot be used in any usual exponential formula.

and designs. However, to make clearer the characteristics of this curve, the data of the last three columns are given, from which the frequency or differential curve was drawn. The percentages of time in Column (4) were read from the curve for the quantities in Column (3), these being taken with the common difference, $\Delta R = 10$ per cent. The quantity, $\frac{\Delta t}{\Delta R}$, was then set down in Column (5), from which the frequency curve, shown in the corner of the Fig. 1, was drawn. The lower limit of this curve is zero while the upper limit is infinity. This is indicated by the fact that a straight line may be used to represent the data as originally plotted. Hence, Equation (*S*) can be used to express the law of frequency variation. From the duration curve it is found that for 25% of the time the run-off will probably be less than 80% of the mean, while for 75% of the time it will be less than about 118 per cent.

TABLE 3.—FREQUENCY DATA FOR SACRAMENTO RIVER, AT RED BLUFF, CALIFORNIA.*

Run-off as percentage of the mean = R.	Percentage of time, t.	R, read from the curve.	t, read from the curve.	$\frac{\Delta t}{\Delta R}$	Run-off as percentage of the mean = R.	Percentage of time, t.	R, read from the curve.	t, read from the curve.	$\frac{\Delta t}{\Delta R}$
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
50.3	2.6	....	....	....	103.3	55.3	120	77.0	0.93
58.8	7.9	30	0.6	0.13	103.3	60.5	130	86.3	0.67
64.6	13.1	40	1.9	0.26	107.4	65.8	140	93.0	0.37
69.3	18.4	50	4.5	0.45	112.5	71.1	150	96.7	0.21
78.3	23.7	60	9.0	0.65	113.4	76.3	160	98.8	0.08
86.4	28.9	70	15.5	0.95	135.9	81.6	170	99.6	....
89.5	34.2	80	25.0	1.20	136.8	86.9	....	....	....
90.4	39.5	90	37.0	1.40	138.9	92.1	....	....	....
99.0	44.7	100	51.0	1.37	161.2	97.4	....	....	....
100.6	50.0	110	64.7	1.23					

* Data from Table 1, *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), pp. 194–196.

Therefore, the run-off will be from 80 to 118% of the mean for 50% of the time. In other words, there is an even chance that the next succeeding record will fall within these limits.

As the maximum point of the frequency curve is so near the 100% ordinate, the mode is nearly equal to the mean and the skewness is small. The abscissa of the median as determined from the duration curve is the intersection of the curve with the "50%-of-time" ordinate, and it is seen to be about 99 per cent. That is, approximately one-half the area under the frequency curve lies on each side of the 99% ordinate.

The range of the average run-off, that is, the range of values which will probably occur about one-half the time, is indicated by the two heavy ordinates to the frequency curve. Their position is determined by finding the intersection of the duration curve, with the 25% and 75% time ordinates. The nearly symmetrical character and the small skewness of the curve shows that,

for any given range on either side of these limits, the probabilities of high and low annual discharge are about equal. These conclusions can also be drawn from the slope of the duration curve or from its equation, which is:

$$t = 100 - 100(10)^{-0.000000006\,R^{3.85}}$$

The exponent, $c = 3.85$, is determined, as will be shown, by the slope of the line with reference to the R-axis. When the slope of the line (or the value of c) is between the values of 2.5 and 4.5 approximately, the skewness will be comparatively small, while high and low records will have nearly equal frequencies. This can also be found easily from the duration curve without determining its equation. Thus, 5% of the time a value of 52% of the mean run-off will not be exceeded, while 95% of the time the run-off will probably be not greater than 145% of the mean. The quantity, $80 - 52 = 28$, is the variation in the value of the mean between 5 and 25% of the time, while the variation in the mean from 75 to 95% of the time is $145 - 118 = 27$. When the ratio of the ranges of these extremes is nearly unity, as in this case, high and low records are about equally probable.

All these conclusions as to the probable range of average conditions and the relative frequency of high and low discharges or run-off, together with the approximate values of each, which are to be expected in perhaps 50 to 100 years, are easily determined from Fig. 1 without the computation of any coefficients, factors, or criteria. Annual maximum flood heights and discharges often seem to follow this simple "linear" frequency law, making the method particularly applicable to the study of flood frequencies and heights.

TABLE 4.—Annual Rainfall at Tientsin, China.*

Year.	Rainfall, in inches, R.	Percentage of time, t.	$R-8$.	Year.	Rainfall, in inches, R.	Percentage of time, t.	$R-8$.
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1902	10.0	1.6	2.0	1915	19.5	51.6	11.5
1920	11.1	4.7	3.1	1918	19.9	54.7	11.9
1900	12.8	7.8	4.8	1919	20.0	57.8	12.0
1899	12.4	10.9	4.4	1892	21.4	60.9	13.4
1922	13.2	14.1	5.2	1916	21.6	64.1	13.6
1895	14.4	17.2	6.4	1893	22.4	67.2	14.4
1909	14.7	20.3	6.7	1901	23.2	70.3	15.2
1913	15.4	23.4	7.4	1896	23.6	73.4	15.6
1906	15.6	26.6	7.6	1904	23.6	76.6	15.6
1898	15.9	29.7	7.9	1897	24.4	79.7	16.4
1903	16.0	32.8	8.0	1911	25.7	82.8	17.7
1907	16.2	35.9	8.2	1908	26.3	85.9	18.3
1917	16.3	39.1	8.3	1914	26.6	89.1	18.6
1910	17.8	42.2	9.8	1891	28.3	92.2	20.3
1921	19.2	45.3	11.2	1894	30.5	95.3	22.5
1905	19.5	48.4	11.5	1912	31.3	98.4	23.3

* Data from records of the British Municipal Council, Tientsin, China.

Table 4 gives the data used in plotting the curves for annual precipitation at Tientsin, China, shown in Fig. 2; and Table 5 the annual run-off, in second-feet per square mile, for the Allegheny River at Red House, N. Y. (Fig. 3). Both these curves are expressed by Equation (A) in which the lower limit is greater than zero. The original data are first plotted as was

done for Fig. 1, but in Figs. 2 and 3 the data shown by the solid dots are such that a smooth curve drawn as an average of the points will be concave upward. In order to extend such a curve beyond the limits of the actual observations the data must be treated so that when replotted they will approximate a straight line. This auxiliary line may then be extended and from it a smooth curve can be plotted graphically through the points as first platted. The necessary treatment is easily and rapidly effected by subtracting a constant quantity from each record of rainfall or run-off as the case may be.

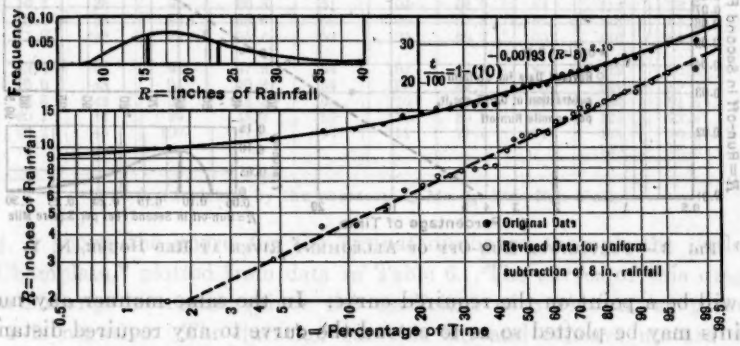

FIG. 2.—CURVES FOR ANNUAL RAINFALL AT TIENTSIN, CHINA.

The constants selected after two or three trials were 8 in. to be subtracted from the precipitation records (Fig. 2) and 0.075 sec-ft. per sq. mile for the run-off records (Fig. 3). The results are shown in Column (4) of Table 4 and Column (3) of Table 5. These quantities were then plotted against their corresponding percentages of time as indicated by the open circles in Figs. 2 and 3. If the constant selected for the first trial is too small, the points will lie along a curve which is flatter than the original one, that is, between the dotted straight lines and the curves. If too large a quantity is used, the auxiliary curve will be reversed, so as to be concave downward and to lie below the position of the dotted straight line.

TABLE 5.—RUN-OFF OF ALLEGHENY RIVER AT RED HOUSE, NEW YORK.*

Run-off in second-feet per square mile, R. (1)	Percentage of time, t. (2)	$R - 0.075$. (3)	Run-off in second-feet per square mile, R. (1)	Percentage of time, t. (2)	$R - 0.075$ (3)
0.085	3.0	0.01	0.135	55.9	0.06
0.085	8.8	0.01	0.155	61.8	0.08
0.105	14.7	0.03	0.16	67.7	0.085
0.11	20.6	0.035	0.165	73.5	0.09
0.115	26.5	0.04	0.185	79.4	0.11
0.12	32.3	0.045	0.195	85.3	0.12
0.12	38.2	0.045	0.24	91.2	0.165
0.125	44.1	0.05	0.245	97.0	0.17
0.13	50.0	0.055			

* Data from Fig. 10, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 170.

Smooth curves can now be plotted through the original points by means of the auxiliary dotted lines. For example (Fig. 2), this curve crosses the line

for a value of $R - 8 = 8$, on the ordinate for $t = 30$ per cent. On the same ordinate locate a point eight units above, that is, on the line for $R = 16$.

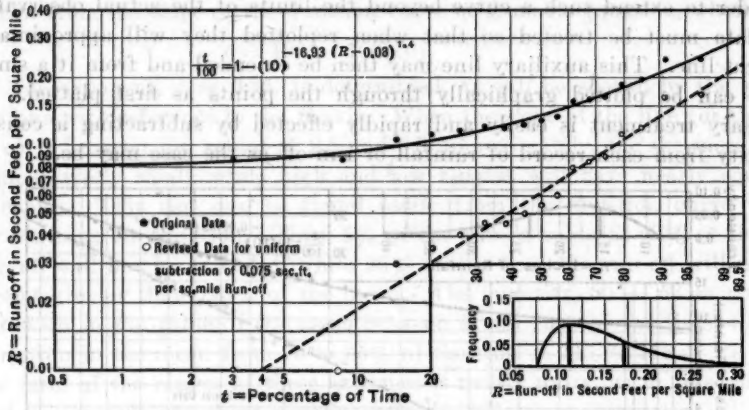
$\frac{t}{100} = 1 - (10)^{-16.93\,(R-0.08)^{1.4}}$
R=Run-off in Second Feet per Square Mile
t=Percentage of Time
● Original Data
○ Revised Data for uniform subtraction of 0.075 sec.ft. per sq.mile Run-off
Frequency
R=Run-off in Second Feet per Square Mile

FIG. 3.—CURVES FOR RUN-OFF OF ALLEGHENY RIVER AT RED HOUSE, N. Y.

This will be a point on the required curve. In the same manner any number of points may be plotted so as to extend the curve to any required distance in either direction. All the curves of Figs. 2 to 6, inclusive, were plotted in this manner.

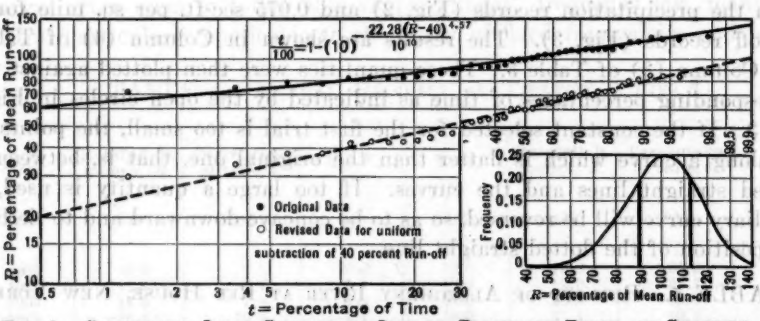
$\frac{t}{100} = 1 - (10)^{-\frac{22.28\,(R-40)^{4.57}}{10^{10}}}$
R=Percentage of Mean Run-off
t=Percentage of Time
● Original Data
○ Revised Data for uniform subtraction of 40 percent Run-off
Frequency
R=Percentage of Mean Run-off

FIG. 4.—CURVES FOR LAKE CHAMPLAIN OUTLET, RICHELIEU RIVER, AT CHAMPLY, QUE., CANADA.

The duration curves in Figs. 2 and 3 are similar, the only apparent difference being in the degree of curvature, Fig. 2 being the flatter. However, there is a marked difference in the left-hand part of the frequency curves. In Fig. 2 this curve begins tangent to the R-axis, while in Fig. 3 it is perpendicular to this axis. This difference is also evident from the equations of the curve. In the first case the value of the exponent, c, is greater than 2, while in the second case it is between 1 and 2. The change takes place at the value, 2, for which value the curve would leave the axis at some definite angle. These characteristics will be demonstrated in the mathematical analysis of the equations. It should be noted that the equations for the curves of Figs. 1, 4, and 5, have values greater than 3.3 for the exponent, c, and, therefore, that in each case the mode is greater than the mean, giving negative skewness.

TABLE 6.—RICHELIEU RIVER OUTLET OF LAKE CHAMPLAIN, AT CHAMPLY, QUEBEC, CANADA.*

Percentage of mean run-off, R.	Percentage of time, t.	R — 40.	Percentage of mean run-off, R.	Percentage of time, t.	R — 40.	Percentage of mean run-off, R.	Percentage of time, t.	R — 40.	Percentage of mean run-off, R.	Percentage of time, t.	R — 40.
70	1.2	30	87	25.6	47	100	50.0	60	111	74.4	71
74	3.6	34	87	28.1	47	101	52.5	61	112	76.8	72
78	6.1	38	91	30.5	51	104	54.9	64	112	79.3	72
78	8.5	38	93	33.0	53	105	57.3	65	113	81.7	73
82	11.0	42	93	35.4	53	105	59.7	65	116	84.1	76
83	13.4	43	94	37.9	54	107	62.1	67	119	86.6	79
83	15.9	43	94	40.3	54	109	64.6	69	121	89.0	81
83	18.3	43	95	42.7	55	110	67.0	70	124	91.5	84
84	20.7	44	99	45.2	59	110	69.5	70	124	93.9	84
85	23.2	45	100	47.5	60	111	71.9	71	124	96.4	84
									129	98.8	89

* Data furnished by the courtesy of Mr. L. Standish Hall.

Fig. 4 shows the curve for the annual discharge through the outlet of Lake Champlain,* plotted from data in Table 6. The curves of this diagram are particularly interesting since the coefficient of variation, as given in Table 1 is negative. The duration curve is flat and concave upward while the frequency curve is high, with the mode greater than the mean and the average range unusually small, only about 22% as against 60% in Fig. 5.

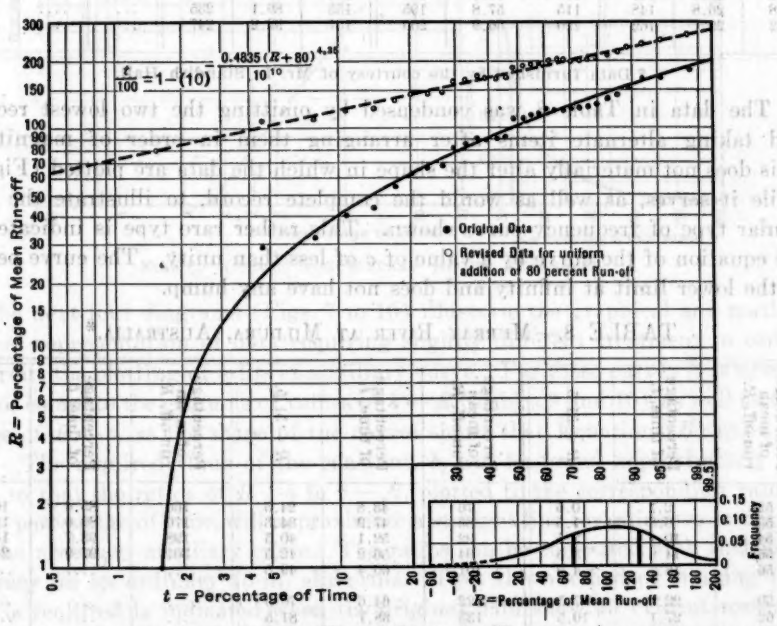
$\frac{t}{100} = 1 - (10)^{-\frac{0.4835\,(R+80)^{4.35}}{10^{10}}}$
R = Percentage of Mean Run-off
t = Percentage of Time
Original Data
Revised Data for uniform addition of 80 percent Run-off
Frequency
R = Percentage of Mean Run-off

FIG. 5.—CURVES FOR MACKENZIE RIVER, AT WARTOOK, AUSTRALIA.

* The writer is indebted to Mr. L. Standish Hall, who kindly furnished the data for the Lake Champlain Outlet, the several Australian rivers, and the Sweetwater River, California, all of which have added greatly to the value of this paper by the variety of conditions illustrated.

Fig. 5 shows a decided difference in the duration curve (plotted from Table 7), which is concave downward. The lower limit is apparently negative and a constant must be added to the values of R to bring the data into suitable form for plotting the auxiliary curve. The frequency curve for this river is comparatively flat with the mode greater than the mean, as in Fig. 4, but with the beginning of the curve at — 80%, shown by the dotted part at the left. These curves should not be interpreted as indicating a possible negative run-off, but approximately zero run-off probably 1½% of the time. Actually a minimum of 1% of the mean was recorded in 32 years' observations. The duration curve was made to pass through this point and averages the remainder of the data remarkably well.

TABLE 7.—MACKENZIE RIVER AT WARTOOK, AUSTRALIA.*

Percentage of mean run-off, R.	Percentage of time, t.	$R + 80$.	Percentage of mean run-off, R.	Percentage of time, t.	$R + 80$.	Percentage of mean run-off, R.	Percentage of time t.	$R + 80$.	Percentage of mean run-off, R.	Percentage of time, t.	$R + 80$.
1	1.6	81	85	32.8	165	121	63.9	201	171	95.3	251
29	4.7	109	90	36.0	170	121	67.2	201	185	98.4	265
32	7.8	112	90	39.1	170	122	70.3	202	...	....	...
41	10.9	121	90	42.2	170	124	73.2	204	...	....	...
44	14.1	124	91	45.3	171	126	76.6	206	...	....	...
55	17.2	135	111	48.5	191	128	79.7	208	...	....	...
63	20.3	143	111	51.6	191	138	82.8	218	...	....	...
67	23.4	147	115	54.7	195	142	85.9	222	...	....	...
68	26.8	148	115	57.8	195	155	89.1	235	...	....	...
82	29.7	162	120	60.9	200	167	92.2	247	...	....	...

* Data furnished by the courtesy of Mr. L. Standish Hall.

The data in Table 8 was condensed by omitting the two lowest records and taking alternate items after arranging them in order of magnitude. This does not materially alter the shape in which the data are plotted (Fig. 6) while it serves, as well as would the complete record, to illustrate the particular type of frequency curve shown. This rather rare type is indicated in the equation of the curve by a value of c of less than unity. The curve begins at the lower limit at infinity and does not have any hump.

TABLE 8.—MURRAY RIVER AT MILDURA, AUSTRALIA.*

Percentage of mean run-off, R.	Percentage of time, t.	$R - 51.5$.	Percentage of mean run-off, R.	Percentage of time, t.	$R - 51.5$.	Percentage of mean run-off, R.	Percentage of time, t.	$R - 51.5$.
52	2.1	0.5	76	43.8	24.5	160	85.4	108.5
53	6.3	1.5	86	47.9	34.5	170	89.6	118.5
54	10.4	2.5	92	52.1	40.5	198	93.7	146.5
55	14.6	3.5	94	56.2	42.5	301	97.9	249.5
56	18.7	4.5	101	60.4	49.5	....	....	.....
59	22.9	7.5	122	64.6	70.5	....	....	.....
62	27.1	10.5	133	68.7	81.5	....	....	.....
63	31.3	11.5	135	72.9	83.5	....	....	.....
68	35.4	16.5	136	77.1	84.5	....	....	.....
72	39.6	20.5	148	81.3	96.5	....	....	.....

* Data condensed from Table 16, *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 246.

Thus far, all the variations in the duration and frequency curves shown can be expressed mathematically by suitable variations in the values and signs of the constants of Equation (*A*). (Equation (*S*) is only a special case of Equation (*A*), in which the constant, *a*, becomes zero.) The limits of these curves are zero and ∞. Of the six curves shown, Figs. 2, 3, and 6 belong to Type III of Pearson's skew frequency curves, while Figs. 1, 4, and 5 belong either to his Type V or VI. (The distinction between these last two types is not clear.)

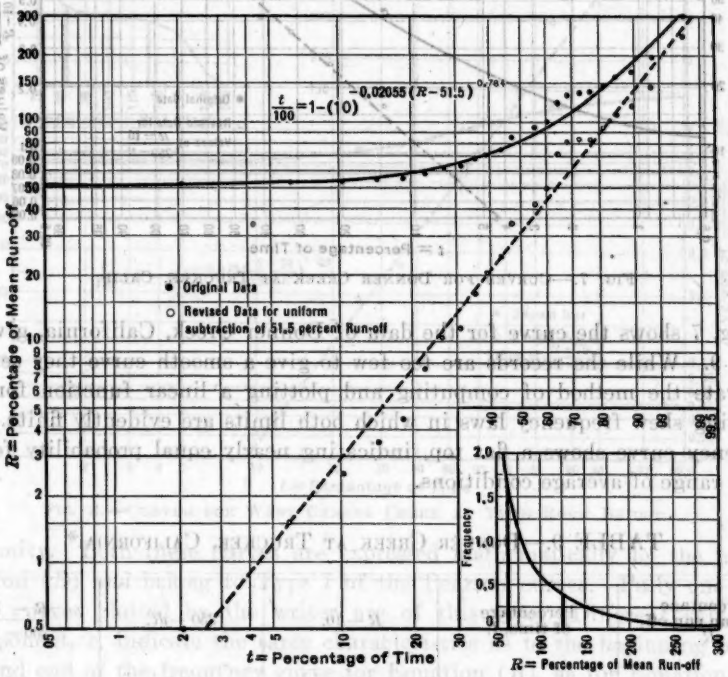
$\frac{t}{100}=1-(10)^{-0.02055(R-51.5)^{0.764}}$
Original Data
Revised Data for uniform subtraction of 51.5 percent Run-off
R = Percentage of Mean Run-off
t = Percentage of Time
Frequency
R = Percentage of Mean Run-off

FIG. 6.—CURVES FOR MURRAY RIVER, AUSTRALIA.

The next four diagrams (Figs. 7 to 10) illustrate the graphical and mathematical representation of data requiring slightly different treatment in order to permit the plotting of a linear auxiliary curve. For these curves it is necessary to compute the columns of values of $b-R$ (Tables 9 and 10), as well as the values of $R-a$, as the shape of the curves shows that Equation (*B*) must be used. The required value of the constant, *b*, can be found approximately by trial, so that the ratios of $R-a$ to $b-R$, plotted to the corresponding values of the percentage of time, will approximate a straight line for guidance in drawing the necessary auxiliary curve. The ratios can be computed with sufficient accuracy on an ordinary 10-in. slide-rule. That this method of treating the data is required is indicated when the original data show an evident reversal in the direction of curvature when plotted on skew frequency paper. The constant, *b*, serves to straighten the right-hand end of the data in the same way that the constant, *a*, was used in the first illustrations to straighten the

left-hand part. A little practice will make it possible to select practical values of these constants, in most cases after two or three preliminary trials.

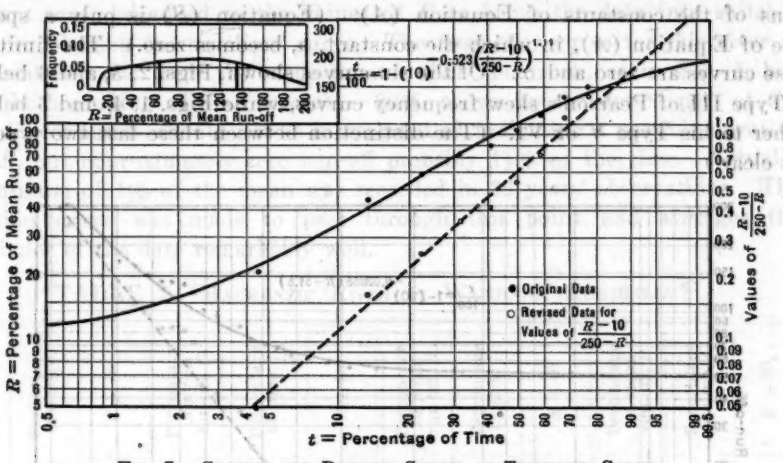
Frequency
R = Percentage of Mean Run-off
$\frac{t}{100} = 1 - (10)^{0.523\left(\frac{R-10}{250-R}\right)^{1.11}}$
R = Percentage of Mean Run-off
t = Percentage of Time
Values of $\frac{R-10}{250-R}$
Original Data
Revised Data for Values of $\frac{R-10}{250-R}$

FIG. 7.—CURVES FOR DONNER CREEK AT TRUCKEE, CALIF.

Fig. 7 shows the curve for the data of Donner Creek, California, given in Table 9. While the records are too few to give a smooth curve they serve to illustrate the method of computing and plotting a linear function for data following skew frequency laws in which both limits are evidently finite. The frequency curve shows a flat top, indicating nearly equal probability for the entire range of average conditions.

TABLE 9.—DONNER CREEK AT TRUCKEE, CALIFORNIA.*

Percentage of mean run-off, R. (1)	Percentage of time, t. (2)	$R - 10$. (3)	$250 - R$. (4)	$\frac{R-10}{250-R}$ (5)
21.5	4.5	11.5	228.5	0.050
44.4	13.6	34.4	205.6	0.167
58.8	22.7	48.8	191.2	0.256
72.9	31.8	62.9	177.1	0.355
79.3	40.9	69.3	170.7	0.406
94.2	50.0	84.3	155.8	0.54
110.3	59.1	100.3	139.7	0.72
134.4	68.2	124.4	115.6	1.08
149.2	77.3	139.2	100.8	1.38
156.3	86.4	146.3	93.7	1.56
178.0	95.5	168.0	72.0	2.33

* Data from Table 1, *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 196.

The curve for West Canada Creek (Fig. 8), from the data of Table 10, seems to be a common type for hydraulic data. The total range between limits is quite small on account of the relatively high value of the minimum. This frequency curve also has a nearly flat top across the average range, but begins at infinity at the left. This merely means that it is the best mathematical

expression that will fit the case and not that there is an infinite probability of the occurrence of the minimum limiting run-off. If the selection of the limits has been properly made it is highly improbable that actually either one would ever be reached. It should be noted that the value of the exponent, c, in the equation for this curve is less than unity while for Donner Creek it is greater

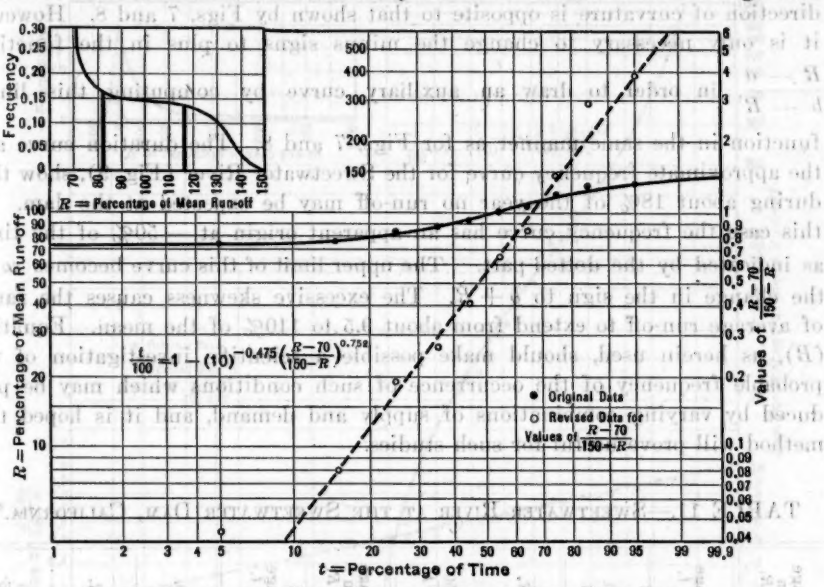
Frequency
R = Percentage of Mean Run-off
R = Percentage of Mean Run-off
t = Percentage of Time
Values of R − 70 / 150 − R
$\frac{t}{100} = 1 - (10)^{-0.475\left(\frac{R-70}{150-R}\right)^{0.75c}}$
Original Data
Revised Data for Values of $\frac{R-70}{150-R}$

FIG. 8.—CURVES FOR WEST CANADA CREEK AT TWIN ROCK BRIDGE.

than unity. Both these curves are expressed mathematically by the use of Equation (B) and belong to Type I of the Pearson curves. Fully one-third of the curves plotted by the writer are of this type. Different values of the exponent, c, indicate the same characteristics as to the beginning of the left-hand end of the frequency curve for Equation (B), as for Equation (A). However, the value of the exponent for zero skewness could not be determined for Equation (B).

TABLE 10.—WEST CANADA CREEK AT TWIN ROCK BRIDGE, NEW YORK.*

Percentage of mean run-off, R. (1)	Percentage of time, t. (2)	$R - 70$. (3)	$150 - R$. (4)	$\frac{R-70}{150-R}$. (5)
73.4	5	3.4	77.6	0.044
75.9	15	5.9	74.1	0.080
83.0	25	13.0	67.0	0.194
86.9	35	16.0	63.1	0.268
93.4	45	23.4	56.6	0.413
101.4	55	31.8	48.6	0.646
106.3	65	36.3	43.7	0.830
117.5	75	47.5	32.5	1.461
129.6	85	59.6	21.4	2.922
132.6	95	62.6	17.4	3.820

* Data from Table 5. *Transactions*, Am Soc. C. E., Vol. LXXXVII (1924), p. 171.

The data for the Sweetwater River at the Sweetwater Dam, California, given in Table 11, presents a most interesting and unusual case. Of the thirty-two records given, five show zero run-off for the year. On the other hand the maximum is nearly ten times the mean. When these data are plotted on skew frequency paper (Fig. 9) they form a reverse curve, but the direction of curvature is opposite to that shown by Figs. 7 and 8. However, it is only necessary to change the minus signs to plus in the function, $\frac{R - a}{b - R}$, in order to draw an auxiliary curve by computing this linear function in the same manner as for Figs. 7 and 8. The duration curve and the approximate frequency curve for the Sweetwater River (Fig. 9), show that during about 18% of the year no run-off may be expected at the dam. In this case the frequency curve has an apparent origin at -50% of the time, as indicated by the dotted part. The upper limit of this curve becomes ∞ by the change in the sign to $b + R$. The excessive skewness causes the range of average run-off to extend from about 9.5 to 110% of the mean. Equation (*B*), as herein used, should make possible a scientific investigation of the probable frequency of the occurrence of such conditions which may be produced by varying combinations of supply and demand, and it is hoped this method will prove useful for such studies.

TABLE 11.—SWEETWATER RIVER AT THE SWEETWATER DAM, CALIFORNIA.*

Percentage of mean run-off, R.	Percentage of time, t.	$R + 50$.	$100 + R$.	$\frac{R + 50}{100 + R}$.	Percentage of mean run-off, R.	Percentage of time, t.	$R + 50$.	$100 + R$.	$\frac{R + 50}{100 + R}$.
0	1.6	50	100	0.500	38	48.4	88	138	0.638
0	4.7	50	100	0.500	42	51.6	92	142	0.648
0	7.8	50	100	0.500	43	54.7	93	143	0.650
0	10.9	50	100	0.500	59	57.8	109	159	0.685
0	14.1	50	100	0.500	63	60.9	113	163	0.693
2	17.2	52	102	0.510	85	64.1	135	185	0.730
5	20.3	55	105	0.523	94	67.2	144	194	0.741
6	23.4	56	106	0.528	99	70.3	149	199	0.748
8	26.6	58	108	0.537	100	73.4	150	200	0.750
8	29.7	58	108	0.537	126	76.6	176	226	0.778
19	32.8	69	119	0.580	123	79.7	183	233	0.785
22	35.9	72	122	0.590	156	82.8	206	256	0.804
25	39.1	75	125	0.600	167	85.9	217	267	0.812
26	42.2	76	126	0.603	185	89.1	235	285	0.825
31	45.3	81	131	0.618	216	92.2	266	316	0.841
					452	95.3	502	552	0.910
					990	98.4	1 040	1 990	0.954

* These data were supplied by the courtesy of Mr. L. Standish Hall.

Special Cases in Hydraulics and Economics.—If the lower limit for the data in Fig. 9 had been positive instead of negative, the curve would have been concave upward at the left with the same strong upward tendency at the right. No hydraulic data have been found to illustrate this condition, but Fig. 10, plotted from data on the valuation of house property in England

and Wales, by Professor Pearson (Table 12) is a good example. Both the integral and differential curves (Fig. 10) have the same general appearance as in Fig. 6, but with much sharper curvature. The data cannot be closely approximated by Equation (*A*).

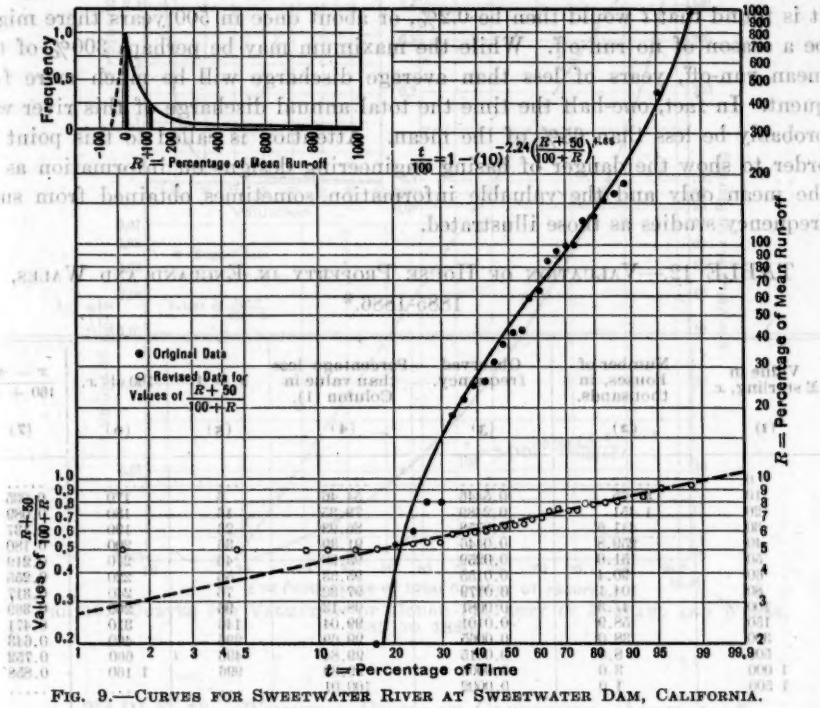

FIG. 9.—CURVES FOR SWEETWATER RIVER AT SWEETWATER DAM, CALIFORNIA.

The last diagram of this series (Fig. 11) illustrates another unusual case. Repeated trials, using numerous different combinations of limits, failed to produce even an approximately satisfactory curve by the process shown in the preceding diagrams. Finally, following a hint in the suggestive papers on skew frequency curves by Dr. Kapteyn (to be referred to later), it was found that by plotting the data (Table 13) from right to left instead of from left to right as in every other instance, a theoretical curve could be drawn which would follow closely four-fifths of the plotted points. In Fig. 11 the values of t are given at the top and increase from right to left while at the bottom those which increase from left to right represent $100\% - t$. It is evident that the two series are symmetrical about the ordinate for 50 per cent. This change amounts to substituting $n - t$ for the left-hand member of the fundamental equations, and the constant term, n, is then eliminated. Fig. 11 shows that the slope of the auxiliary curve is negative and hence the sign of the exponent, c, is negative. While for about one-fifth the data at the upper end the curve

does not follow the plotted points closely, it does indicate the same general tendency as to direction.* Evidently there is some cause or set of causes producing this deviation in the records, which cannot be expressed by such a simple law as that which seems to fit the remainder of the data. The lower limit of the frequency curve is apparently at — 10 per cent. Hence zero run-off may be expected. By substituting zero for R, $f(R)$ reduces to 0.025, and it is found that t would then be 0.2%, or about once in 500 years there might be a season of no run-off. While the maximum may be perhaps 300% of the mean run-off, years of less than average discharge will be much more frequent. In fact, one-half the time the total annual discharge of this river will probably be less than 65% of the mean. Attention is called to this point in order to show the danger of basing engineering designs on information as to the mean only and the valuable information sometimes obtained from such frequency studies as those illustrated.

TABLE 12.—VALUATION OF HOUSE PROPERTY IN ENGLAND AND WALES, 1885-1886.*

Value in £ sterling, x. (1)	Number of houses, in thousands. (2)	Observed frequency. (3)	Percentage less than value in Column (1). (4)	$x - 4$ (5)	$160 + x$. (6)	$\frac{x-4}{160+x}$. (7)
0	.....	.....	.....	...	.....	.....
10	3 175	0.5446	54.46	6	170	0.035
20	1 451	0.2489	79.35	16	180	0.089
30	441.6	0.0758	86.93	26	190	0.137
40	259.8	0.0446	91.39	36	200	0.180
50	151.0	0.0259	93.98	46	210	0.219
60	90.4	0.0155	95.53	56	220	0.255
80	104.1	0.0179	97.32	76	240	0.317
100	47.3	0·0081	98.13	96	260	0.369
150	58.9	0.0101	99.04	146	310	0.471
300	38.0	0.0065	99.69	296	460	0.643
500	8.8	0.0015	99.84	496	660	0.752
1 000	3.0	0.0005	99.89	996	1 160	0.858
1 500	1.0	0.0002	100.01	...	.....	.....

* Original data from *Philosophical Transactions*, Vol. 186, p. 396; reprinted in Kapteyn's "Skew Frequency Curves" (First Paper), p. 35.

The equations of the duration curves for all the data illustrated are given on the diagrams. Equations for the frequency curves can be obtained by the ordinary process of differentiation. The general forms for these second equations have already been given, being such that the constants can be readily computed from those of the duration curve and the equations can also be easily formed without using the actual process of differentiation.

The form of skew frequency blank diagram given in Fig. 12 is identical with those used in all the foregoing examples. An engineer wishing to try out this method or experiment with his own data will find this blank convenient for his use. A trial will demonstrate the simplicity of the principles involved and, it is hoped, the discussions here given will enable any one successfully to draw skew frequency curves fitting his own case.

* It was afterward found that much closer agreement with the data at the upper end of the curve of Fig. 11 could be obtained by using the most general form of Equation (B), involving five constants. (See dotted curve and its equation, Fig. 11.)

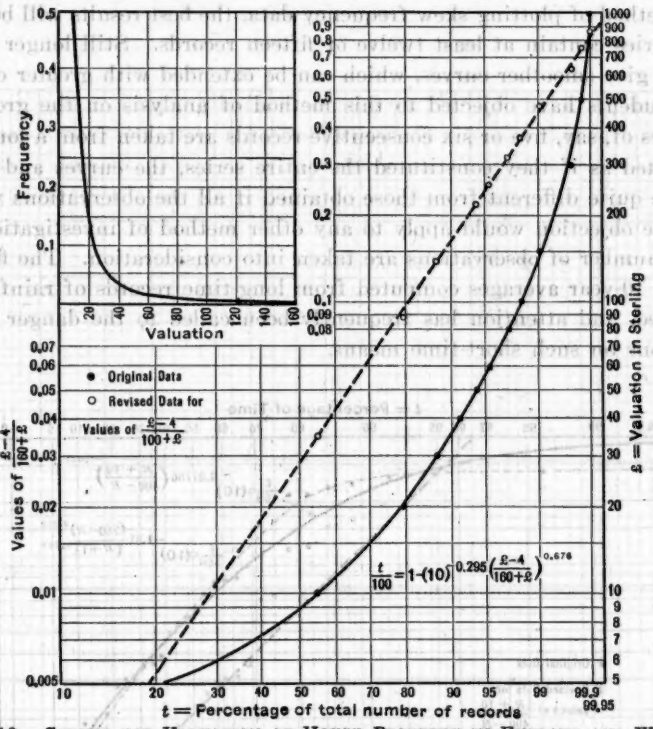

FIG. 10.—CURVES FOR VALUATION OF HOUSE PROPERTY IN ENGLAND AND WALES, 1885 TO 1886.

TABLE 13.—WIMMERA RIVER AT GLENORCHY, AUSTRALIA.*

Percentage of mean run-off, R.	Percentage of time, t.	$100-t$.	$R+10$.	$400-R$.	$\frac{R+10}{400-R}$	Percentage of mean run-off, R.	Percentage of time, t.	$100-t$.	$R+10$.	$400-R$.	$\frac{R+10}{400-R}$
5	1.7	98.3	15	395	0.038	65	51.7	48.3	75	335	0.224
10	5.0	95.0	20	390	0.051	75	55.0	45.0	85	325	0.262
16	8.3	91.7	26	384	0.068	85	58.3	41.7	95	315	0.302
19	11.7	88.3	29	381	0.076	86	61.7	38.3	96	314	0.306
20	15.0	85.0	30	380	0.079	88	65.0	35.0	98	312	0.314
21	18.3	81.7	31	379	0.082	99	68.3	31.7	109	301	0.362
28	21.7	78.3	38	372	0.102	159	71.7	28.3	169	241	0.702
31	25.0	75.0	41	369	0.111	164	75.0	25.0	174	236	0.737
31	28.3	71.7	41	369	0.111	215	78.3	21.7	225	185	1.216
36	31.7	68.3	46	364	0.126	216	81.7	18.3	226	184	1.228
36	35.0	65.0	46	364	0.126	238	85.0	15.0	248	162	1.531
39	38.3	61.7	49	361	0.136	245	88.3	11.7	255	155	1.645
49	41.7	58.3	59	351	0.168	262	91.7	8.3	272	138	1.970
54	45.0	55.0	64	346	0.185	274	95.0	5.0	284	126	2.255
59	48.3	51.7	69	341	0.202	278	98.3	1.7	288	122	2.360

* These data were furnished by the courtesy of Mr. L. Standish Hall.

Extension of Short-Term Record Series.—In the practical use of the foregoing method of plotting skew frequency data, the best results will be obtained if the series contain at least twelve or fifteen records. Still longer series are likely to give smoother curves, which can be extended with greater confidence. Some students have objected to this method of analysis on the ground that, if a series of, say, five or six consecutive records are taken from a longer series and treated as if they constituted the entire series, the curves and equations might be quite different from those obtained if all the observations were used. The same objection would apply to any other method of investigation if only a small number of observations are taken into consideration. The fluctuation in 5 and 10-year averages computed from long-time records of rainfall is well recognized, and attention has frequently been called to the danger of basing conclusions on such short-time means.

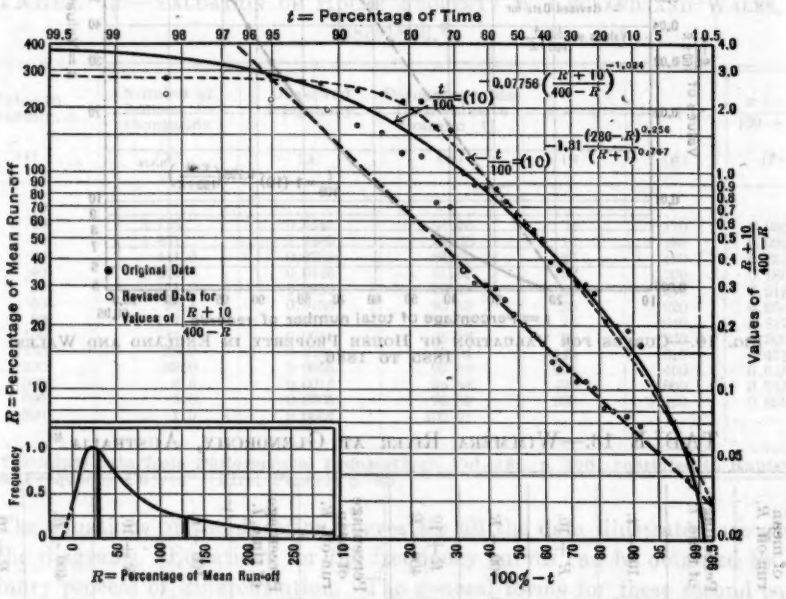

FIG. 11.—CURVES FOR WIMMERA RIVER AT GLENORCHY, AUSTRALIA.

It is possible to avoid this difficulty to some extent even when there are only from five to ten records available if there is a much longer record at another point on the same or a neighboring stream, provided the difference in conditions is not too great. A method similar to that indicated* by Joel B. Cox, Assoc. M. Am. Soc. C. E., has also been used by other engineers in dealing with short-time records. Its application to the foregoing method of treating hydraulic data is given here in order to make this paper a complete reference work for those who may not be familiar with its use.

* "Periodic Fluctuations in Rainfall in Hawaii," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 461.

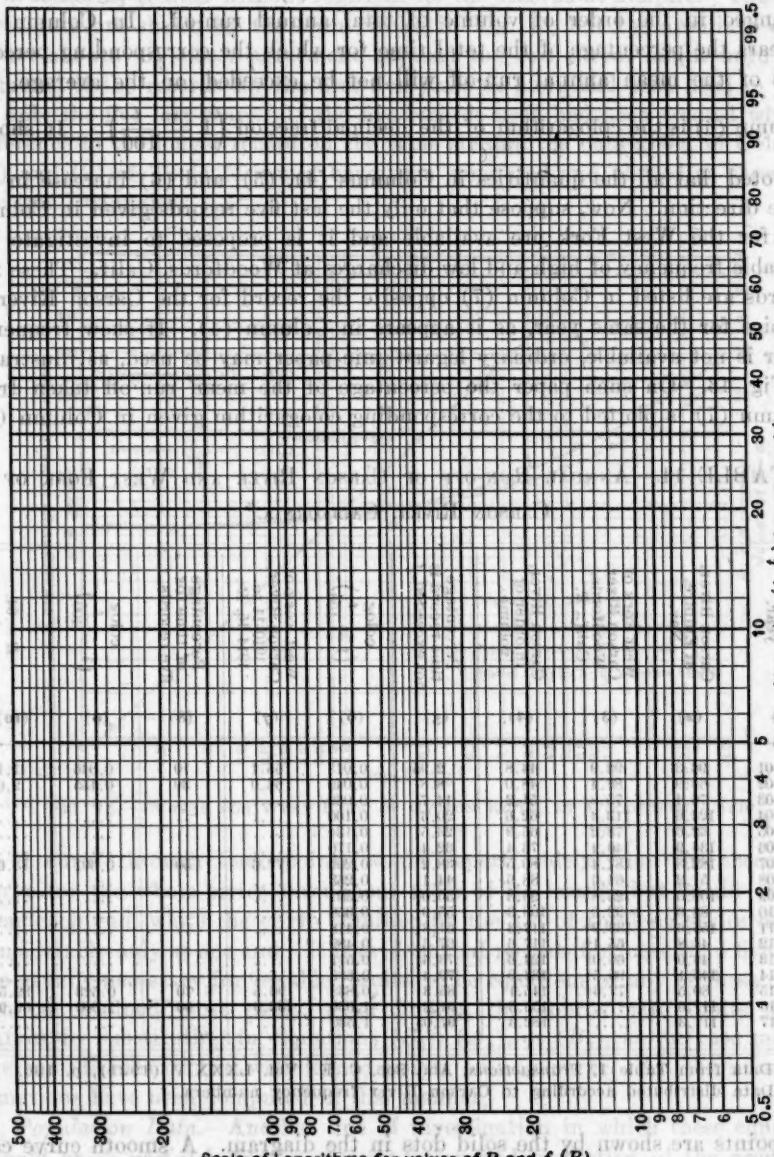

FIG. 12.—SPECIALLY RULED CHART FOR PLOTTING SKEW FREQUENCY DATA.

In Table 14 are given run-off data for the Carson River, at Empire, Nev., and for the West Fork of the Carson River at Woodfords, Calif. Columns (1), (2), and (3) are taken directly from Mr. Hall's paper previously mentioned. In Column (4) the data of Column (2) for the Carson River is rearranged in the order of volume of total annual run-off. In Column (5) appears the percentage of the total time for which the corresponding percentages of the mean annual run-off will not be exceeded, on the average. In Column (6) is the cologarithm of the decimal fraction $\left(1 - \frac{t}{100}\right)$. It should be noted that all the quantities in Columns (4), (5), and (6) increase in the same direction. Now, suppose that only the last five records given in Column (3) for the West Fork are available and it is proposed to investigate the probable frequency of high and low discharges at Woodfords, Calif. These five records are listed in Column (7) opposite the record for the Carson River at Empire for the same year, as it appears in Column (4). If skew frequency paper is not available, ordinary logarithmic paper may be used, as illustrated in Fig. 13. On such paper the percentage of the mean run-off taken from Column (7) is plotted to the corresponding cologarithm given in Column (6).

TABLE 14.—Annual Run-off of Carson River and West Fork of Carson River, California.*

Seasonal year. (1)	Carson River, at Empire, Nev. (2)	West Fork of Carson River, Woodfords, Calif., R. (3)	Carson River, in order of volume. (4)	Percentage time record is not exceeded, t. (5)	colog $\left(1 - \frac{t}{100}\right)$. (6)	West Fork of Carson River, 1910-11 to 1914-15, R (7)	Percentage of time for five records, t. (8)	colog $\left(1 - \frac{t}{100}\right)$. (9)	$R - 64$. (10)
1900-01	96.3	92.9	44.8	2.95	0.013	65.1	10	0.046	1.1
1901-02	65.9	88.4	48.0	8.8	0.040	66.0	30	0.155	2.0
1902-03	76.4	75.8	51.2	14.7	0.069	....	..	.....	....
1903-04	124.6	113.3	62.6	20.6	0.100	....	..	.....	....
1904-05	62.6	73.2	65.9	26.5	0.134	....	..	.....	....
1905-06	134.9	146.4	76.4	32.4	0.170	....	..	.....	....
1906-07	182.3	187.4	80.5	38.2	0.209	77.6	50	0.301	13.6
1907-08	51.2	64.6	88.6	44.1	0.253	....	..	.....	....
1908-09	108.3	125.8	96.3	50.0	0.301	....	..	.....	....
1909-10	88.6	92.0	108.3	55.9	0.356	....	..	.....	....
1910-11	155.6	133.9†	117.3	61.8	0.418	....	..	.....	....
1911-12	44.8	65.1†	117.6	67.6	0.489	....	..	.....	....
1912-13	48.0	66.0†	124.6	73.5	0.577	....	..	.....	....
1913-14	145.1	96.5†	134.9	79.4	0.686	....	..	.....	....
1914-15	80.5	77.6†	145.1	85.3	0.833	96.5	70	0.523	32.5
1915-16	117.6	.....	155.6	91.2	1.056	133.9	90	1.000	69.9
1916-17	117.3	.....	182.3	97.05	1.530	....	..	.....	....

* Data from Table 1, *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 196.
† Data distributed according to Carson River frequency numbers.

The points are shown by the solid dots in the diagram. A smooth curve can evidently be drawn through four of these points. In order to plot this theoretical curve it is first necessary to plot the auxiliary curve. It is found by trial that if 64 is subtracted from each quantity in Column (7), the first, third, and last points will fall on a straight line when the quantities, $R - 64$, in

Column (10) are plotted to the corresponding cologarithms from Column (6). This is shown by the dotted straight line and the crossed circles in Fig. 13, from which the dotted curve was then plotted. The curve shown by the solid line was plotted in the ordinary manner, using the full record of fifteen years at Woodfords, treated without reference to the records at Empire. The complete data are shown by the open circles in Fig. 13, but the auxiliary curve is not shown. A closer agreement between the two curves might have been obtained if the records for the first five years or more had been used in place of only the five last years. If the five records used had been treated without reference to the longer record on the same stream, the data given in Columns (8) and (9) of Table 14 would have been computed and plotted as shown by the small triangles.

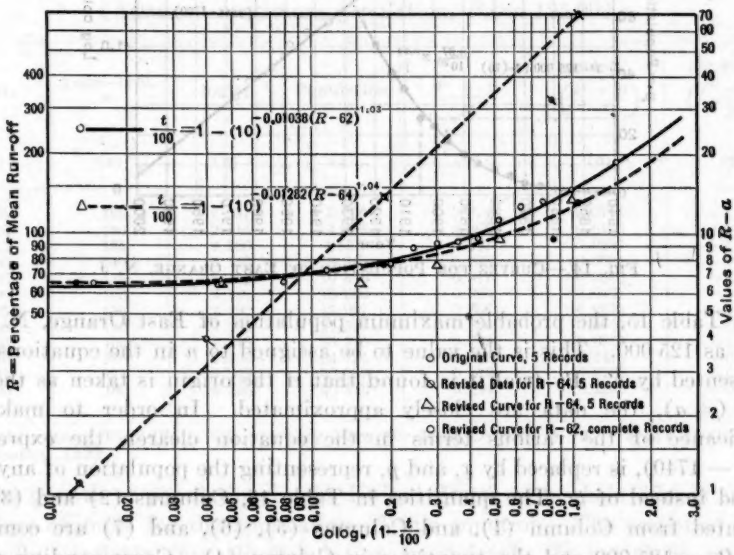
R = Percentage of Mean Run-off
Values of R - a
Colog. (1 - t/100)
Original Curve, 5 Records
Revised Data for R - 64, 5 Records
Revised Curve for R - 64, 5 Records
Revised Curve for R - 62, complete Records

FIG. 13.—CURVES FOR WEST FORK OF CARSON RIVER, AT WOODFORDS, CALIF.

A curve through these points would depart widely from the theoretical curve for the whole series throughout most of its length, but would cross it near each end, while the curve obtained by the aid of the long series departs considerably only at one end. It is evident that records for more than five years are usually necessary for reliable results. The equations of any of these curves can be obtained most easily, after finding the approximate limits of the curves, by substituting in Equations (S_2), (A_2), or (B_2), as the case may be, the co-ordinates of two well-defined points on any curve. All the numerical equations have been computed in this manner.

Population Data.—Another line of investigation in which these equations should prove useful is in the representation and prediction of the growth of municipal and general population.* Their application to such studies is shown

* Experience has shown that population growth seems to follow a law best represented by an ogee curve, just as has been found to be the case with hydraulic data.

in Figs. 14, 15, and 16. These diagrams also illustrate the straight line plotting of the logarithmic equations, (A_2) and (B_2), on ordinary co-ordinate paper.

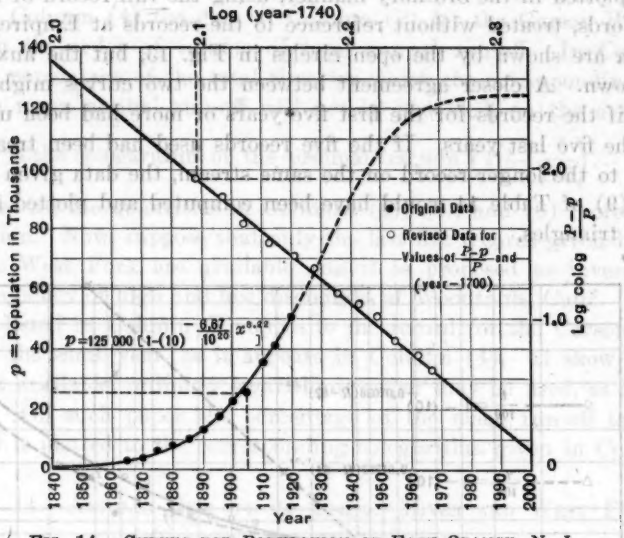

FIG. 14.—CURVES FOR POPULATION OF EAST ORANGE, N. J.

In Table 15, the probable maximum population of East Orange, N. J., is taken as 125 000. This is the value to be assigned to n in the equations, here represented by P. By trial it is found that if the origin is taken as the year 1740 $(= a)$, the data are closely approximated. In order to make the significance of the various terms in the equation clearer, the expression, (year — 1740), is replaced by x, and p, representing the population of any year, is used instead of t. The quantities in Table 15, Columns (2) and (3), are computed from Column (1), and Columns (5), (6), and (7) are computed with $P = 125\,000$ and the quantities in Column (4). Corresponding values are plotted from Columns (3) and (7) as shown by the circles in Fig. 14. The auxiliary curve can then be drawn and fits the data quite well. The scale of log colog $\left(\frac{P-p}{P}\right)$ was reversed from the usual direction so that the auxiliary curve would not parallel the general direction of the final curve, hence the slope of the auxiliary curve is not negative as might be thought.

By substituting the co-ordinates of the first and last points on the auxiliary curve in Equation (A_2) the values of the remaining constants are found and the following approximate equation can be written:

$$p = 125\,000\left[1 - (10)^{\frac{-6.67}{10}\,(\text{yr.}-1740)^{6.22}}\right]$$

The reverse curve shown in Fig. 14 can now be plotted from this equation or graphically by readings taken from the auxiliary curve. The value of the exponent, c, in this equation is large and hence the accuracy of the

plotting may be increased by using different lengths for the units of the scales for log x and log colog $\frac{P-p}{P}$.

This type of curve is easily plotted if a reasonable estimate can be made for the probable maximum total population that can be supported by the area under consideration. An approximate method to assist in arriving at this estimate will be given in connection with the next illustrative example. It is to be noted that the reverse curve follows the economic laws, now generally recognized, of an increasing rate of growth up to a certain point followed by a gradually decreasing rate.

TABLE 15.—POPULATION OF EAST ORANGE, NEW JERSEY.*

(Assumed maximum population, P, = 125 000.)

Year.	Year—1740, x.	log x.	Population $= p$.	$\frac{P-p}{P}$	colog. $\left(\frac{P-p}{P}\right)$	log colog. $\left(\frac{P-p}{P}\right)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1865	125	2.097	3 206	0.974	0.0114	−1.943
1870	130	2.114	4 315	0.965	0.0155	−1.810
1875	135	2.130	6 497	0.947	0.0237	−1.625
1880	140	2.146	8 349	0.933	0.0301	−1.521
1885	145	2.161	10 328	0.917	0.0376	−1.425
1890	150	2.176	13 282	0.893	0.0491	−1.309
1895	155	2.190	17 927	0.856	0.0675	−1.171
1900	160	2.204	21 506	0.828	0.0820	−1.086
1905	165	2.218	25 175	0.799	0.0975	−1.011
1910	170	2.230	34 731	0.722	0.1415	−0.849
1915	175	2.243	40 969	0.672	0.1726	−0.763
1920	180	2.255	50 712	0.594	0.2262	−0.646

* Data from Fig. 17, p. 22, of "City Plan for East Orange, N. J." report of City Plan Commission, 1922.

The growth of population in the United States as a whole affords another excellent example for the application of this equation. The statistics are so generally available that the tabulation is omitted. It would be made in exactly the same manner as for the preceding case. The dotted line in Fig. 15 is an auxiliary curve based on the assumption that the maximum population which can be supported by the area of the United States is 400 000 000 and that the curve begins with the year 1620.

This line fits the data quite well along the central part but there is a slight departure in the points as plotted to the left of the middle, which can be reduced by taking an earlier year of origin. The deviation of the direction indicated by the last four points at the right from the direction of the dotted line may be due to an over-estimate of the value of the limit. The equation for the population curve as obtained from this auxiliary curve is:

$$p = 400\left[1 - (10)^{\frac{-5174.4\,(y-1620)^{6.23}}{10^{20}}}\right]$$

in which, p is the population, in millions, and y is the corresponding year.

With the values of the constants of this equation substituted in the formula for the mode it is found that, with a limit of 400 000 000 for the total

population, the probable year of maximum rate of growth would be 1970. Examination of the statistics shows that the average rate of increase was greatest for the 10-year period from 1900 to 1910. There was a marked decrease of rate during the next 10-year period which was doubtless due, at least in part, to the practical suspension of immigration and other economic results caused by the World War. While the decrease in rate is, therefore, probably more or less temporary, it will be assumed for the purposes of illustration that the year of maximum rate of increase was 1914, that the population for that year, as found by interpolation, was 97 500 000 and that the average annual increase of 1 600 000 during the previous decade also applied for that year. It was found by trial that with the year of origin kept

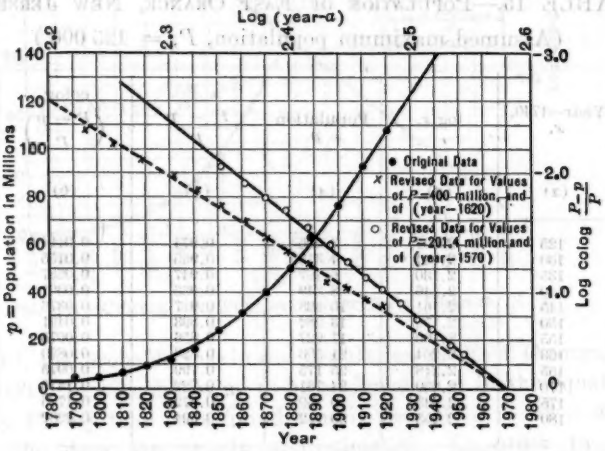

FIG. 15.—CURVES FOR POPULATION OF UNITED STATES.

constant, a large change in the value of P would cause a relatively small change in the value of c. Now assume that a and c remain constant and are found by trial by using any reasonable value of P; if the data show that the probable year of maximum rate of increase has been passed, and if the values of p and p' (population and annual increase for the given year) are computed for that year, then an approximate value for P can be obtained as follows:

$$p = P - P\,e^{-k\,(y-a)^c}$$

$$\text{Mode} = (y_{\text{max.}} - a) = \left(\frac{c-1}{c\,k}\right)^{\frac{1}{c}}$$

$$P = \frac{p'}{c\,k\,(y-a)^{c-1}\,e^{-k\,(y-a)^c}}$$

$$p' = c\,k\,P\,(y-a)^{c-1}\,e^{-k\,(y-a)}$$

$$(y_{\text{max.}} - a)^c = \frac{c-1}{c\,k}$$

Then,

$$p = P - \frac{p'}{c\,k\,(y-a)^{c-1}} = P - \frac{p'}{c\,k\left(\frac{c-1}{c\,k}\right)(y-a)}$$

Hence, substituting $y_{max.}$ for y, $p_{max.}$ for p, and $p_{max.}'$ for p', the resulting formula is:

$$P = p_{max.} + \frac{p_{max.}'\,(y_{max.} - a)}{(c - 1)} \qquad (10)$$

in which, $y_{max.}$ is the year of maximum increase, while $p_{max.}$ and $p_{max.}'$ are the corresponding population and rate of increase.

Substituting in Equation (10) the values proposed for the constants, but with 1570 instead of 1620, the value of P is found to be 201 400 000 for the limiting population. With these values for a and P points were computed and the second auxiliary curve shown by the solid line was drawn. This line fits the data perhaps a little more closely than the first one, although there is little to choose between them. The curve drawn through the original data as shown by the dots in Fig. 15, is plotted from the second auxiliary curve and is really a good fit. Its equation is:

$$p = 200\left[1 - (10)^{\frac{-0.181\,(y - 1570)^{1.96}}{10^{20}}}\right]$$

Other authors have suggested* the desirability of an equation giving an unsymmetrical curve on account of a possible (if not probable) skewness in the curve for the rate of growth of the United States. The equations herein presented are believed to offer the means for providing any required skewness in the rate curve and to give an integral curve of comparatively simple and usable form. The numerical equation for p is given only for the purpose of illustration and should not be considered as the writer's proposal for basing estimates as to the future population. The value of any such equation depends on the reliability with which the upper limit of population can be estimated and this is more a problem in economics than in engineering. For the United States it is perhaps more probable that the limit is 400 000 000 than 200 000 000.

A population curve somewhat more difficult to represent mathematically is given in Fig. 16. It shows that the growth of New Bedford, Mass., may be considered as the result of the simultaneous operation of two distinct laws depending on entirely different and independent sets of causes. The heavy line drawn to average the plotted points can be resolved into two curves. The first is shown by the light solid line, the "curve of" normal growth. This curve and its equation are of the same form as those shown on Figs. 14 and 15. The dotted curve represents approximately the rise and fall of the population engaged in the whaling industry. This business had such a marked effect on the life of the city that it may be considered independently, as governed by distinct laws. An equation can be formed for each of these secondary curves. That for the total population is then written by adding the right-hand members of the first two equations. If the total population of any year is represented by t and the corresponding year by y, then the equation of the final curve is:

$$t = 170 - 154\,(10)^{\frac{-0.004735\,(y - 1660)^{18.3}}{10^{40}}} - 16\,(10)^{-0.4865\left[\frac{y - 1770}{(1855 - y)^2}\right]^{0.488}}$$

* *Engineering News-Record*, December 1, 1921, p. 889.

The equation of the dotted curve is interesting inasmuch as it is the general form of Equation (G) with the exponent, d, equal to $2c$. While the upper limit of the year in Equation (G) is ∞, the corresponding value of t is not n but passes through the maximum value, n, when $y = b$, and then decreases toward the limit zero. Evidently the whaling industry must have reached its zenith at New Bedford about 1855 and has been declining ever since.

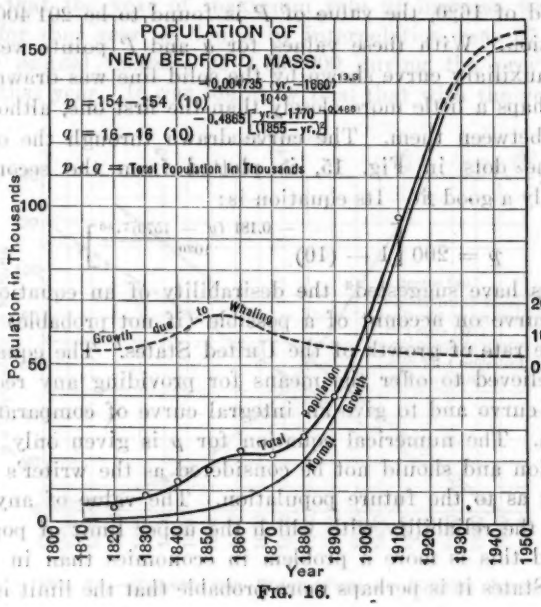

FIG. 16.

Figs. 14, 15, and 16 illustrate possibilities as to the use of the equations in studies for extensions of municipal water supplies and sewage works, and for the future needs of transportation and communication. It is hoped that, with the suggestions made by Kapteyn and Van Uven as to the significance of the various terms in the right-hand member of the equations, it will be possible to arrive at a surer knowledge of the causes producing changes in population and to estimate more accurately their effects.

The following possible application is suggestive only, and is made subject to revision after further study and discussion. The population in any district evidently tends to increase with time, due to births and the arrival of new families. It also depends on the date of origin, the effect of which is constant; an increase in the numerical value of this date tends to decrease the results of the causes of increase of population.* Likewise removals caused by death or otherwise tend to decrease the population, such removals increasing more or less in proportion to the time. If the relation between the causes of increase and decrease in population are fixed and constant, Equation (A) may be used to express the law of their resultant effect. If, however, this

* To express the same idea in non-mathematical terms—the assumption of a later date results in an attendant relative decrease in population.

relation is not a fixed one, Equation (*B*) is required, since it makes it possible to show the result of some fixed cause affecting the tendencies to decrease population, which cause may depend on some fixed date. If the causes that tend toward a reduction in population are such that their total result will at some time become greater than the result of the causes tending toward an increase, then Equation (*G*) as modified for the case of New Bedford may be used. One or more of these equations may be used to show the combined result of a great variety of causes affecting phenomena following skew frequency laws.

Biological Application.—A further test of the writer's method was made by fitting a curve to the biological data given in Table 16. The resulting curve differed from any previously investigated, requiring Equation (*C*) for its representation.

The data consist of measurements of the lengths of stalks of flax grown under special conditions. While the conditions were such as to produce an unusual skew distribution of the quantities measured, Dr. Kapteyn, from his analysis of the data, was able to state the general nature and effect of the causes producing the results observed. Similar conclusions can also be drawn from the curves and equations shown on Fig. 17, resulting from the writer's methods. It is stated by Dr. Kapteyn that no figure could be given (perhaps from lack of time, as the work was then in the press) and no equations are given. On the other hand the methods here given permit not only the construction of the duration and frequency curves but also the formation of their equations.

TABLE 16.—LENGTHS OF STALKS OF FLAX.*

Length of stalks, x.	Number of stalks less than length in Column (1).	Percentage of total, $\frac{y}{100}$.	$220 - x$.	$\frac{1}{220 - x}$.	Length of stalks, x.	Number of stalks less than length in Column (1).	Percentage of total, $\frac{y}{100}$.	$220 - x$.	$\frac{1}{220 - x}$.
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
10	163	12.2	210	0.00476	120	567	42.4	100	0.0100
20	180	13.5	200	0.00500	130	669	50.0	90	0.0111
30	197	14.7	190	0.00526	140	797	59.6	80	0.0125
40	215	16.1	180	0.00556	150	956	71.5	70	0.0143
50	239	17.9	170	0.00588	160	1 094	81.8	60	0.0167
60	265	19.8	160	0.00625	170	1 208	90.3	50	0.0200
70	302	22.6	150	0.00667	180	1 280	95.6	40	0.0250
80	343	25.6	140	0.00715	190	1 320	98.7	30	0.0333
90	383	28.6	130	0.0077	200	1 333	99.6	20	0.0500
100	442	33.0	120	0.0083	210	1 336	99.8	10	0.1000
110	490	36.6	110	0.0091					

* Data taken from Appendix to Chapter III, Kapteyn's "Skew Frequency Curves" (Second Paper), p. 69.

The interpretation of the results observed, can best be given in the words of the author:

"There seems to be an almost insuperable impediment against growth. Those individuals, however, who succeed in overcoming the first difficulty

then begin to grow very rapidly indeed, the rapidity increasing till the size 25 is reached. After that the growth begins to diminish; it gradually decreases, to only half of the maximum growth for the individuals of the size 100 and below one-tenth of the maximum growth for the individuals of size 170." * * *

"All this proves to be in good agreement with what has been really observed. Dr. Tammes writes: 'The case I sent you is as follows: the quantities communicated are stalk-lengths of *Linum crepitans*, a variety of the ordinary flax. They were measured, at a moment in which the growth had not yet ceased, * * *. The seeds were sown in a great deep flower-pot. Their number was purposely taken very high, so that they were *extremely crowded*. At starting, therefore, the difficulty for each seed was to get a root into the soil. It seems allowable to assume that all the seeds germinated. This has necessarily entailed an intense struggle, and many individuals must not have succeeded, or not sufficiently succeeded. For those who really got their root in the soil, there now came a good time. There was plenty of food for a good many of very small plants. The case, however, changed when the plants, becoming greater, required more room. Then a second struggle ensued, *viz.*, the struggle for the available food in the too narrow room. The plants now became more and more impeded in their growth'."

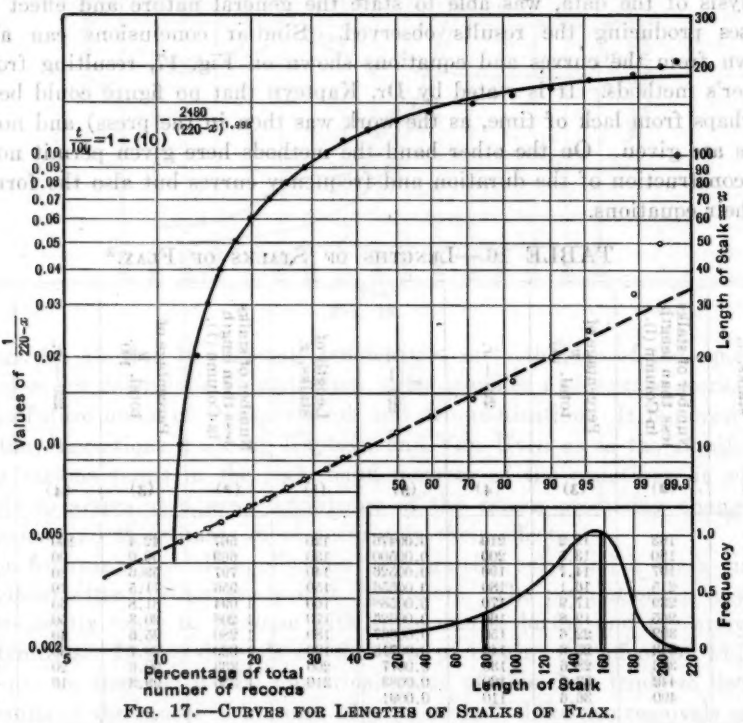

FIG. 17.—CURVES FOR LENGTHS OF STALKS OF FLAX.

These facts are also shown by the curves and equations of Fig. 17 as follows: The difficulty in germinating is indicated by the fact that the frequency curve begins at a height of about 0.11 above the x-axis and by the form of the equation, which has a theoretical lower limit of $-\infty$, giving a finite frequency for a length of zero. The rapid early growth is shown by

the fact that the frequency curve is nearly parallel to the axis at the left, which indicates strong reaction to the causes of growth. This was made possible while the plants were small by the presence of plenty of food. Soon the confinement of the flower-pot and the depletion of the food in the soil began to retard growth; this is shown by the rise of the frequency curve. The large number of individuals between the lengths of 140 and 170 is shown by the height of the frequency curve at the mode and the steepness of the sides of the hump. Hence, for individuals of this size the reaction to the causes of growth was comparatively weak and the second struggle resulted.

While the departure of the duration curve from two or three of the points at the upper end of the curve would make it impossible to extend the curve in this direction, this departure applies only to about 1% of the total number of individuals observed and therefore does not detract from the value of the conclusions to be drawn from the curves as a whole. The close agreement of the theoretical curve with 99% of the data would lead to the conclusion that the equation is adequate for the purpose of an interpolation formula, and it is hoped that the method may prove useful in biology and statistics as well as in engineering investigations.

General Discussion

The foregoing illustrations show that a considerable variety of conditions governing the occurrence of phenomena which follow skew frequency laws of distribution can be investigated by the writer's simple methods. The cases illustrated cover such a general range of conditions that it would seem that any engineer could soon acquire facility in making frequency studies.

In previous papers and discussions published by the Society, numerous passages call attention to the limitations and difficulties in adapting Pearson's skew frequency curves for use with short series of observations of rainfall and run-off data, the problem which it has been the writer's object to overcome, if possible, by new methods of investigations. For example, Mr. Hazen states:*

"Data represented by skew probability curves are capable of being handled by mathematical analysis, but the methods are complicated. * * * for stream-flow data, there are two reasons why the mathematical analysis is not advisable: First, the difficulty of applying the methods is too great for some engineers; and, second, and more important, such short-term records as are available are not long enough to serve as a basis for a conclusive determination of the shape of the curve."

This undoubtedly refers to the proper type of the Pearson probability curve, the selection of which, therefore, often seems to be a matter of judgment and may be somewhat approximate, provided it fulfills the majority of the requirements. Five equations were required by Professor Pearson to cover the practical range of skew frequency variations.

Likewise, R. E. Horton, M. Am. Soc. C. E., states:†

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 214.
† *Loc. cit.*, p. 228.

"Tolley and others have shown that none of Professor Pearson's curves seems to be very satisfactory for representation of rainfall variation."

* * * * *

"The generalized law of error * * * like the Gaussian law of error, and most frequency curves utilized by biologists and statisticians, can only be integrated for particular cases. For this reason it is not wholly satisfactory for engineering work where integral frequencies, rather than probabilities of individual events, are mostly required * * *."

This criticism of the Pearson frequency equations, that they cannot generally be integrated for application to engineering problems, is entirely obviated by the use of the writer's equations.

Another serious objection to the use of the Pearson frequency curves is the uncertainty as to the accuracy of the constants when based on coefficients or criteria computed from a series of not more than twenty or thirty terms which show considerable irregularity and departure from a smooth curve. This is usually the case with observations of rainfall and run-off. In discussing the accuracy of the coefficient of skew which it is shown may be from 17.5 to 61.7% in error, for records from 20 years down to 10 years in length, Mr. Foster states:*

"An actual record will not correspond precisely to the theoretical curve; and the coefficients, as computed from an actual record, therefore, would probably show greater variations than are indicated in Table 3."

* * * * *

"(2) The coefficient of skew is greatly affected by the length of the series, and only a very long record would give a value for c_s, which would be considered very accurate."

In his closing discussion, Mr. Foster continues in the same vein:†

"For ordinary engineering work, however, the observations are very limited in number. As a result, although the coefficient of variation can be computed with a fair degree of precision, the coefficient of skew can be determined only approximately."

Mr. Hazen, Mr. Hall,‡ and Frederic H. Hapgood,§ Jun. Am. Soc. C. E., all bring out much the same idea in their discussions of Mr. Foster's paper.

In using the methods of plotting duration or integral frequency curves as described herein, and in obtaining the equations of these curves, it is not necessary to compute either of these coefficients unless they are desired for the purpose of classification or comparison.

That there are certain limitations and difficulties in applying the writer's equations, especially when both limits of the curve are finite, is admitted. However, these difficulties will be found to be more theoretical than practical, or else they will be found to be inherent in the very nature of such fluctuating phenomena as rainfall and run-off, and therefore would apply to all methods of analysis. Numerous trials have shown that curves plotted with these equations will fit data following skew frequency laws of distribution as closely as those based on any other equations heretofore published.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 166.
† *Loc. cit.*, p. 197.
‡ *Loc. cit.*, p. 180.
§ *Loc. cit.*, p. 176.

After developing the equations and method of plotting data on skew frequency paper, it was submitted to a member of the Society for suggestions and criticism. Through his kindness the writer received, more than a year after the formulas had been independently deduced, the little known but valuable and suggestive papers on skew frequency curves by Kapteyn and Van Uven.* They show that the function of R, which seems to be much the same herein as in their papers, indicates the character and effect of the causes producing the variations in the observed values of the phenomena. They state in the Second Paper (1) that when the records are distributed according to the "normal Gaussian law" (plot as a straight line on Hazen's probability paper), the "elementary increment† is independent of the value of R"; (2) when the data plot as a straight line on skew frequency paper (for example, Fig. 1), the lower limit of the curve is zero and $f(R) = R$ (Equation (S)), and "the elementary increment is proportional to the value of R", (3) when the data, as plotted on skew frequency paper, fall on a curve (for example, Figs. 2 and 3) that can be made straight by the addition or subtraction of a constant (a in the foregoing discussion), "the elementary increment consists of a part proportional to the attained value of R (which is positive for a positive value of R), and of another part independent from R (which is negative for a positive value of a)"; and (4) if the data are best fitted by Equation (C), "the elementary increment consists of a part independent of R (which is positive for positive values of b) and of another part proportional to R (which is negative for positive values of R)".‡ It should be noted that the effects of the causes producing a finite lower limit are of opposite sign to the effects of causes producing a finite upper limit.

Equation (B) covers a condition common in hydraulic data and not discussed by Kapteyn and Van Uven. It provides for the simultaneous results from independent sets of causes of which some affect the lower limit and others, the upper limit. Equation (B) seems to be especially applicable to the mathematical expression of the variation in the frequencies of river discharges, when viewed in the light of the foregoing quotations. Consider the principal factors affecting variations in run-off. It is in general proportional to rainfall; a positive increment in the latter tends to produce a positive increment in the former. Absorption, deep seepage, and transpiration are all positive demands which are, more or less, both constant and independent of

* "Skew Frequency Curves in Biology and Statistics" (First Paper), by Dr. J. C. Kapteyn, Professor of Astronomy at Groningen, 1903; "Skew Frequency Curves in Biology and Statistics" (Second Paper), by Dr. J. C. Kapteyn and Dr. M. J. Van Uven, Professor of Mathematics at Groningen, 1916.

† *Loc. cit.*, Second Paper, pp. 31–32, as follows:

"The rate of increase of some organism under the influence of some cause depends not only on the intensity of that cause, but also on the degree in which the organism reacts upon it. For example, in some cases the rate of increase of a plant may be considered to be proportional to the area of some organs of the plant. By measuring the diameter, x, of the organ concerned we find the rate of increase proportional not only to the intensity of the cause of growth (rain, solar heat, etc.), but also to the second power of the measured quantity, x. In this case we call x^2 the "reaction-function." Thus, the increment of the plant, particularly of the measured quantity, x, is proportional as well to the intensity of the cause as to the particular value of the reaction-function, $\Psi(x)$. * * *

"The increment within some period of growth may be considered as a multiple, m, of an elementary increment. This multiple being proportional to the intensity of the cause, the elementary increment is, in its turn, proportional to the reaction-function."

‡ In these quotations the nomenclature of Kapteyn and Van Uven has been changed to conform with that of this paper.

the amount of run-off and which have a most decided effect in reducing it (see Kapteyn and Van Uven's Deduction (3)). Evaporation from land and water surfaces depends in large part on the amount of moisture available, the loss from this source increasing with an increase in run-off. At the same time topography has a large effect, while this cause of variation is entirely independent of any changes in the actual amount of the run-off (see Deduction (4)).

While this interpretation of the terms of the function of R may require modification, it is at least suggestive of a possible use that can be made in scientific investigations of such data. The plotting of two or three sets of annual flood heights or discharges to fit Equation (S) would seem to indicate that all but the causes producing effects proportional to R (see Deduction (2)) are of such minor importance as to be negligible. The writer's development of these equations was entirely empirical but the interpretation of $f(R)$, as expressing the results of the action of various causes affecting observed phenomena, gives them a semi-rational basis which materially increases their value.

In using these equations for the study of rainfall and run-off data, it is necessary, on account of the irregularity of the data, to assume the values of the upper and lower limits of the curve. This calls for the exercise of the engineer's judgment, guided by any indications as to a tendency toward definite limits when the data are plotted on "logarithmic" or skew frequency paper. This may be considered by some as an insurmountable objection to the use of these equations. It has been found, however, that two or three trials will generally give satisfactory results. It may be noted that Kapteyn and Van Uven also find it necessary to estimate the values of the limits for their curves.

If the data plot as a particularly smooth curve, especially at the lower end, a geometrical method* may be used by which the approximate values of the lower limit, a, of the curve can theoretically be determined. It is probable that a similar method could be used at the upper end of the curve to determine the value of the upper limit, b, if the data were sufficiently regular. All the values of the limits of the curves shown in the diagrams were found by trial with little trouble.

If sufficient data are available and the investigation is of considerable importance, approximate values for the constants of the equations can be determined by the graphical method here given and corrections to give more exact values can then be computed by the methods of least squares, but this should seldom be necessary.

Conclusions

1.—The graphical analysis of "integral skew frequency" or "duration curves", as herein presented, is developed especially for the examination of records consisting of from about ten to fifty observations. With fewer than

* See "Logarithmic Discharge Curves," in "King's Hand Book of Hydraulics," pp. 267–268.

ten the results are liable to great uncertainty, while with more than fifty the data must be grouped for convenience in plotting.

2.—The equations presented cover practically the entire range of conditions necessary for the analysis of hydraulic and many other types of skew frequency data. The field is much more general than that heretofore available for practical use in engineering design.

3.—The equations may be useful in such studies as the growth of population.

4.—By the method suggested, data may be plotted as a linear function of the independent variable and hence the curve can be extended to a reasonable distance beyond the range of actual observations with much more confidence than if curves of either single or reverse curvature were required.

5.—Data may be plotted and curves drawn on "logarithmic" or "skew frequency" paper without the necessity of computing coefficients or criteria of uncertain accuracy, or the use of special tables of coefficients. The only mathematical aids required are an ordinary slide-rule and a table of four-place logarithms.

6.—The operations required by this method are so easily performed that for purposes of design any engineer can make scientific investigations of data which to the ordinary investigator seem to follow obscure skew frequency laws of distribution.

7.—With irregular hydraulic data, the accuracy of the method depends on the judgment and skill of the engineer in selecting the best values for the limits of the curves, and in drawing the line that best represents the plotted points. The other constants of the equations are found graphically or by simple computation.

8.—It is possible to analyze the causes of, and estimate their effects on, phenomena studied by the use of these equations, and to resolve complex results into simpler forms, which are more easily investigated.

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DISTRIBUTION OF REINFORCING STEEL IN CONCRETE BEAMS AND SLABS

By Boyd S. Myers,* M. Am. Soc. C. E.

Synopsis

The object of this paper is to determine the correct amount of reinforcing steel required in the top of concrete beams and slabs at the supports for varying degrees of continuity. The discussion is based on the established fact that the area of steel required in a simply supported concrete beam is determined by the equation, $M = \frac{W l^2}{8}$, from which, by assuming various degrees of continuity, the area of steel in the top, over the supports, is found. The area of steel in the bottom at mid-span plus the average area in the top, over the supports, is shown to be equal to the area required in the bottom of a simply supported span of the same length carrying the same load.

Many city building codes and other recommendations require the total area of steel in concrete beams to be calculated from the equation, the moment, $M = \frac{W l^2}{6}$. The writer contends that this moment should be $M = \frac{W l^2}{8}$, giving an area of steel that will adequately and satisfactorily care for all the bending stresses.

General Principles

Although reinforced concrete structures have been used extensively for the past twenty-five years, and the general theory of their design has been quite fully developed, yet engineering literature, including the publications of the Society, give meager discussions of the proper distribution of reinforcing steel in concrete beams and slabs. Believing that this subject is as important as the correct area of steel to be used, the writer presents this paper in the hope that the discussion will lead to the establishment of the correct constants to use in the design of monolithic concrete structures.

The following general conditions should be kept in mind:

1.—Concrete beams and slabs are mechanical compositions of sand, cement, rock, and steel, and, consequently, are non-homogeneous.

2.—They usually have T-sections at the center of the span, and rectangular sections at the supports, and, therefore, have varying moments of inertia.

Note.—Written discussion on this paper will be closed in November, 1926. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

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3.—Therefore, they cannot readily be analyzed by the three-moment theorem or similar theory.

4.—In monolithic concrete structures it is permissible to consider slabs and beams as fixed at their supports. An interior beam framing into a column at each end, in line with similar beams in the adjacent spans, unquestionably is fixed at its supports. A single-span beam framing into concrete columns should be considered as partly fixed at the supports, as will be shown later.

Likewise, a beam framing into girders instead of columns with beams of equal or greater size in the adjacent spans should be considered as fixed at the supports. If the beams in the adjacent spans are smaller, however, the interior beam should be considered as partly fixed, as will be developed later. It will be assumed that concrete beams in monolithic structures are fixed or partly fixed at their supports and the bending moments produced at the supports, based on certain reasonable assumptions, will be determined.

Simply Supported Beams

Let $B\,B$ (Fig. 1) represent a simply supported beam of span, l, carrying a uniform load of W lb. per lin. ft. Evidently, the equal reactions at the supports, B and B, are $\frac{W\,l}{2}$. It is also evident that the maximum bending moment occurs at the center line of the span and is equal to:

$$M_c = \left(\frac{W\,l}{2} \times \frac{l}{2}\right) + \frac{W\left(\frac{l}{2}\right)^2}{2} = \frac{W\,l^2}{8}$$

The variation of the bending moment may be shown graphically by a parabola, $B\,C\,B$ (Fig. 2), drawn through the points of support, B. The vertical ordinates from the base line, $B\,D\,B$, to the parabola, $B\,C\,B$, shows the intensity of the bending moment at any point along the span length of the beam, $B\,B$.

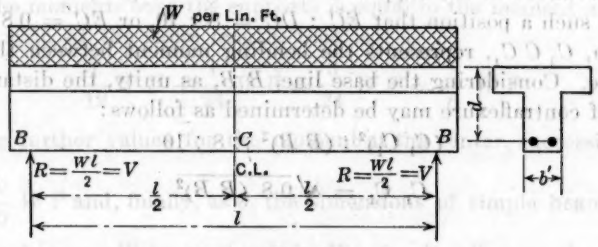

Fig. 1.

Beams Fixed at Both Supports

Assume that instead of being simply supported, this beam, $B\,B$ (Fig. 2), is fixed at the supports. Evidently, there will be a bending moment at the support inducing a zone of tension in the top of the beam and a zone of compression in the bottom. Then, as there is tension in the bottom at the center

line of the span, and compression in the bottom at the support, there must be a point of zero stress somewhere between the center of the beam and the support. This point near each support is generally known as the point of contraflexure. The part of the beam between the points of contraflexure may be considered as a simple span, and the part from the point of contraflexure to the support, as a cantilever beam.

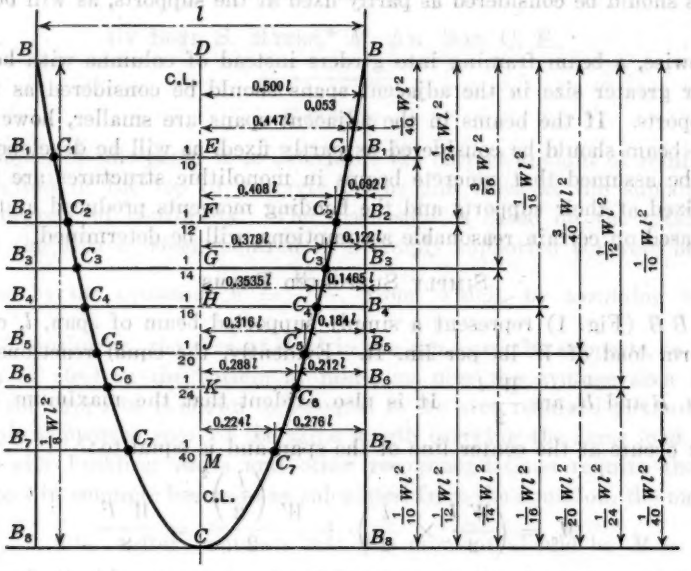

FIG. 2.

Assume that the bending moment at the center of this beam, $B\ B$, is $\frac{W\,l^2}{10}$, locate the points of contraflexure and determine the bending moments which are produced at the supports. Referring to Fig. 2, draw the base line, $B_1\ E\ B_1$, in such a position that $EC : DC = 8 : 10$, or $EC = 0.8\ DC$. Then the parabola, $C_1\ C\ C_1$, represents the bending moment between the points of contraflexure. Considering the base line, $B\ B$, as unity, the distance between the points of contraflexure may be determined as follows:

$$(C_1\ C_1)^2 : (B\ B)^2 :: 8 : 10$$

$$C_1\ C_1 = \sqrt{0.8\ (B\ B)^2}$$

but,

$$B\ B = l = \text{unity}$$

therefore,

$$C_1\ C_1 = \sqrt{0.8} \times l = 0.894\ l$$

Then, the distance, $E\ C_1$, from the center line to the point of contraflexure is $0.447l$, and the distance, $B_1\ C_1$, from the point of contraflexure to the support is $0.053l$. Now, considering $C_1\ C_1$ as a simple span carrying the load,

W, per linear foot, the reaction at C_1 is $0.447Wl$; then the moment produced on the cantilever beam, $C_1 B_1$, is,

$$(0.447\ W\ l \times 0.053\ l) + \frac{W\ (0.053\ l)^2}{2} = 0.0236\ W\ l^2 + 0.0014\ W\ l^2$$

$$= 0.025\ W\ l^2 = \frac{1}{40}\ W\ l^2$$

It will be noted that the sum of the moment at the center of the span and the average of the moments over the supports is equal to the moment at the center of a simple span of the same length, that is,

$$\frac{1}{10}\ W\ l^2 + \frac{1}{40}\ W\ l^2 = \frac{5}{40}\ W\ l^2 = \frac{1}{8}\ W\ l^2$$

Also, that the moment at the support is one-fourth the moment at the center of the span.

In like manner, assume that the bending moment at the center of the beam, $B\ B$, is $\frac{1}{12}\ W\ l^2$, represented by $F\ C$ (Fig. 2). Then,

$$F\ C = \frac{8}{12}\ D\ C = 0.667\ D\ C$$

and, considering the base line, BB, as unity, the distance, $C_2\ C_2$, between the points of contraflexure is found to be:

$$\sqrt{0.667} \times l = 0.816\ l$$

Considering $C_2\ C_2$ as a simple span carrying the load, W, per foot of span, with a reaction at $C_2 = 0.408Wl$, and $B_2\ C_2$ as a cantilever, the moment at B_2 will be,

$$(0.408\ W\ l \times 0.092\ l) + \frac{W\ (0.092\ l)^2}{2} = 0.0375\ W\ l^2 + 0.0042\ W\ l^2$$

$$= 0.0417\ W\ l^2 = \frac{1}{24}\ W\ l^2$$

Notice, again, that the sum of the moment at the center of the span and the average of the moments over the supports is equal to the moment at the center of a simple span of the same length, that is,

$$\frac{1}{12}\ W\ l^2 + \frac{1}{24}\ W\ l^2 = \frac{3}{24}\ W\ l^2 = \frac{1}{8}\ W\ l^2$$

Assuming further values for the moment at the center, successively, as $\frac{1}{14}$, $\frac{1}{16}$, $\frac{1}{20}$, $\frac{1}{24}$, $\frac{1}{40}\ W\ l^2$ and, finally, as 0, the dimensions of simple beams and cantilevers in each case will be as shown in Fig. 2. In all cases the sum of the center moment and average moment at the supports is equivalent to the moment for a simple beam of the same span.

Since the bending moments at the supports and at the center of the span determine the area of steel required at those points, the foregoing discussion shows that the designer has a wide latitude in the distribution of the steel in the interior spans of a monolithic concrete structure. He can place in the

bottom of the beams all the steel required by the equation, $M = \frac{1}{8} W l$, and thus take care of the positive bending stresses, but, on account of the monolithic construction, unsightly cracks would develop over the supports and near the columns and these might be serious enough to affect the strength of the structure. Again, he can place all this steel in the top over the supports and detail the beams as cantilevers to develop the resulting tension, and he would have a theoretically satisfactory design. Or, he can place any part of the steel required by this equation in the bottom of the beam at the center of the span, and the remainder in the top of the beam at the supports, detailing the beam accordingly, and the design will care for all the bending stresses adequately and satisfactorily.

Considerations Affecting Distribution

The distribution of the steel in beams and slabs should be governed by the following considerations:

A.—The area of steel in the top of a beam framing into a column should be a minimum consistent with good designing, so that the bending moment produced in the columns either by unbalanced loads, by unequal spans, or by a beam framing into one face of the column only, may be a minimum. The columns perform the primary function of supporting the floor loads and should not be burdened with unnecessary bending which would necessitate a material increase in their size and cost.

B.—In practically all cases, the beam has a T-section at the center of the span, and a rectangular section at the support. Therefore, at the support, the concrete can develop the full stress in only approximately one-half as much steel as at the center of the span. If the area of steel in the top at the support is equal to or greater than the area placed in the bottom at the center, it is necessary to increase the depth of the beam considerably at the supports, in order to develop full stress in the top steel. This materially increases the cost of building forms and partitions to conform to the irregular and sloping soffits of the beams.

C.—Beams framing into girders from opposite sides and having approximately equal spans should have over the supports a minimum area of steel consistent with good designing. In this case, each beam is fixed by its companion, and by the torsional resistance of the girders. Evidently, the greater the area of top steel, the greater will be the torsional stresses produced by unbalanced loads, or by unequal spans.

D.—After a careful study of the foregoing theoretical and practical considerations, it should be evident that in only one type of construction is it logical and practical to place the same area of steel in the top over the support as in the bottom at the center of the span. This type consists of slabs of approximately equal spans forming parts of the slab-and-beam type of construction. The slab has the same depth at the support as at the center of the span, and, consequently, will develop the same area of steel at both

points. The interior slab spans then should be designed for $M = \frac{1}{16} W l^2$. If however, the area of steel over the supports is made three-fourths of the area placed in the bottom at the center of the span, the interior spans should be designed for $M = \frac{1}{14} W l^2$. Similarly, for one-half as much in the top as in the bottom, the interior slab spans should be designed for $M = \frac{1}{12} W l^2$.

Unless a beam has a rectangular section throughout its entire length it is impractical and illogical to place the same area of steel in the top over the supports as is placed in the bottom at the center of the span. This condition is of such rare occurrence that it need not be considered. It is uneconomical to place more steel in the top over the supports than the rectangular beam section will develop; it is costly and undesirable either to extend the straight bars in the bottom of beams into the adjacent spans doubly reinforcing the beams at the supports or to build heavy concrete haunches at the supports, to develop the top steel.

The bending produced in the columns by a large area in the top of the beams at the supports is a vital factor in the design. The wall columns with beams framing into them at right angles to the wall line would be subjected to a serious bending stress. This bending stress is taken by a section of the column extending approximately a half story both below and above the beam, provided the column bars extend about 40 diameters below and above the rough floor for bond. The interior columns would experience bending from either unbalanced loads or unequal spans. The wall columns should not be subjected to a bending greater than that produced by placing an area of steel in the top at the support of about one-fourth that in the bottom at mid-span. For ordinary loads and for spans up to about 25 ft., the bending in the columns would not be serious and can be easily cared for in the design based on the foregoing assumptions.

Proper Moment Coefficients

A careful study of these theoretical and practical considerations points to the following conclusions:

1.—Single-span beams framing into brick walls should be considered as simply or freely supported, and should be designed for a moment, $M = \frac{1}{8} W l^2$, at mid-span.

2.—Single-span beams framing into concrete columns should be considered partly fixed and should be designed for moments, $M = \frac{1}{10} W l^2$, at the center, and $M = \frac{1}{40} W l^2$, at the supports.

3.—End-span beams framing into concrete columns should have one-fourth as much steel in the top at the end support and one-half as much at the

interior support as in the bottom of the beam at mid-span. The principle previously developed that the total area of steel in any beam is equivalent to that required by the equation, $M = \frac{1}{8} W l^2$, applies to this case. Assuming that $M = \frac{1}{11} W l^2$ at mid-span, $M = \frac{1}{22} W l^2$ at the interior support, and $\frac{1}{44} W l^2$ at the exterior support, so that:

$$\frac{1}{11} W l^2 + \frac{\left(\frac{1}{22} W l^2 + \frac{1}{44} W l^2\right)}{2} = \frac{1}{8} W l^2$$

4.—End-span beams framing into brick walls or into concrete lintel beams should be considered as freely or simply supported at the wall end. These beams should be designed for $M = \frac{1}{10} W l^2$ at mid-span, and for $M = \frac{1}{20} W l^2$ at the interior support, whence, as before,

$$\frac{1}{10} W l^2 + \frac{\frac{1}{20} W l^2}{2} = \frac{1}{8} W l^2$$

5.—Interior-span beams should have an area of steel in the top at the supports equal to one-half that in the bottom of the beam at mid-span. These beams should be designed for $M = \frac{1}{12} W l^2$ at mid-span and $M = \frac{1}{24} W l^2$ at the supports.

6.—End-span slabs should have the same area of steel over the interior support as in the bottom at mid-span. It is common practice to bend up alternate slab bars, so the area in the top at the exterior support would be one-half the area of the steel in the bottom at mid-span. The end-span slabs should be designed for $M = \frac{1}{14} W l^2$ at mid-span, $M = \frac{1}{14} W l^2$ at the interior support, and $M = \frac{1}{28} W l^2$ at the exterior support. Then, as recommended:

$$\frac{1}{14} W l^2 + \frac{\frac{1}{14} W l^2 + \frac{1}{28} W l^2}{2} = \frac{1}{8} W l^2$$

If the end span is built at least 8 in. into a solid brick wall or if it is built monolithic with a fair-sized lintel beam, the exterior support will develop top steel equivalent to $M = \frac{1}{28} W l^2$, unless the slab span is unusually large. If the exterior bearing is considered a simple support, the end-slab span should be designed for $M = \frac{1}{12} W l^2$ at mid-span and $M = \frac{1}{12} W l^2$ at the interior support. Again,

$$\frac{1}{12} W l^2 + \frac{\frac{1}{12} W l^2}{2} = \frac{1}{8} W l^2$$

This last case departs from a strictly monolithic construction, and a careful analysis will show that the moment constants given will be deficient by about 3 per cent. This will not affect the design materially, but, if accurate results are desired, the end slab span with a simple exterior support should be designed for $M = \frac{3}{35} W l^2$ at mid-span and $M = \frac{3}{35} W l^2$ at the interior support.

Beams Fixed at One Support and Partly Fixed at the Other Support

Beams only partly fixed at one support depart somewhat from the condition of full continuity, and, therefore, may not yield to the application of the $\frac{1}{8}$-moment principle without small errors. Take, for example, an end-span slab having $M = \frac{1}{14} W l^2$ at mid-span, $M = \frac{1}{14} W l^2$ at the interior support, and $M = \frac{1}{28} W l^2$ at the exterior support. Referring to Fig. 3, let $B\ D\ B$ be the base

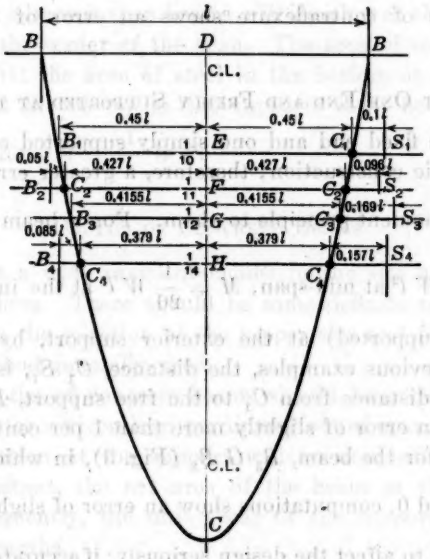

FIG. 3.

line on which is drawn a parabola, $B\ C\ B$, representing the bending moment produced on a simple span beam by a load, W, per foot of span, giving a maximum moment, $M = \frac{1}{8} W l^2$. Draw the line, $B_4\ S_4$, in such a position that $H\ C = \frac{8}{14} D\ C$. The parabola, $C_4\ C\ C_4$, represents the bending moment between the points of contraflexure, $C_4\ C_4$. Then, $C_4\ C_4 = 0.756\ l$ and the reactions at C_4 from the load, W, are $0.378\ W\ l$. Let $Y\ L$ be the distance

from C_4 to the support, S_4, Y being an unknown fraction. Then, assuming that the moment produced at the support, $S_4 = \frac{1}{14} W l^2$,

$$W l (Y l) + \frac{W (Y l)^2}{2} = \frac{1}{14} W l^2 = 0.071428 W l^2$$

from which, $Y = 0.157$.

Similarly, assuming that the bending moment at the exterior support is $\frac{1}{28} W l^2$, the distance, $B_4 C_4$, is found to be equal to 0.085. As a check the distance, $C_4 C_4$, between the points of contraflexure is computed as,

$$1.000 - (0.157 + 0.085) l = 0.758 l$$

showing an error of about one-half of 1 per cent.

Following the same procedure for a beam, $C_2 C_2$ (Fig. 3), having mid-span and end moments of $\frac{1}{11} W l^2$ and $\frac{1}{22} W l^2$, respectively, the corresponding dimensions on Fig. 3 are deduced. In this case a check on the distance between the points of contraflexure shows an error of less than one-third of 1 per cent.

Beams Fixed at One End and Freely Supported at the Other End

Beams with one fixed end and one simply supported end depart radically from true monolithic construction; therefore, a greater error may be expected in applying the $\frac{1}{8}$-moment principle to them. For a beam, $B_1 E C_1$ (Fig. 3), for which $M = \frac{1}{10} W l^2$ at mid-span, $M = \frac{1}{20} W l^2$ at the interior support, and $M = 0$ (simply supported) at the exterior support, by a similar line of reasoning to the previous examples, the distance, $C_1 S_1$, is found to be equal to 0.1 l. Then the distance from C_1 to the free support, B_1, is equal to 0.9 l instead of 0.894 l, an error of slightly more than 1 per cent.

In like manner for the beam, $B_3 G B_3$ (Fig. 3), in which the moments are $\frac{1}{12} W l^2$, $\frac{1}{12} W l^2$, and 0, computations show an error of slightly more than 3%, which is not enough to affect the design seriously; if accurate results are desired, however, this end span should be designed for $M = \frac{3}{35} W l^2$ at mid-span and $M = \frac{3}{35} W l^2$ at the interior support.

Spans of Unequal Length

The $\frac{1}{8}$-moment principle is useful in designing adjacent spans of unequal lengths. With two spans, freely supported at the exterior supports, design

the shorter span for $M = \frac{1}{10} W l^2$ at mid-span and $M = \frac{1}{20} W l^2$ at the interior support. Compute the area of steel required in the longer span for $M = \frac{1}{8} W l^2$ and deduct from this one-half the area that the shorter span beam will develop in the top at the interior support.

For three spans, consisting of a short span in the middle and two longer end spans freely supported at the exterior supports, make the beam section of the short span the same size as for the longer end spans; design the end spans for $M = \frac{1}{10} W l^2$ at mid-span and $M = \frac{1}{20} W l^2$ at the interior support and carry the top steel the entire length of the short middle span. If conditions are reversed and there are two short end spans with a long interior span, design the end spans in the usual way and determine the area of steel for the interior span for $M = \frac{1}{8} W l^2$. Deduct from this the average area of top steel that the adjacent span beams will develop at the supports to give the area of steel in the center of the span. The area of steel can be checked from the relation that the area of steel in the bottom at mid-span plus the average of the areas in the top at the supports must be equal to the area required by the equation, $M = \frac{1}{8} W l^2$.

Depths of Concrete Beams

The designer has a wide range of choice in the size and shape of structural concrete members. There should be some definite principles or standards to assist him in the selection of the proper size and form, instead of the "random guess" method generally used.

The effective depths of concrete beams should be a definite ratio of the span length. Then, for a given load per linear foot of span, if the span length is doubled, the reaction at the support will be doubled, and if the width of the beam is constant, the net area of the beam at the support will be doubled, and, consequently, the unit shear at the support will remain constant for all span lengths.

Referring to Fig. 1, let,

l = length of span, in feet.
W = load per linear foot of span.
b' = width of the stem, in inches.
d = effective depth of beam, in inches.
j = ratio of lever arm of resisting couple to depth, d.
V = total shear at support $= \frac{W l}{2}$.
x = ratio of d, in inches, to l, in feet.

The net area of the beam section, $b\ d = b\ x\ l$, and the maximum unit shear, v, at the support is:

$$v = \frac{V}{b'\,j\,a} = \frac{\frac{W\,l}{2}}{b'\,j\,x\,l} = \frac{W\,l}{2\,b'\,j\,x}$$

Notice that the factor, l, cancels out of the general equation, showing that the unit shear at the support is constant for all span lengths for a given load, W, provided b' and x remain the same. This principle makes it possible to draw simple shear diagrams which will show for any ratio, x, $\left(= \frac{d}{l}\right)$, the width of beam necessary to keep the unit shear at the support within any maximum specified. This diagram (Fig. 4) also will show at a glance the

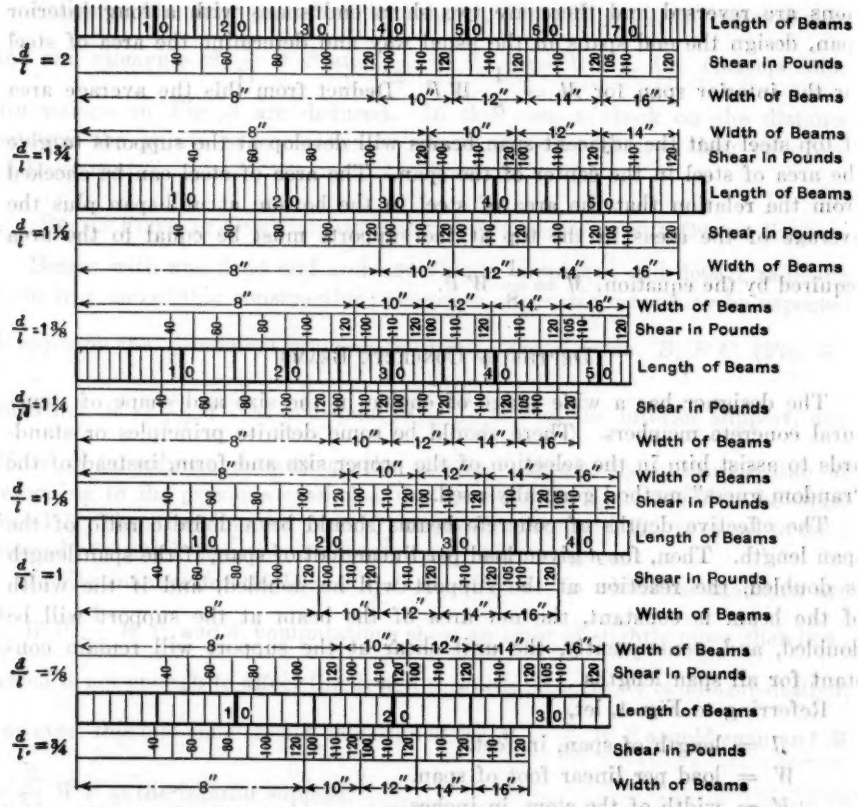

FIG. 4.—SHEARS CORRESPONDING TO DIFFERENT LENGTHS AND WIDTHS OF BEAMS FOR VARIOUS VALUES OF RATIO OF LENGTH, l, IN FEET, TO EFFECTIVE DEPTH, d, IN INCHES.

correct size of beam for any given load, W, and the unit shear produced at the support by that load for any span length. This principle would have an ideal application to the metric system of measurements. It gives satisfactory results with American units—feet and inches. The following ratios

for x are recommended: $\frac{3}{4}$, $\frac{7}{8}$, 1, $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 in. per ft. of span. These will give ranges of depth from one-sixteenth (for $\frac{3}{4}$ in. per ft.), to one-sixth of the span (for 2 in. per ft. of span) or nine different depths of beam for any load and span length. Fig. 4 shows this shear diagram for a maximum unit shear in the concrete at the support of 120 lb. per sq. in.

For example, a concrete beam is to be designed with a clear span of 24 ft., carrying a total load of 2 800 lb. per lin. ft. Referring to Fig. 4, for an effective depth $= \frac{3}{4}$ in. per ft. of span, that is, for $x = \frac{3}{4}$, it is seen at a glance that a beam 16 in. wide and 20 in. deep is required to carry this load, and that the shear at the support would be about 117 lb. per sq. in. If d were taken as $\frac{7}{8}$ in. per ft. of span, the beam would be 14 in. wide and 23 in. deep and the shear at the support would be 114 lb. per sq. in.

Design of Shear Reinforcement

The determination of the size, number, and spacing of stirrups to properly care for the shearing stresses in concrete beams and girders is one of the most tedious and time-consuming problems which the designer has to solve, and one for which no practical or rational method is presented in any of the technical literature on reinforced concrete designing. One textbook, published in 1923, states: "It is usually satisfactory to select a certain size of stirrup and calculate the spacing at different points along the beam." This involves a pure guess as to the number and spacing of the stirrups. Guesswork should not be countenanced in the field of design.

In this discussion the following standard symbols will be used:

s = spacing of stirrups, in inches.
N = total number of stirrups per beam.
A_v = area of the vertical members of a stirrup, in square inches.
A_s = total area of vertical shear steel, in square inches $= N A_s$.
f_s = stress on stirrups, in pounds per square inch.
j = ratio of lever arm of resisting couple to the effective depth of the beam = 0.875 average.
d = effective depth of beam, in inches.
b' = width of stem of T-beam, in inches.
V = total shear on Section $b' d$, in pounds per square inch.
v_1 = unit shear on Section $b' d$ carried by stirrups, in pounds per square inch.
v_2 = allowable unit shear on plain concrete = 40 lb. per sq. in.
V_1 = total shear carried by stirrups $= v_1 b' d$.
V_2 = total shear carried by plain concrete $= v_2 b' d$.
l = clear span of beam, in inches.
l' = distance, in inches, from support to the point where the total shear on Section $b' d = 40$ lb. per sq. in.

In Fig. 5, the triangle, $A B C$, represents the shear diagram for one-half a beam carrying a uniformly distributed load. The area, $D B C E$, represents the shear carried by plain concrete and the area, $A D E$, the shear carried by

the stirrups. Note that $\frac{V'_1}{2}$ is the area of the diagram representing the shear carried by stirrups for one end of the beam.

From similar triangles:

$$l' : V_1 :: \frac{l}{2} : V \quad \ldots\ldots (1)$$

From Equation (1),

$$l' = \frac{V_1}{2\,V} \times l = \frac{v_1\, l}{2\,V} \quad \ldots\ldots (2)$$

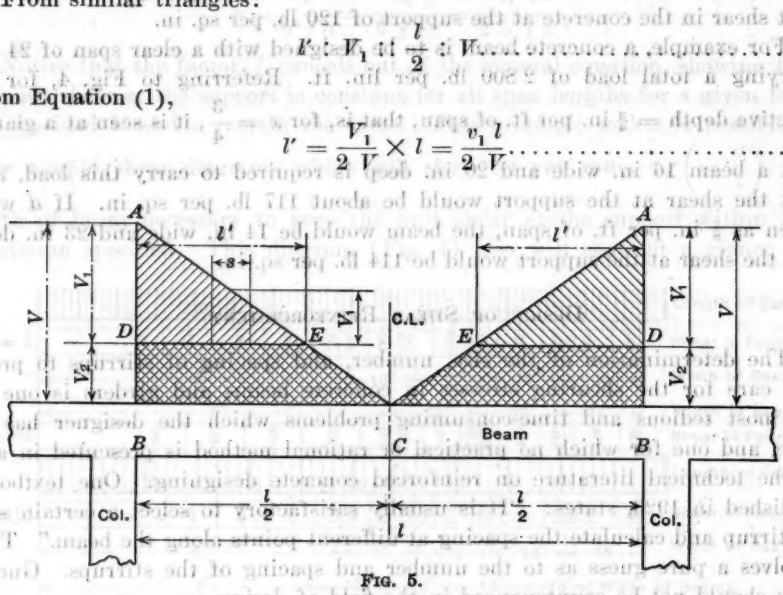

FIG. 5.

The equation generally used for determining the spacing of stirrups is,

$$s = \frac{A_v f_s j\, d}{V_1} \quad \ldots\ldots (3)$$

Equation (3) may take the form,

$$V_1 s = A_v f_s j\, d$$

$V_1 s$ represents the area of the shear diagram carried by one stirrup; but $V_1 s = A_v f_s j\, d$, therefore, $A_v f_s j\, d$ may be taken as the area of the shear diagram carried by one stirrup. Consequently, the total number of stirrups required per beam is,

$$N = \frac{V_1 l'}{A_v f_s j\, d} \quad \ldots\ldots (4)$$

but,

$$V_1 = v_1 b' d$$

therefore,

$$N = \frac{v_1 b' d\, l'}{A_v f_s j\, d} = \frac{v_1 b' l'}{A_v f_s j} \quad \ldots\ldots (5)$$

Note that d disappears from Equation (5) and that, consequently, for any given value of v_1, the number of stirrups required is independent of the depth of the beam. The depth of the beam influences the number, however, by its effect on the value of v_1.

In applying Equation (5) it is necessary to assume the size of stirrups to be used in order to ascertain the value of A_v. Equation (5) may be transformed to give the total area of stirrups as follows:

$$A_s = N A_v = \frac{v_1 b' l'}{f_s j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

from which A_s or N may be found.

To illustrate the application of these equations, assume a beam with a clear span of 20 ft., 14 in. wide, and 20 in. deep, carrying a total uniform load of 3 000 lb. per lin. ft. The reaction at each support is 30 000 lb.; the shear per square inch on Section $b'\ d$ at the support is 119 lb. Now, applying Equation (2):

$$l' = \frac{v_1\ l}{2\ V} = 80 \text{ in.}$$

Applying Equation (5), and assuming ½-in. round stirrups:

$$N = \frac{v_1\ b'\ l'}{A_v\ f_s\ j} = \frac{79 \times 14 \times 80}{0.39 \times 16\,000 \times 0.875} = \text{sixteen } \tfrac{1}{2}\text{-in. round stirrups}$$

Or, by applying Equation (6),

$$A_s = \frac{v_1\ b'\ l'}{f_s\ j} = \frac{79 \times 14 \times 80}{16\,000 \times 0.875} = 6.32 \text{ sq. in.}$$

Spacing of Stirrups

Except in rare cases, the bent longitudinal bars should not be considered as shear reinforcement. They vary much in size, in their position in the beam with respect to the support, and in the angle of the bent portion. If the plain concrete can carry, say, 40 lb. per sq. in. at the support, it can carry 40 lb. throughout the entire length of the beam. The assumption that the concrete may carry one-third of the shearing stresses at any point in the beam is supported by no logical basis. The plain concrete is usually considered to take a shearing stress of 40 lb. per sq. in. on the section, $b\ d$, and stirrups are used to care for the additional shearing stress. They should be spaced so that they are subjected to a theoretically uniform stress. Each stirrup should care for its proportional area of the shear diagram.

The area, $A\ D\ E$ (Fig. 6), represents the shear carried by the stirrups at one end of the beam of Fig. 5. Theoretically, using sixteen ½-in. round stirrups in the beam, eight at each end, this triangle, $A\ D\ E$, should be divided into eight equal areas and a stirrup placed at the center of gravity of each area; but, if $A\ D\ E$ is divided into sixteen equal areas, stirrups placed at each odd numbered line of division will practically coincide with the eight gravity lines mentioned. Since one of the functions of a stirrup is to give a vertical support to the beam bars, there should be an extra half space adjacent to the support, giving $N + 1$ spaces in all.

This spacing may be accomplished by the following analytical method. From plane geometry the areas of similar right triangles are proportional to the squares of the respective sides, and, conversely, the sides of similar right

triangles are proportional to the square roots of the respective areas. Therefore, referring to Fig. 6,

$$y_1 : l' :: \sqrt{1} : \sqrt{N+1}, \text{ or, } y_1 = \frac{80}{\sqrt{17}} = 19.4 \text{ in.}$$

In the same manner:

$$y_1 : y_3 :: \sqrt{1} : \sqrt{3}, \text{ or } y_3 = 33.6 \text{ in. etc.}$$

All such computations giving the spacings shown on Fig. 6 may be accomplished by a single setting of the slide-rule.

If the slide-rule is read to the nearest inch, no stirrup will be more than ½ in. from its theoretical position, and this will be much more accurate than the actual location.

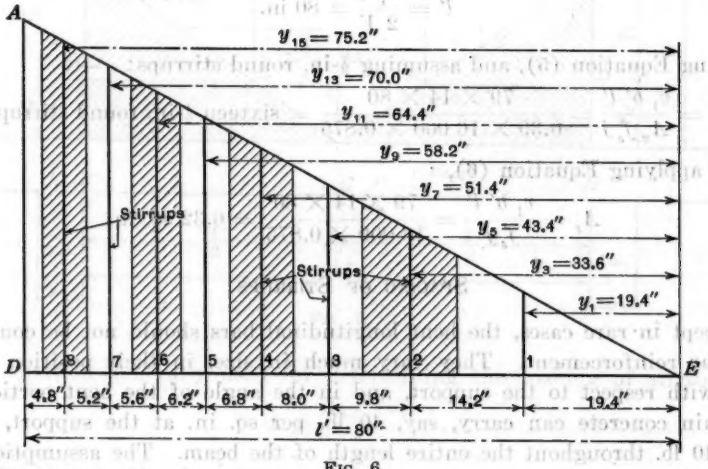

FIG. 6.

It should be noted that in all cases of fixed beams, the part of the beam from the support to the point of contraflexure acts as a cantilever beam and, consequently, its stirrups should be inverted. In spans with one end freely supported and the other end fixed, the reactions are, respectively, 0.55 W l and 0.45 W l. The unit shears are determined in the usual manner; the stirrups at a free end are the same as those for a simple span the length of which is 0.9 l, and the stirrups at the interior support are the same as those in an interior span the length of which is 1.1 l.

It is hoped that the foregoing ideas on arranging longitudinal and shear reinforcement will clarify this general question or at least stimulate thought and expression of opinion from which some advance will result. If this one point of view provokes discussion tending to rationalize the design of concrete beams and slabs, it will have served a useful purpose. The moment coefficients and design diagrams represent ideas on reinforcement that have proved extremely serviceable to the writer. He hopes that other engineers will contribute from their own funds of thought and thus swell the store of useful information to the end that concrete design may assume a more rational and, at the same time, a more economical and practical aspect.

SNOW REMOVAL PROBLEMS OF ORGANIZATION AND OPERATION*

By George E. Hamlin,† M. Am. Soc. C. E.

The operation of snow removal, like construction or maintenance, must be approached from the viewpoint of organization or personnel as well as from that of equipment, and to this personnel the greatest attention must be given. It is only by reason of a thoroughly competent, efficient, and loyal organization that satisfactory results can be produced in the limited time allowed for the opening to operation of the main and connecting road systems under the charge of a State Highway Department if travel is to be continued with the minimum inconvenience.

Through its very nature, long hours, extreme weather conditions, and personal discomfort enter largely into the work. It is only the desire of the organization to fulfill its function and its "not-to-be-beaten-at-any-cost" attitude that will cause the men to "carry on" through day and night, through storm and zero weather, continuously if necessary, until the sections under their charge are free for travel.

The experience of several years in the development of a snow removal organization in Connecticut has been utilized in the general recommendations for this maintenance problem.

In planning a snow removal campaign, it must be considered that owing to the general lack of surplus maintenance funds, only average conditions can be considered and planned for. The purchase of equipment alone represents the expenditure of many thousands of dollars and in the average locality, such equipment also must be suitable for types of work other than snow removal. Thus the highest type of snow-fighting equipment cannot always be economically furnished.

The personnel must be largely the same as that employed in yearly maintenance with the addition of units required to operate efficiently under the extreme conditions which may occur. Each foreman should be in touch with additional men who, in the case of urgent need, will be used to reinforce the regular patrol force or replace it when continuous operation becomes necessary.

To the regular organization, however, must be delegated the brunt of the work. These are the men who are familiar with both summer and winter conditions in their districts. Each landmark along the road is known to them almost as if it were in their own dooryard. A tendency for drifts to form under certain wind conditions is remembered and measures are taken to

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* Presented at the meeting of the Highway Division, Salt Lake City, Utah, July 9, 1925.

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counteract this. Each foreman is certain of the ability of his crew to operate more quickly and efficiently than the crew in the adjoining section and this feeling is largely held by the men themselves. It is their section, under their charge in winter as well as in summer—"The next gang has never beaten us yet! Come on, boys!" Where this feeling exists the larger part of the personnel problem has been solved.

The regular maintenance organization for carrying on snow removal will include:

1.—A Maintenance Engineer and Assistants at Headquarters;
2.—A District Maintenance Engineer in charge of a Supervision District; and
3.—Foremen and laborers on each Patrol Section.

The Maintenance Engineer is in full charge of the snow removal program. During the progress of clearing snow, he is necessarily confined wholly to his office in order to receive reports and to give instructions incident to the changing conditions or the severity of the storm in any particular locality. To him each District Maintenance Engineer reports by telephone regularly during the storm, giving the location and condition of equipment, the progress of snow removal, and recommendations as to transfer of additional equipment.

On his desk is a map of the State and as each District Engineer reports, the information is transferred graphically to the map, making it an accurate progress chart. On this map is also recorded additional information reported by the District Engineer, including the depth of snow and the location of drifts. Ordinarily, two operations of a plow will clear the roadway to a sufficient width to permit the passing of traffic. On the map, a single line denotes that a single lane has been opened. After the next passage of the plow an additional line is added to represent the opening of the road to two-way traffic. All roads are opened to this width before removing any snow from the shoulders and gutters, in order to open up waterways and provide room for the snow of future storms.

The Assistant Maintenance Engineers function directly under the Maintenance Engineer. They inspect the equipment and the progress of the work and keep in frequent touch by telephone with the main office. Each of these men has a State-wide knowledge of conditions and is fully capable of taking complete charge in emergency, thus relieving and supplementing the District Maintenance Engineer in a snow-congested district.

The foremen report by telephone as frequently as possible to the District Engineer. This is necessary, not only to have information available for the traveling public, but to permit the transfer of any particular organization unit, under necessity, to a point where conditions require immediate attention.

A chart in the main office also carries the locations of each of the snow removal units, these being represented by buttons inscribed with the type and character of such unit. When particularly adverse conditions are reported from a certain district, or a snow removal unit goes out of commission, which sometimes will occur when many of such units are in operation, an examination of the location chart in combination with the progress chart will deter-

mine the best method of transfer. Orders to the District Engineer are telephoned and frequently within ½ hour the relief unit is on its way to the new location. The merchant or property owner with a telephone connection along the plow route has proven invaluable in getting in touch with the foreman in the shortest time.

The necessary equipment will vary largely with the location and average climatic conditions. In Southern New England, the maintenance trucks are utilized for most of the snow removal. Each truck of 2 tons capacity or more is fitted with a push plow with a 10-ft. blade, is in charge of the regular foreman, and is assigned to the section which is ordinarily patroled and maintained.

The trucks usually start operating when the snow has reached the depth of 2 to 3 in. and are used continuously until the roads are cleared, sometimes for a period of 36 hours or more. The trunk line roads have first place in the program, being followed by the State Aid or secondary roads.

Each truck carries from two to six laborers, depending on the severity of the storm, and the necessity for shoveling at some particular location. In hilly sections, where more snow can be anticipated, tractors fitted with V-type plows are utilized in addition to the truck equipment. These tractors must be designed for mobile use so that they may be transferred to the location of the greatest depth of snow during any particular storm.

Plow equipment should be properly located for convenience of operation, so that no time need be lost by necessity of distance of travel. Average snow conditions will enter largely into this location.

Familiarity with the section to be kept open is one of the greatest assets in snow removal. During a blinding snowstorm with the temperature near the zero mark, it may be difficult even to locate the highway. The foreman who is in charge of this district throughout the year will often recognize topographical features and thus be enabled to minimize delays.

It is desirable to furnish each District Engineer with one or more additional plows and a supply of parts to be transported by light truck on telephone notification. Where possible, it is also advantageous to keep in reserve one or two trucks in each district to replace damaged equipment. A small reserve of men for emergency use is provided by the foreman.

In Connecticut, a total of 145 trucks are fitted with snow plows to clear about 1 800 miles of highway, an average of 13 miles per truck. This average, however, does not in any way represent the mileage of any particular truck, which is governed wholly by the importance of the road and the conditions in each locality. On the heavily traveled main thoroughfares between large cities a truck may be scheduled for less than 5 miles of plowing, while on the less important highways and connecting State Aid roads, the distance may be 20 miles or more.

The work of each truck is scheduled in accordance with the importance and necessity of traffic. On some sections of the main Boston-New York highway plows are worked in batteries, as in this way mechanical trouble occurring to one truck does not necessarily tie up any particular section of this important highway for a considerable period.

Mobility of equipment is secondary only to personnel. From telephone reports received from the foremen through the District Engineer, to the Maintenance Engineer, information is rapidly developed in case of emergency, so that equipment may be transferred and centered at any particularly congested location.

During the three seasons, 1922-25, the cost of snow removal in the State of Connecticut has been as given in Table 1.

TABLE 1.

Year.	Total cost.	Average cost per mile.	Mileage cleared.
1922-23	$175 021.78	$116.07	1 500
1923-24	53 800.20	30.35	1 800
1924-25	70 453.36	39.75	1 800

During the winter of 1924-25, while the average cost for the State was $39.75 per mile, an analysis shows a cost of $15.42 per mile in a district located in the southeastern part of the State adjoining Long Island Sound, as compared with $95.32 per mile in the northwestern section in which the lower Berkshire Hills furnish more severe weather conditions.

For several years Connecticut has erected snow fences in exposed locations where drifting conditions can be expected. Usually these justify their cost and hence the mileage has been increased each year. The location should be carefully chosen in accordance with prevailing wind direction, otherwise snow may be banked in the highway by a contrary wind and travel blocked at this point. By the removal of brush along the sides of a highway, frequently the tendency for drift forming can be eliminated. Experiments will often determine the solution at minimum expense.

Each Maintenance Engineer is suffering with the complaint of all maintenance engineers—lack of equipment beyond ordinary requirements. A severe snowstorm with an accompanying high wind will develop this complaint almost on the moment into an acute attack requiring immediate operation. This is the danger point. The crisis always passes rapidly. There may remain local difficulties to be taken care of, but during the height of the attack these may be forgotten. Their solution will follow in natural course when traffic is rolling with the least inconvenience over the length of improved highways in the State.

The investment in such highways represents one of the State's greatest business assets. The inconvenience and expense connected with their non-use even for a short period is enormous. Snow removal must be a definite part of each highway program if the maximum value from the investment is to be obtained.

FINISHING AND CURING OF CONCRETE ROADS*

By C. L. McKesson,† Assoc. M. Am. Soc. C. E.

Synopsis

Separate sections are devoted to the finishing and to the curing of concrete roads.

Finishing Concrete.—Modern practice is reviewed with a description of acceptable appliances.

Data are given regarding tests for determining the relative efficiency of finishing machines of two well known types.

The use of the Vialog is recommended as an aid in securing a smooth pavement surface.

Curing Concrete.—The results of recent investigations are set forth in detail, including considerable research data not heretofore published.

The investigations described indicate that the period of watering may be much reduced from present standard practice. The expense of long continued watering constitutes an important item of cost in the construction of concrete roads, and any substantial reduction in the watering period therefore would result in an appreciable saving.

Experiments indicate that the average time during which the pavement is kept closed to traffic may be reduced. This reduction in the curing period would result in a lower cost for maintaining detours and in an even greater advantage to the motoring public through the reduced cost of the operation of vehicles over rough and sometimes lengthy detours.

General

In 1925, the United States Bureau of Public Roads fixed the total mileage of concrete pavements in the United States at approximately 34 000, and each year many miles are being added to this total.

During the period of development of this type of construction, highway engineers have been fully alive to their responsibility in the expenditure of the huge sums of public funds for this work, and many separate agencies comprising the engineering bureaus of Governmental departments, State highway organizations, municipal engineers, colleges, and universities, various technical societies, the Portland Cement Association, and a host of private investigators, have been diligently studying the problems involved in this popular type of construction.

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* Presented at the meeting of the Highway Division, Salt Lake City, Utah, July 9, 1925.

† Materials and Research Engr., State Highway Comm., Sacramento, Calif.

Many of the problems of design and construction have been solved and specification requirements are now quite well standardized, including the use of a rich concrete (1 : 1¼ : 3 to 1 : 2 : 4) in a one-course pavement. Thickened edges and longitudinal center joints have been adopted by many of the States. The time of mixing has been fixed at 1 min. by thirty-four States, the others requiring 1¼ to 1½ min. The grading of aggregate has not been standardized and perhaps will never be, owing to widely differing characteristics of materials in various sections.

The finishing of concrete pavements has been the subject of intensive study by all highway engineers because roughness is so easily discerned both by engineers and by the traveling public. News of innovations in finishing methods and equipment has traveled fast, and as a result methods of finishing and appliances have become fairly well standardized.

Finishing Concrete

Hand-Finishing.—A heavy hand template or strike-board is required by nearly every State. The design of this template is usually specified, likewise the weight as from 11 to 20 lb. per ft. of length. A weight of 15 lb. per lin. ft. is adequate and is not too heavy for manipulation. Such a template should be rigid in construction and made so that it can be adjusted to eliminate sag and to provide the desired crown in the pavement.

A straddle-float for finishing at joints is required by thirty-two States. The latest and most efficient type has a metal frame and renewable or metal-protected boards on the finishing face. Such a float should be about 6 ft. long and approximately 5 in. wide with a handle long enough to permit its operation from the side of the pavement.

Rollers are required by many States. These rollers, 8 to 10 in. in diameter and 4 to 6 ft. long, have been considered useful in removing the surplus water which collects on the surface of the concrete after tamping and floating. The California Highway Commission, by decreasing the quantity of mixing water to the limit consistent with proper finishing, has largely eliminated this surplus water so that the roller is no longer regarded as essential.

Belts have been widely used and have given good results. A ¾-in. hose used in the same manner as a belt has been found satisfactory on some jobs. A flexible finishing board about 12 in. wide with a length equalling the width of the pavement and provided with "plow" handles is used in some States and is believed by them to excel the belt in that it does not cup and sag and in that its movements can be controlled more accurately. It is the writer's belief that the finishing board used transversely tends to preserve the crown better and that it gives a smoother surface than a belt. This same type of finishing board gives excellent results when operated longitudinally instead of transversely, the operators working from bridges. When used longitudinally, the equipment is usually known as the "longitudinal float" and in some cases it is made rigid by the addition of a 2 by 4 or 2 by 6-in. piece nailed to the upper surface.

Straight-edges from 6 to 10 ft. in length are necessary on every project if satisfactory finishing is to be secured. A split straight-edge 6 ft. long is a

valuable instrument for checking up joints. The handle should be light and of sufficient length to enable the user to test an entire joint from one side of the pavement. Such straight-edges, made from aluminum, are now available. They have the merit of lightness and freedom from warping. The State of Washington used such a straight-edge with a short section of projecting "fin" or knife-edge. Whenever the straight-edge is placed on the fresh concrete, the "fin" leaves a slight but unmistakable mark on the pavement and thus creates a permanent record of its use. Inspectors are instructed to test each joint carefully, and the presence of these marks on the pavement furnishes indisputable evidence as to their compliance with instructions. A satisfactory finish is the result of the use of proper appliances by skilled men, and specifications should clearly describe both methods and equipment.

Well-mixed concrete of uniform consistency is necessary for first-class work. Coarse sand and dryer consistencies of concrete somewhat increase the finishing difficulties but complaints from the finishers should not be justification for the use of fine sand or sloppy concrete. A vast amount of research work has shown that fine sand or a wet mix results in inferior strengths and wearing qualities in a pavement. The California Highway Commission has adopted the truncated cone slump test for controlling consistency and specifies coarse sand. A maximum slump of 1 in. for machine finishing has been found practicable. These requirements have not resulted in added cost and they insure a concrete of compressive strength of 3 000 to 5 000 lb. in 28 days and with a satisfactorily smooth surface as evidenced by Vialog readings.

Machine Finishing.—Some of the foregoing discussion of equipment and methods applies equally well to machine finishing. Nine States now require machine finishing; in eleven it is optional with the contractor. One State is reported as prohibiting machine finishing and a number of States regulate the use by special provisions. In mountainous regions some difficulty is experienced in the use of mechanical finishing equipment due to the frequent necessity for widening pavements, on curves and the use of a flat cross-section on superelevated curves. Frequent expansion joints somewhat hamper the operation of mechanical finishers, but on sections having long tangents mechanical finishers are efficient and economical.

In California the question has been raised as to the relative efficiency of the so-called "tamping machines" as compared with "finishing" or "screed" machines not provided with a tamping strike-board. The latter type depends on the transverse movement of the finishing board and the compacting action of the surplus material carried ahead of the blade that serves as a strike-board. The California Highway Commission made a somewhat limited study to ascertain, if possible, the relative efficiency of these two types of equipment and of hand-finishing.

Sections selected on a paving project near Sacramento were finished with the "tamper type", with the "finishing" machines, by hand, and one section by the tamper and finisher used in succession.

Cores were drilled from these sections and the voids in the samples determined. The cores were then tested for compressive strength and finally Vialog readings were taken to determine the relative roughness of the various

sections. The average weight per cubic foot of the concrete was compared with the weight of concrete in 6 by 12-in. cylinders, made according to the standard requirements of the American Society for Testing Materials at the time the concrete was being laid. Table 1 gives the results.

TABLE 1.—COMPARISON OF TESTS ON CORES FINISHED BY DIFFERENT METHODS.

Method of finishing.	Cores.			Weight, 6 by 12-in. cylinder from same station, in pounds per cubic foot.	Relative density, assuming cylinder = 100%.	Average compressive strength of cores, in pounds per square inch.	Compressive strength cylinder from same station, in pounds per square foot.	Strength ratio: cores to test cylinders, percentage.	Vialog record, inches per mile.
	Percentage of voids.	Specific gravity.	Weight per cubic foot.						
Tamping machine	8.82	2.29	142.45	148	96.25	3 645	2 578	142	20.3
Hand-tamped and finished.	9.72	2.28	142.09	146	97.32	3 303	2 593	128	....
Tamping machine and finishing machine	10.81	2.26	140.99	147	95.91	3 347	2 593	129	....
Finishing machine	10.88	2.24	139.24	146	95.37	3 840	2 640	145	21.7

In these tests the tamping type machine had a slight advantage as to the density and smoothness of pavement, and the "finishing" machine had a trifling advantage as to strength ratio. The showing was considered as somewhat favoring the tamping type but the results did not justify the exclusion of the non-tamping type of finisher.

Relative Smoothness Determinations.—The Vialog has been used by the California Highway Commission as a means for determining the relative smoothnesses of various sections of pavement. This is an instrument attached to an automobile to determine the degree of roughness of the road by recording the vertical motion of the automobile body with reference to its front axle. The utmost care is necessary if the results are to have any real significance. The condition and inflation of tires must be uniform at all times. Standardization tests must be made at frequent intervals and skill and uniformity in the operation of the car are essential.

The Vialog, however, provides the only means available for measuring surface irregularities over long stretches of pavement, and in spite of its defects its use has been found to be well worth while.

Vialog readings taken at frequent intervals during construction, stimulate the field forces in their efforts to secure a smooth pavement. Some check is also possible as to the relative efficiency of various finishing methods and equipment. During the 1924 construction season, the Vialog readings indicated surface roughnesses on concrete pavements varying from 8.9 to 30.5 in. per mile. Vialog readings taken during the 1925 construction showed a substantial decrease in roughness, in one instance to as low as 6 in. per mile, with the Vialog operated at the standard speed of 20 miles per hour.

Curing of Concrete

Present Practice.—As has been pointed out the practice with reference to the finishing of concrete has been more or less standardized throughout the United States; but the same cannot be said of curing.

All the State Highway Departments specify covering with canvas or burlap immediately after placing and subsequent sprinkling, to prevent hair-cracks until the pavement is ready for covering with earth or for ponding. In many States, covering with wet earth and ponding are considered as alternatives optional with the contractor, but in some of the States either one or the other method is definitely specified. Calcium chloride curing is now permitted in several of the States.

The wide variations in curing practice are in the length of time which the pavement must be kept wet, the number of days during which it must be kept covered, and the number of days which must elapse before it is opened to traffic. The time of watering varies from 8 to 18 days and the time of opening to traffic, from 13 to 30 days.

Long-continued watering of concrete involves an important item in the cost of construction. Keeping the pavement closed to traffic during the curing period involves the expense of maintenance of detours and quite often inconvenience and increased operating expense to automobile owners. It will be agreed that the watering and curing periods should be reduced to the minimum consistent with safe practice.

Recent Curing Studies.—Since 1920, several elaborate studies have been made for the purpose of determining the relative efficiency of various methods of curing. The State Highway Department of Illinois experimented quite extensively with the use of calcium chloride and other deliquescent salts, and a detailed report covering their investigations was presented by H. F. Clemmer, M. Am. Soc. C. E., and Mr. Fred Burggraf* in 1923.

In 1923, the Structural Materials Research Laboratory of Lewis Institute, Chicago, Ill., conducted a series of curing tests. The California Highway Commission, in co-operation with this Laboratory, likewise conducted an extensive series of curing tests at Sacramento, in 1924.†

The California Highway Commission, in 1924, conducted an independent series of field investigations for the determination of the relative efficiency of wet earth curing and calcium chloride curing.‡ With the results of the previous tests at hand and with the desire of definitely justifying certain tentative conclusions previously reached, the California Highway Commission during 1925 conducted an additional series of experiments.

As the data from all these experiments will be referred to in the following discussion of the curing methods, it seems desirable to include a brief description of the scope of each of these studies.

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* *Proceedings*, Am. Soc. for Testing Materials, Vol. 23, Pt. 2, 1923.

† The results of this series have been covered in a report published as *Bulletin No. 15*, by the Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.

‡ "The Curing Concrete with Calcium Chloride," by C. L. McKesson, Assoc. M. Am. Soc. C. E., *California Highways*, Vol. 2, p. 5, February, 1925; *Public Roads*, Vol. 6, No. 1, March, 1925.

The Illinois series included tests of more than 500 concrete slabs, 8 by 12 by 30 in., subjected to various methods of curing. The co-operative series conducted by the California Highway Commission and the Structural Materials Research Laboratory included flexural strength tests of more than 500 beams, 7 by 10 by 38 in., cured with an earth covering watered for varying periods, with curing paper, and with calcium chloride. This co-operative investigation will, hereinafter, be referred to as the Sacramento Tests. The field tests, series of 1925, by the California Highway Commission, include 105 cores drilled from pavement cured by covering with earth and watering 0, 3, 7, 8, 10, 12, and 14 days and tested at 14, 21, and 28 days. Sixty-four reference specimens consisting of cylinders, 6 by 12 in., were also made from the same concrete in connection with this series and cured in the laboratory in water for 28 days.

Water Curing.—To analyze and correlate the results of the various investigations, some consideration should first be given to the objective to be attained. This objective consists in the provision of sufficient water to carry on hydration during a stated curing period. It is well known that hydration is rapid in the earlier stages of curing and that it proceeds more slowly after the first few weeks. It has been shown by W. K. Hatt,* M. Am. Soc. C. E., and by others, that the presence in concrete of free moisture results in a reduction in strength. It would seem reasonable to assume that the most efficient method of curing would be that which would provide sufficient water for hydration during a given curing period without a surplus remaining as free moisture at the time of testing.

In the research investigations in which the California Highway Commission has been interested, special attention has been given to the moisture content in the concrete at various stages of curing, and it is believed that the assumption just stated has been verified in principle. If the assumption is correct that the quantity of water necessary for hydration varies with the length of the curing period, and that the presence at the time of test of an excess of water in the concrete results in lowered strengths, it must follow that the period of watering should vary with the length of the curing period. All the tests herein discussed indicate that this is true.

The length of the curing period is the time necessary for concrete to develop sufficient strength to permit of its safe use for the purpose for which it was constructed. In the design of concrete for structures, the ultimate compressive strength is frequently assumed to be 2 200 to 2 500 lb. per sq. in. In pavement, stresses and loadings are much more uncertain, but concrete having a compressive strength of 2 000 to 3 000 lb. per sq. in. is usually considered safe for use. The necessary curing period for concrete in pavements or in structures is, therefore, a period of time which, with suitable curing, will enable it to develop these strengths.

Concrete made from satisfactory materials continues to develop strength for years after the termination of the curing period. This increasing strength adds to the factor of safety. It is not necessary, and it may not be possible,

* "Effect of Moisture on Concrete," *Proceedings*, Am. Soc. C. E., May, 1925, Papers and Discussions, p. 757.

to supply water during a curing period in sufficient quantity to carry on hydration for a period of years.

It has been shown* that if hydration in concrete is stopped due to a lack of moisture, it will resume when water is applied even after a long lapse of time, and that thereafter the concrete will increase in strength more or less normally. Concrete in roads will be subjected to occasional rainfall after the curing period ends and moisture for continued hydration will thus become available from time to time. In the curing of concrete roads, therefore, the engineer is most concerned in supplying the water necessary to develop the required strength in the earliest possible time.

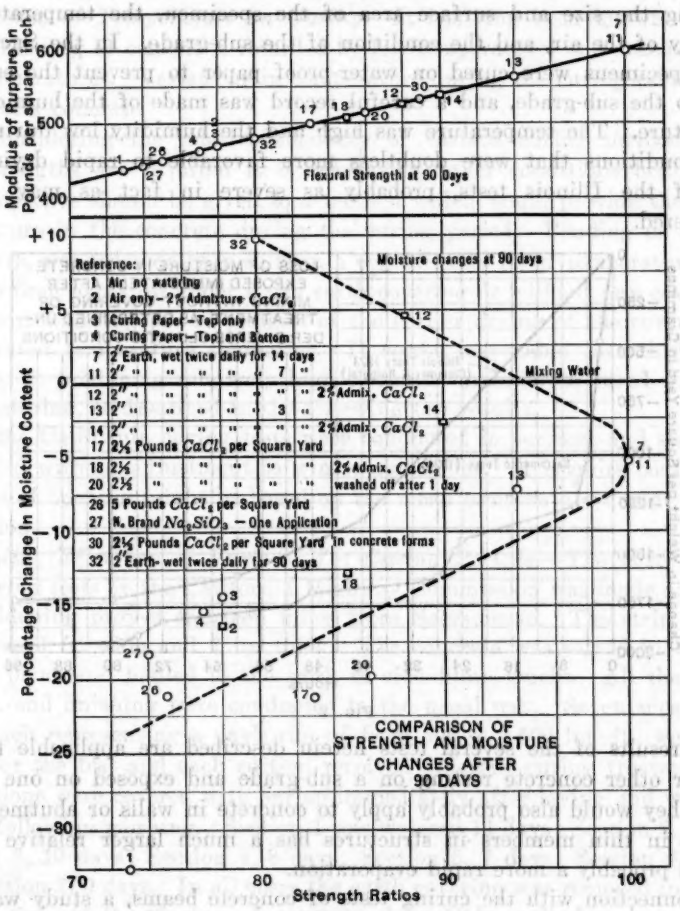
Modulus of Rupture in Pounds per square inch
600
500
400
Flexural Strength at 90 Days
+10
+5
0
−5
−10
−15
−20
−25
−30
Percentage Change In Moisture Content
Moisture changes at 90 days
Mixing Water
Reference:
1 Air no watering
2 Air only – 2% Admixture $CaCl_2$
3 Curing Paper – Top only
4 Curing Paper – Top and Bottom
7 2" Earth, wet twice daily for 14 days
11 2" " " " " 7 "
12 2" " " " " " " 2% Admix. $CaCl_2$
13 2" " " " " 3 " " "
14 2" " " " " " " 2% Admix. $CaCl_2$
17 2½ Pounds $CaCl_2$ per Square Yard
18 2½ " " " " " 2% Admix. $CaCl_2$
20 2½ " " " " " washed off after 1 day
26 5 Pounds $CaCl_2$ per Square Yard
27 N. Brand Na_2SiO_3 – One Application
30 2½ Pounds $CaCl_2$ per Square Yard in concrete forms
32 2" Earth- wet twice daily for 90 days
COMPARISON OF STRENGTH AND MOISTURE CHANGES AFTER 90 DAYS
70
80
90
100
Strength Ratios

FIG. 1.

Quantity of Water for Curing.—In the Sacramento Tests, concrete of normal consistency contained mixing water amounting to about 8% of the combined weight of the cement, sand, and gravel. It was found that the

* "Effect of Age and Condition of Storage on Compressive Strength of Concrete," by H. F. Gonnerman, *Proceedings*, Am. Concrete Inst., 1918.

original mixing water, if conserved, was sufficient to carry hydration through a 90-day curing period. The relative strengths obtained by some of the more important methods of curing are shown in Fig. 1, which also shows the change in moisture content in the concrete measured in terms of original mixing water. It will be seen that concrete which had no watering lost about 32% of its moisture while concrete watered only three days, lost only 7.1% of the original mixing water. This indicates that treatment during the first three days after the placing of concrete has much to do with subsequent strengths.

The rate of loss of moisture is dependent on a number of conditions, including the size and surface area of the specimen, the temperature and humidity of the air, and the condition of the sub-grade. In the Sacramento Tests, specimens were cured on water-proof paper to prevent the escape of water to the sub-grade, and a careful record was made of the humidity and temperature. The temperature was high and the humidity low during these tests—conditions that were doubtless more favorable to rapid drying than those of the Illinois tests, probably as severe in fact as may ever be encountered.

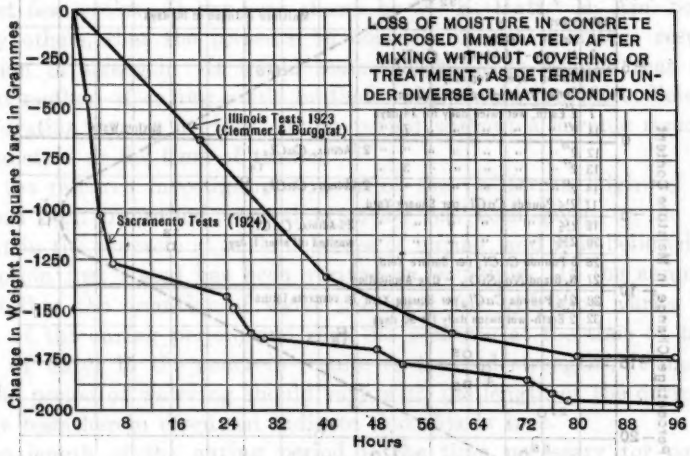

FIG. 2.

The results of the several tests herein described are applicable to pavements or other concrete resting on a sub-grade and exposed on one surface only. They would also probably apply to concrete in walls or abutments, but concrete in thin members in structures has a much larger relative surface area and probably a more rapid evaporation.

In connection with the curing tests of concrete beams, a study was made of the loss of water in concrete placed in pans and exposed in sunshine. A similar test was included in the Illinois series. The results of both tests are shown in Fig. 2.

In both tests, the loss of moisture was rapid during the first three days of exposure. After that, it was much less. As would be expected, the early loss was greater in Sacramento than in Illinois.

In Fig. 1, moisture changes in the concrete during the 90-day curing period are expressed as percentages of original (mixing) water content. It will be seen that watering up to the time of test (90 days) resulted in an increase of moisture over the quantity used in mixing (see Item 32). This method of curing provided free moisture not necessary for hydration and appreciably decreased the strength. These tests indicate that at 90 days there is a well-defined relation between moisture content and flexural strength in concrete. A comparison of the upper and lower curves in Fig. 1 indicates that a substantial increase or decrease in water content (based on mixing water) is reflected by a decrease in strength.

Influence of Temperature on Curing.—It has long been known that the early hardening of cement is retarded by low temperatures and accelerated by high temperatures. Variations in temperature during the curing period similarly affects concrete.

In considering the relative efficiency of various curing methods where early strengths are required it is reasonable, other conditions being equal, that preference should be given to a method which would provide the highest temperature in the concrete during the curing period. Ponding or covering with earth, wet twice daily, results in a considerably lower temperature in the concrete than exposure under a dry earth covering or without any covering.

A short watering period permits of the earlier drying of the covering and of increased temperature thereafter. It seems reasonable that this had its influence in producing relatively high strengths in concrete cured with little or no watering, as described in the following test results.

All the California curing tests were conducted in summer and should not be applied without adjustment to winter conditions. It is quite possible that the time of opening might be based on the mean temperature during the curing period.

Relative Efficiency of Various Wet Curing Periods.—The 1925 series of field curing tests by the California Highway Commission was made on a pavement-widening project fourteen miles from Sacramento. The strip of pavement was 6 ft. wide and 7 in. thick. The concrete was mixed in a central mixing plant and hauled to the work in side-dump trucks. All the mixing, placing, and finishing were conducted in the usual way. Seven separate sections, each representing a day's run of from 500 to 1 500 lin. ft., were designated for the test and each section received different curing treatment. All the sections were covered with burlap immediately after laying and with earth on the following morning. Section 1 was watered 14 days; Section 2, 12 days; Section 3, 10 days; Section 4, 8 days; Section 5, 7 days; Section 6, 3 days; and Section 7, 0 days. In all cases the earth covering was removed in 14 days. At the time of the placing of concrete, 6 by 12-in. cylinders in pairs were taken at five stations in each section. The 10 specimens, representing each section, were cured in water in the laboratory for 27 days and broken on the twenty-eighth day.

One core was drilled at each of the five stations in each section for 14, 21, and 28-day tests. Each age and condition was thus represented in the

test by five cores. Some honeycombed or otherwise defective cores were obtained, but these were rejected and drilling was continued until a full set of apparently perfect cores was secured.

TABLE 2.—SACRAMENTO CURING TESTS (1925) FOR COMPRESSIVE STRENGTHS.

(6 by 12-Inch Moulded Cylinders and 4½ by 7-Inch Drilled Cores. Concrete Mix, 1 : 1.9 : 3.8, Using 6 Sacks of Cement per Cubic Yard.)

Number.	Watering period, in days.	Strengths.						
		14 Days.			21 Days.		28 Days.	
		*28 days, 6 by 12-in. cylinder.	†Cores, in pounds per square inch.	‡Strength ratio.	†Cores, in pounds per square inch.	‡Strength ratio.	†Cores, in pounds per square inch.	‡Strength ratio.
Section 1	14	3 296	3 285	100	3 873	118	4 229	128
" 2	12	3 105	3 468	112	4 103	132	4 414	142
" 3	10	2 930	3 123	107	3 694	126	4 225	144
" 4	8	2 968	3 366	113	3 483	117	3 877	130
" 5	7	3 118	3 073	99	3 596	115	4 101	132
" 6	3	3 356	3 948	118	4 323	129	4 068	121
" 7	0	3 378	3 329	99	3 436	102	3 814	113
Averages	..	3 164	3 370	106	3 787	119	4 104	130

* 6 by 12-in. cylinders made in accordance with A. S. T. M. standards, and cured in water for 28 days. Each strength is an average of ten specimens, except Section 7 in which only four specimens were made.

† Cores drilled with Calyx shot drill. The cores tested were free from visible defects. Each strength is an average of five cores.

‡ Strength ratios are ratios of core strengths to 28-day strength of cylinders from the same section.

Table 2 gives the results of the tests of these cores and Fig. 3 indicates the relative efficiency of various periods of watering. Curing without water gave high strength ratios in the early tests showing that mixing water, conserved to some extent by a covering of dry loose earth, provided nearly enough water to insure proper hydration through the early curing periods. In drilling, it was found that the surface of the concrete cured by this method was noticeably softer than that from sections which had received water treatment. This also verifies the observations* of D. A. Abrams, M. Am. Soc. C. E., and the results of the Sacramento Curing Tests in which it was found that concrete which received no water had only about three-fourths of the surface hardness of similar concrete cured in water even for 3 days.

Watering for three days gave the highest strength in the 14-day tests. This verifies some of the Illinois Tests and almost exactly checks the Sacramento Test of 1924. It is quite apparent that 3-day watering provides all the moisture necessary to carry on hydration through a 14-day, or even a 21-day curing period. At the same time, it permits of a higher temperature in the concrete at an early period in the curing and thus accelerates early hardening.

* "Wear Tests of Concrete," by D. A. Abrams, *Bulletin 10*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.

It does not provide any free moisture to reduce the strength at the time of the test.

Watering for seven and eight days gave satisfactory strengths at all ages. The strengths and strength ratios for these two treatments were about the same except for the 14-day tests. In this average, one of the five cores broke at an unaccountably low stress, thus reducing the average somewhat.

Watering for ten and twelve days gave about the same strengths as the shorter watering periods in 14-day tests, but showed a large gain in the 21 and 28-day tests.

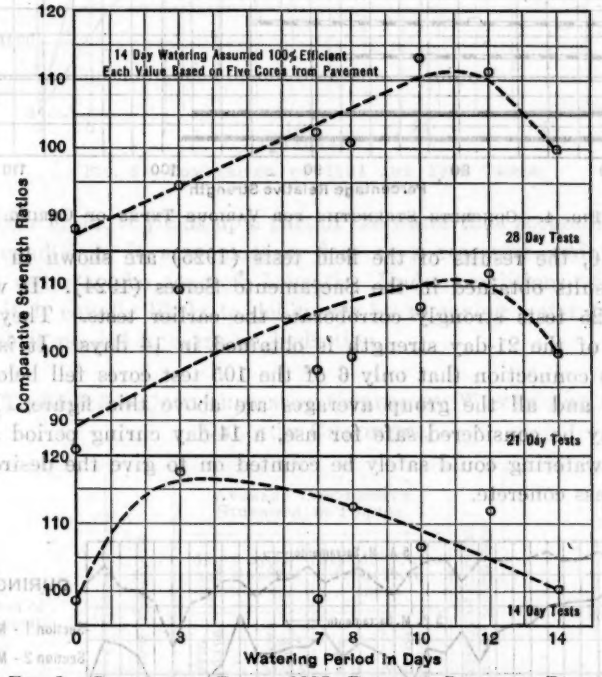

FIG. 3.—SACRAMENTO TESTS, 1925, RELATIVE STRENGTH RATIOS FOR VARIOUS PERIODS OF WATERING.

Watering for fourteen days, heretofore accepted as the standard in California, fell behind other periods of watering as to strength in almost every instance. It made a little better showing than curing without water, but was decidedly superior in surface hardness.

Fig. 4 shows a comparison of the results of this test with those obtained in the Sacramento Tests of 1924. It also shows the results obtained in Series 1 of the Illinois Tests.

Fig. 5 shows the variation of the temperature and humidity during the period covered by the field tests (1925).

It will be seen that the curing period included some very hot dry weather. There was no rainfall after the expiration of the watering periods and only 0.05 in. during the entire period of the test.

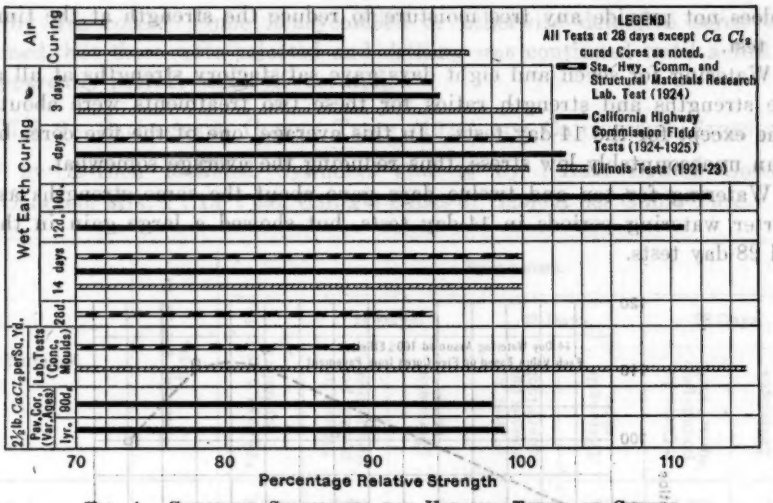

FIG. 4.—CONCRETE STRENGTHS FOR VARIOUS TYPES OF CURING.

In Fig. 6, the results of the field tests (1925) are shown in comparison with the results obtained in the Sacramento Series (1924). It will be seen that the 1925 tests strongly corroborate the earlier tests. They show that about 90% of the 21-day strength is obtained in 14 days. It is worthy of note in this connection that only 6 of the 105 test cores fell below 3 000 lb. in strength and all the group averages are above this figure. If 3 000-lb. concrete may be considered safe for use, a 14-day curing period and 3 days or more of watering could safely be counted on to give the desired strength with first-class concrete.

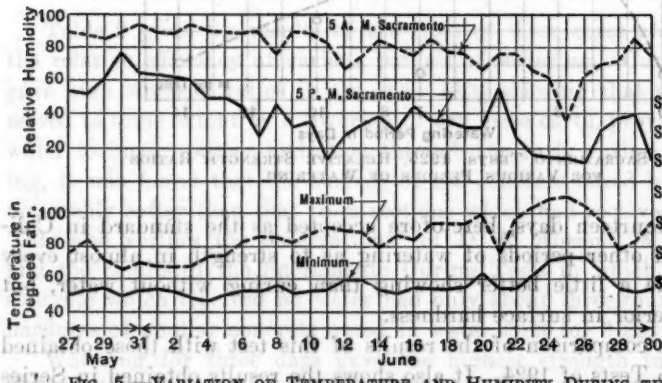
Relative Humidity
5 A. M. Sacramento
5 P. M. Sacramento
Temperature in Degrees Fahr.
Maximum
Minimum
27 29 31 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
May
June
CURING PERIODS
Section 1 - May 27 to June 24
Section 2 - May 28 to June 25
Section 3 - May 29 to June 26
Section 4 - May 30 to June 27
Section 5 - June 1 to June 29
Section 6 - June 2 to June 30
Section 7 - June 3 to July 1

FIG. 5.—VARIATION OF TEMPERATURE AND HUMIDITY DURING THE SACRAMENTO TESTS, 1925.

Calcium Chloride Curing.—A study of the foregoing results and the knowledge that mixing water, if conserved, is sufficient to carry on hydration, even through a 90-day period, throws some light on the subject of the curing of concrete by the application of calcium chloride. Calcium chloride deliquesces readily when the atmosphere has a high relative humidity. At low humidities,

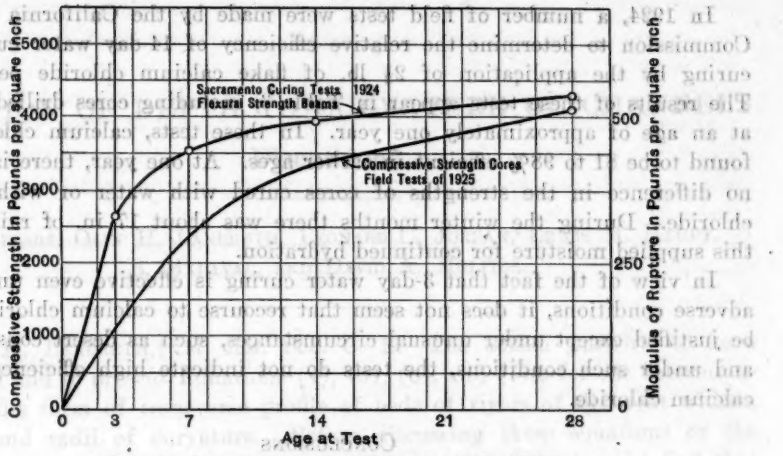

FIG. 6.—COMPARISON OF 1924 AND 1925 TESTS.

probably 40 or lower, it yields up a part of the water thus accumulated. Under favorable conditions it acts more or less as a reservoir and perhaps supplies some water to the concrete beneath. Under conditions of low relative humidity, it is equally likely to deliquesce by removing water from the concrete, which is needed to carry on hydration.

TABLE 3.—COMPARISON OF CURING BY WATER AND BY CALCIUM CHLORIDE.

Location of experiment.	Number of samples.	Average Compressive Strength, in Pounds.				Efficiency of calcium chloride,* percentage.	Remarks.
		3 by 6 by 12-Inch Moulded Specimens.		4½-Inch Cores from Pavement.			
		Cured with water, 28 days.	Cured with calcium chloride, 28 days.	Cured with water.	Cured with calcium chloride.		
Sacto-Stockton Highway, near Sacto, Calif.	24	.....	.....	4 292	4 218	98	Hot and dry. Water-cured concrete tested at 57 days. Concrete cured with calcium chloride tested at 70 days.
	20	.....	.....	5 064	5 042	99	Cores drilled and tested at age of 1 year.
Ventura-Los Angeles Highway, near Calabasas, Calif.	32	3 064	2 783	.....	.....	91	Hot and dry.
	28	.....	.....	4 057	3 294	81	
Los Angeles Highway, near Newbury Park.	12	.....	.....	3 491	3 093	88	Hot and dry.
Eureka-Arcata Highway at Arcata, Calif.	72	3 887	3 545	.....	.....	92	Experiment in October. Weather cool; humidity high (70 to 90 per cent). Some rain.

* Efficiency of calcium chloride curing, assuming water curing to be 100% efficient.

In 1924, a number of field tests were made by the California Highway Commission to determine the relative efficiency of 14-day water curing and curing by the application of 2½ lb. of flake calcium chloride per sq. yd. The results of these tests appear in Table 3, including cores drilled recently at an age of approximately one year. In these tests, calcium chloride was found to be 81 to 98% efficient in earlier ages. At one year, there is little or no difference in the strengths of cores cured with water or with calcium chloride. During the winter months there was about 17 in. of rainfall and this supplied moisture for continued hydration.

In view of the fact that 3-day water curing is effective even under most adverse conditions, it does not seem that recourse to calcium chloride would be justified except under unusual circumstances, such as desert construction; and under such conditions, the tests do not indicate high efficiency for the calcium chloride.

Conclusions

These test data seem to justify the following conclusions:

1.—That 7 days of watering is ample to insure proper hydration in concrete through a curing period of 28 or 90 days.

2.—That this watering period might be reduced to 3 days if the concrete is of first-class quality, thereby effecting a substantial saving.

3.—That, during summer weather, pavement constructed of first-class concrete may safely be opened to traffic in 14 days. (During cold weather, the hardening of concrete is retarded and the time of opening should depend on the actual conditions in each case.)

Acknowledgment

Grateful acknowledgment is made of the assistance rendered to the writer in connection with the planning and conduct of the foregoing tests by R. M. Morton, Assoc. M. Am. Soc. C. E., State Highway Engineer, T. E. Stanton, M. Am. Soc. C. E., Assistant State Highway Engineer, C. S. Pope, M. Am. Soc. C. E., Construction Engineer, all of the California Highway Commission, and of the courtesy of the Structural Materials Research Laboratory of Chicago, Professor Abrams in charge, in permitting reference to the results of the co-operative tests made at Sacramento in 1924 by the California Highway Commission and the Structural Materials Research Laboratory prior to the publication of the report of such tests. H. F. Gonnerman, Associate Engineer, represented the Structural Materials Research Laboratory in the Sacramento Curing Tests in 1924 and in the preparation of the report of that series of tests. The writer takes pleasure in acknowledging his valuable assistance in connection with the curing tests and in the interpretation of the results. F. T. Maddocks, Testing Engineer, and W. B. Stonebraker, Assistant Testing Engineer, of the California Highway Commission, also rendered able assistance in connection with the tests and in the preparation of data for this paper.

RELATION OF DEPTH TO CURVATURE OF CHANNELS

Discussion*

By Messrs. Olin H. Landreth, Leonard C. Jordan, Lewis M. Haupt, B. Malaval, and David A. Molitor.

Olin H. Landreth,† M. Am. Soc. C. E.—The author purports to have developed and to present Equations (4), (5), (6), and (7),‡ for the determination of the form of transverse profile of beds of rivers of different widths, depths, and radii of curvature. Before discussing these equations or the author's conclusions, it appears necessary to draw attention to the fact that water flowing in a channel is not different from other material in that it follows the fundamental laws of mechanics and of motion. If it moves in a straight line at a given linear velocity it possesses kinetic energy which, if disturbed, may be converted into kinetic energy in another direction, into pressure, or into both. If the water is constrained to move in a circular path at the same linear or tangential velocity it possesses the same kinetic energy in the tangential direction and, in addition, it exerts pressure radially outward due to its centrifugal force, which is nothing more than the radial component of its kinetic energy in the curved path.

Although the stream, as a whole, is constrained to follow the curved channel, like a railroad train running around a curve, the individual particles of water do not follow this simple curved path, as the author points out. Years ago a French engineer, by using filaments of colored liquid, found that in channel bends the particles of water on the surface passed to the concave side, descended down the bank, crossed over the bottom, and arose to the surface again on the convex side of the stream, thus, when taken in conjunction with their longitudinal motion, following a helicoidal path. The speaker, more than thirty-five years ago, suspected and experimentally showed this helicoidal motion to exist in a number of curves on the Cumberland River, in Tennessee; he used small blocks of wood, loaded to be of the same density as water, and painted white so as to be visible at considerable depths.

This radially outward flow of water particles in their helicoidal paths causes them to accumulate when they reach the outer or concave bank and thus to raise the surface of the stream to a higher elevation than at the inner bank. In other words, the stream surface has a transverse slope rising from the inner to the outer bank. This difference of elevation of the water surface at the two banks is given quantitatively by the Gockinga formula, the author's

* Discussion on the paper by H. C. Ripley, M. Am. Soc. C. E., continued from March, 1926, *Proceedings*.

† Cons. Engr., New York, N. Y.

‡ *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1909.

Equation (1).* It will be noted that this transverse slope varies directly as the square of the longitudinal velocity of the stream, a relation which later will be seen to be significant.

It is this centrifugal force, and this helicoidal movement of the water particles, which enable a stream flowing in a curved channel, through either alluvial or at least erodable material forming its banks, to erode its outer or concave bank, to increase its channel depth near its outer or concave bank, to transfer the eroded material to the opposite inner or convex bank, and thus as a whole to change its channel location. Centrifugal force varies directly as the square of the linear velocity and inversely as the radius of curvature of the stream.

With the water particles moving in such helicoidal paths, it is easy to see just how the mechanical agencies operate to produce the forms of channel cross-section which are found to exist under such conditions: A particle of erodable bank material, whether humus, silt, sand, gravel, or cobble, on the outer or concave bank of the channel has two forces which tend to erode it from its bank setting and carry it to the bottom. These are (*a*) its natural gravity; and (*b*) the downward component of friction and of impact of the successive water particles which flow past it. These two agencies concur to loosen and to carry particles down to the bottom. On the bottom the radially inward friction and impact of the water particles are usually adequate without the assistance of gravity (which is now ineffective) to transfer the eroded bank material radially (or more properly, diagonally) inward toward, or partly up, the inner bank, and if the velocity component of the water particles in the diagonally inward direction is sufficient to erode the bottom, without the assistance of gravity, the channel will be deepened near the concave or outer bank. When the water particles, however, begin to flow toward the surface, their erosive power, which is now diagonally upward, finds itself in opposition to the downward force of gravity, so that after a certain point is reached not only are the water particles unable to loosen the bank material, but they are generally unable even to transport the loose eroded material far up the ascending path on the inner bank, and as a consequence they deposit the material on that bank, or sometimes before the bank is reached, forming a bar extending diagonally out into or across the river. This causes the transverse slope of that bank to be much flatter and more irregular than the slope of the outer bank which is sometimes nearly vertical and, as a consequence, unstable and subject to slides.

For any given stream, channel, and degree of curvature, the eroding and transferring power of the water particles depends entirely on their velocity in their helicoidal paths. If the stream is a slow, sluggish one, this velocity will be slow and the eroding and transferring power will be slight, no matter what the curvature may be, and as a consequence the maximum depth of the channel will be less than if the velocity were greater. Also the location of the deepest part of the channel will not be so near the outer bank as if the velocity were greater. In other words, not only the eroding power but also the form of the

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1907.

transverse profile is clearly dependent on, and, therefore, a function of, the linear velocity of the flow.

One would expect, therefore, to find in the author's equations, which purport to determine the shape of cross-section, a term to indicate either the linear velocity or the longitudinal slope of the stream. An examination, however, of Equations (4), (5), (6), and (7) will show that nowhere in them does a term indicating the linear velocity or longitudinal slope of the stream appear.

This omission would seem to be a fatal defect, since the equations in question as given would predict the same depths and the same form of transverse cross-section for the channel of a stream, whether it had a flowing velocity of 0.1 ft. per sec., or 10 ft. per sec., which every engineer conversant with river hydraulics knows is not the case. This omission and defect doubtless explains why some of the author's calculated or predicted curves of transverse profile do not approach more closely to the profiles as actually observed, as in Figs. 1(*E*), 2(*B*), 3(*B*), 4(*C*), 5(*C*), and 6(*C*),* none of which would appear to present strong proof of the correctness of the equations from which they were produced.

LEONARD C. JORDAN,† ASSOC. M. AM. SOC. C. E.—Having little direct knowledge of this subject, the speaker has been somewhat amazed to find a mathematical determination of a matter which is usually regarded as indefinite and independent of rules and formulas. The cross-sections of the streams noted in the paper seem to serve as a reliable proof of the dependability of the formula. The matters of grade and velocity have already been mentioned. The speaker fails to see just how they are included in the formula. Likewise, the factor of material through which the stream runs surely is given no consideration in the mathematical prediction of the shape of channel. Is it to be assumed that even in solid rock the stream, if given sufficient time, would wear its channel to the calculated shape?

On the other hand, what about the application of the formula to those sandy bedded streams of the Western prairies? In this case there are at least two conditions to consider. After a flood has subsided the sand is left in a shape which possibly is in agreement with the formula for some definite depth of water, probably for the greatest depth possible without any considerable erosion.

During the violent floods to which these streams are subject there is an entirely different condition, one which probably is no more obedient to any formula than it is to the desires of the engineer who is trying to cross the river with his camp wagons and is wondering what type of bridge and foundations will be required for his new railroad line. When, due to flood conditions, the stream attains sufficient velocity, it whips up the sand of its bed until the mixture of water and sand flows several times as deep as the dry-weather channel. Further flood waters will add greatly to the velocity, erosion, and depth until the channel extends downward to unknown distances.

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, pp. 1924–1925, 1927–1930.

† Cons. and Designing Engr., New Rochelle, N. Y.

An example of such floods is the case of the South Platte River in Denver, Colo., which went on a "rampage," tore out a railroad bridge, and carried away a locomotive. After the waters had subsided to the usual small trickle, the sandy bottom appeared precisely as before, but the locomotive was never seen again. It had gone with the sand and had found a place down in the bottom of the deep flood channel where it could wait until the next flood carried it farther. Residents of Denver vouch for the truth of this story.

These freak conditions are well known to men who have had sad experiences with them. They most certainly throw serious obstacles in the way of a complete and universal application of the formula. The question naturally arises: Can it be possible that the channel conforms to the formula at all stages from the water-and-sand, deep-flood, condition to the shape observed in dry weather?

The author is to be congratulated on the boldness of his delving into the realms of the supposedly unknown factors governing river erosion during floods. His paper is certain to benefit those of the profession who are engaged in the problems of stream control, bridge construction, and dredging.

LEWIS M. HAUPT,* M. AM. SOC. C. E. (by letter).†—This remarkable analysis is of great practical importance to the welfare of humanity and is the climax of time-honored research in the development of commercial channels. It solves the problem of the requisite canalization of streams to secure and maintain ample channels at least cost, without dredging for maintenance.

Hitherto, reliance has been placed mainly on dredging and jetties or revetments regardless of the reactions of the directing banks, as their relation to the depths and erosion were not formulated, whereas by the application of the author's equations, accurate and relatively permanent results may be obtained for securing and maintaining the requisite channels in navigable waters by reaction.

When the many variable properties of water are considered, it would seem impossible to include them in a single algebraic equation applicable to ever-changing conditions. Water is at times a liquid, a solid, or a gas, depending on temperature, and in violation of physical laws, it is lighter when solid than fluid, and in air it moves with high velocities, causing drifts of both rock and earth, thus creating alluvial plains and obstructing channels. Moreover, it is incompressible so that when driven by high winds it is extremely destructive. These properties would seem to render abortive any attempts to reduce its energies to formulas for their utilization; and yet the tests to which the Ripley formulas have been applied under variable conditions indicate their great practical value as aids to navigation as well as to the protection of banks from erosion.

It is also apparent that if the paths of rivers were straight between parallel banks there would be no reaction and the channels would have to be adjusted to the ever-varying volumes from floods and tributaries while the variable quan-

* Cons. Engr., Physical Hydrography, Cynwyd, Pa.
† Received by the Secretary, March 8, 1926.

tities of sediment ejected into such artificial channels, at junctions, would soon create bars and render the stream unnavigable without constant dredging.

The efficiency of the reaction principle in creating automatic channels was recognized by the author when he was in local charge of the construction of jetties in the Texas District. He designed a curved jetty for the Galveston entrance (which was not built) and, later, he contracted for the reaction breakwater at Aransas Pass, which has opened and maintained that channel to date, thus fully demonstrating its utility, as shown in Fig. 2 (*B*).* Similar proposals were made for the bars at the Passes of the Mississippi in 1902 (not built), and for the Rio Grande do Sul, Brazil, in 1887, but more conservative counsels prevailed with disastrous results and great expense (see Fig. 6† (*A*), (*B*), and (*C*), and Fig. 7‡ (*A*) and (*B*), and Fig. 10§ (*A*) and (*B*).

From the citations made in the paper and the results of years of experience at great expense, it is evident that a relatively permanent deep-water channel could have been obtained at these two localities with less than one-half the cost and without dredging; and there would be no advance of the bars seaward, as is demonstrated by the curved jetty at Aransas Pass opened in 1907 and in use ever since, at an inlet which had been abandoned for many years.

It appears that where there is a prevailing littoral drift and large tidal basins it is practicable, by the application of these formulas, to create permanent navigable channels of any reasonable depths at less than one-half the cost for construction, and practically nothing for maintenance. The author is entitled to the highest commendation for this contribution to engineering knowledge.

B. MALAVAL,‖ ESQ. (by letter).¶—The writer, after carefully studying the methods of application of the author's formulas, has tried to verify them by applying them to different rivers, as he considers they will have great value in the regulation of the rivers of France, especially in the estuaries.

For some sections of the Lower Seine at its entrance into the estuary, and also at various places on Rio Grande do Sul, there is approximate agreement between the real and calculated sections. In the estuary of the Seine, a deep channel follows the south shore, independent of the great central channel of the estuary, called the Honfleur Channel.

The concave shore has a radius of 7 000 m. and, without any work except what Nature has done, gives a maximum depth of 11.25 m. at high water of neap tides, whereas that calculated from Equation (7)** gives a maximum depth of 10.86 m. The cross-profile calculated from the author's formula follows almost everywhere the real one. For a point 1 km. away, with the same radius of curvature and the same maximum depth, the calculation gives a depth of 10.90 m., with a cross-profile following the real one and in a remarkable manner.

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* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1925.
† *Loc. cit.*, p. 1930.
‡ *Loc. cit.*, p. 1931.
§ *Loc. cit.*, p. 1934.
‖ *ex*-Director, French Company, Port of Rio Grande do Sul, Brazil, Paris, France.
¶ Received by the Secretary, April 14, 1926.
** *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1909.

The writer is convinced that a project for the improvement of the Seine in which a dike is used to prolong the natural curve of 7 000-m. radius down stream would remove the obstacles to navigation from this side. Still it would be necessary to study the placing of another dike up stream in accordance with the principles that Mr. Ripley has established, in order to create a passage connecting the existing works.

The cross-profiles (Fig. 6* and Fig. 9†), calculated from maps of Rio Grande do Sul of different dates, give also the most satisfactory results; one gives a depth of 12.53 m. instead of 13 m. in a curve of 7 500-m. radius, and a second gives another close approximation to the actual.

The writer is in perfect accord with the author when he states that it is not practicable to secure a permanent improvement in the navigable channel of a river by dredging. At Rio Grande do Sul, all the dredges accomplished absolutely nothing. How could it be otherwise when at times on the crest of the bar, with velocities of 2 m. per sec., erosions of several thousand cubic meters per hour occur?

It would be interesting to know how to avoid the cross-over bar in sinuous parts of rivers. Probably it would be necessary to confine the stream between two dikes for a length at least equal to the width of the river and return finally to the single curved dike.

In straight reaches it suffices to narrow the stream by spur jetties to obtain the given depth by Mr. Ripley's Equation (4)‡ when R = infinity.

The studies made at Rio Grande do Sul and the absence of rules for determining the curve of jetties as a function of the depth to be obtained, emphasize the importance of the formulas which Mr. Ripley has determined, and prove that they will render the greatest service to engineers who are engaged in the improvement of rivers. Mr. Ripley is to be congratulated for his discovery.

DAVID A. MOLITOR,§ M. AM. SOC. C. E. (by letter).‖—The paper discusses a subject about which relatively little is known. Such information as may have been published at various times is not readily accessible.

No very exact results can be expected from formulas of the kind treated by the author, chiefly because they depend on conditions which can scarcely be duplicated in any two rivers. As shown in the paper, they appear to accord with flow conditions in alluvial bottoms, but may be greatly in error when applied to other sand or clay bottoms, and would probably be inapplicable to channels in rock, as for the Niagara and Ottawa Rivers.

Hence any statement as to the percentage of error resulting from the use of the formulas can apply only to the particular examples given, without in any way reflecting a general measure of accuracy to be expected in other untried cases. However, approximate solutions to such problems are all that

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1930.
† *Loc. cit.*, p. 1933.
‡ *Loc. cit.*, p. 1909.
§ Cons. Engr., Detroit, Mich.
‖ Received by the Secretary, April 19, 1926.

can reasonably be expected, so that the formulas may often serve a useful purpose, especially where errors as high as 20% may be unimportant.

The author presents formulas for two conditions of flow. The first formula (Equations (4) and (5))* is Mitchell's formula, Equation (3),† devised for the Delaware River in 1878, in which the author substituted literal dimensions for the numerical values. He then states that the formula has become general in its application to streams of all sizes and characters of channel. One would naturally expect that such a formula necessitates various values for the numerical coefficient, 17.52, to fit the variations in the bottoms of different rivers. Even if this formula does give results according well with observed cross-sections of several large rivers for values of R between $40\sqrt{A}$ and $50\sqrt{A}$, that fact is undoubtedly attributable to a uniformity in the alluvial bottoms of the particular rivers considered. Other results obtained from the use of the formula may not agree with observed cross-sections and some decided modifications of the numerical constant would be required.

The second formula (Equation (6) or Equation (7)),* is the author's modification of Mitchell's Equation (3) made applicable to curved channels occupying only part of a waterway—such channels as may result at high-water stages when the stream spreads out beyond its normal banks sometimes to great distances. Under such conditions the contracted channel becomes the only important subject to be considered. The author then employs the expedient of depressing the water to some lower level approaching the normal width of the stream, and deals with a reduced area of average depth, D_a, and depth at center, $P = 1.65\ D_a$. A new numerical constant, 26.26, is now introduced to fit this condition of a partial flow area.

Here, again, it is to be expected that the two numerical factors, 1.65 and 26.28, may require considerable modifications depending on the amount by which the water level is depressed below the abnormal high level. The same would apply when the bottom material differs from that cited in the paper.

Another expedient might have been employed, by restricting the excessive width to approximately a normal cross-section between elevated banks, retaining the actual high-water level and solving the problem by the use of the previous formulas with $D = 1.445\ D_a$. This would certainly fit the hydraulic conditions better than depressing the high-water level.

The formulas might have been expressed in more workable shape with all limitations and special cases collected together for ready reference. The writer begs leave to submit such a compilation of the author's data with a view of enhancing the value of the paper and assisting the reader.

The fact that such a complex problem with so many variable factors left unconsidered, or possibly covered by numerical coefficients, should be amenable to treatment by any formula of whatever kind, seems quite remarkable. The subject-matter of the paper is, therefore, of considerable value to the profession and the author's efforts deserve hearty appreciation.

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(01) * *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1909.
† *Loc. cit.*, p. 1908.

Streams with Curved Channels Occupying the Entire Widths of Their Waterways (Fig. 11).—

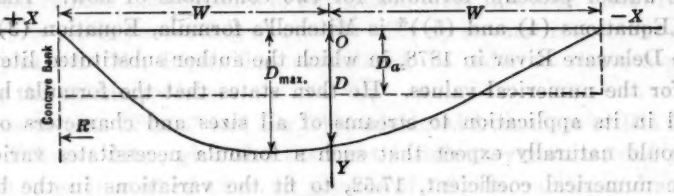

FIG. 11.

Let A = the area of cross-section in square feet.

D_a = the mean depth, in feet = $\frac{A}{2\,w}$.

D = the depth at center of stream, in feet = 1.445 D_a.

R = the radius of curvature of concave bank of stream, in feet.

Mitchell's formula expressed in terms of these dimensions becomes:

$$y = D\left(1 - \frac{x^2}{W^2}\right) + \frac{17.5\,D}{R}\left(1 - \frac{x^2}{W^2}\right)x$$

$$= D\left(1 - \frac{x^2}{W^2}\right)\left(1 + \frac{17.5\,x}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

In this formula, y becomes maximum, or $y = D_{max.}$, when,

$$+ x = \sqrt{\frac{W^2}{3} + \left(\frac{R}{52.56}\right)^2} - \frac{R}{52.56} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

For dimensions, in meters, the constant, 17.5, in Equation (9) becomes 5.34 and the constant, 52.56, in Equation (9*a*) becomes 16.02.

Limitations.—when $R < 40\sqrt{A}$, there will be no further deepening of the channel and hence the minimum $R = 40\sqrt{A}$ will govern. When $R > 50\sqrt{A}$, the channel retains its shape as for a straight channel.

Areas computed by Equation (9) are about 4% too small, and on cross-over bars the maximum depth is actually about 15% less than that given by the formula.

Streams with Curved Channels Occupying Only Parts of Their Waterways; also Channels Alongside Single Curved Jetties (Fig. 12).—

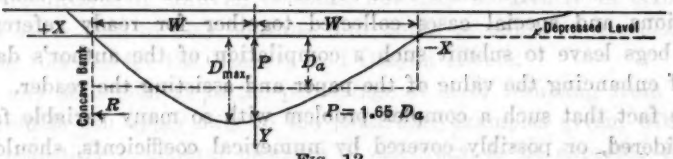

FIG. 12.

$$y = P\left(1 - \frac{x^2}{W^2}\right) + \frac{26.28\,P}{R}\left(1 - \frac{x^2}{W^2}\right)x$$

$$= P\left(1 - \frac{x^2}{W^2}\right)\left(1 + \frac{26.28\,x}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In this formula, y becomes maximum, or $y = D_{max.}$, when,

$$+ x = \sqrt{\frac{W^2}{3} + \left(\frac{R}{78.8}\right)^2} - \frac{R}{78.8} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

For dimensions, in meters, use 8.01 for 26.28 and 24.0 for 78.8.

Limitations.—when $R < 40\sqrt{A}$, there will be no further deepening of the channel. Equations (10) and (10*a*) are good for values of $R > 40\sqrt{A}$ to $R < 110\sqrt{A}$.

The width of the channel at mean depth as given by Equation (10) is 20% too large. The values of A, by Equation (10) are generally 8% in excess of the actual values.

STRESSES IN HELICALLY REINFORCED CONCRETE COLUMNS

Discussion*

By Messrs. J. H. Griffith, John Tucker, Jr., and Jacob Feld.

J. H. Griffith,† M. Am. Soc. C. E. (by letter).‡—The authors deal with the question of the causes of failure in concrete, more particularly in plain and reinforced concrete columns. Is failure in concrete to be attributed primarily to shearing, tensile, or compressive strain? What are the influences of internal friction, helical reinforcing, vertical rodding, etc.? A perusal of the data obtained in past years shows quite conclusively that most failures in plain concrete have obtained under mongrel or ambiguous conditions of strain. The data must accordingly be regarded as possessing apparent rather than rigorous scientific import. The refinements of strict experimental technique usual in scientific testing have been too often neglected for the quite laudable aim of facilitating common sense working conditions. Thus, the real cause of failure has been destined to remain a moot point among engineers.

As a simple example of ambiguous straining the case of the test with the "shear-tool" so commonly used for concrete, timber, and steel, may be cited. The apparent pure shearing failure is, of course, in reality a combination of shearing, compression, and tensile strains of indeterminate amounts. Another type of experiment in the same category is the so-called punching test. Its unique position may be compared to that of the "yellow primrose by the shore" once quoted so aptly by Mr. George B. Grant in "A Treatise on Gear Wheels". It gives the punching strength and nothing more. For, again, the effect of shearing proper is completely masked by a combination of stresses the independent effects of which it is entirely impossible either to calculate or to measure.

Indeterminate data in the case of cylinder crushing are numerous, the ambiguities arising from the presence of end "cappings" of variable thicknesses and degrees of plasticity, unknown axial eccentricities, and the consequent introduction of terminal stress couples, indeterminate radial frictional restraints from the platens of testing machines, sluggish universal bearings, etc. Accordingly, the relatively wide variations found in platting the data must be "camouflaged" by averaging the data in groups on the graph. The

* Discussion on the paper by A. W. Zesiger, M. Am. Soc. C. E., and E. J. Affeldt, Assoc. M. Am. Soc. C. E., continued from May, 1926, *Proceedings*.

† Prof. of Experimental Eng.; Engr. in Chg. of Civ. Eng. Laboratories, Iowa State Coll., Ames, Iowa.

‡ Received by the Secretary, February 26, 1926.

"stream of tendency" is thus narrowed to give the semblance of a statistical "law". Critical observers, however, who have made a comparative study in this field in the last twenty-five years, are aware that these laws are in reality too often mere zones of observations. In view of the many interesting points raised by the authors it would seem desirable that the perennial doubt which is made to encompass this whole subject of concrete test data should be dispelled by the occasional definite attainment of an objective. Why does the material fail and how?

The scientific method for obtaining a law, which is quite generally the practice among investigators, is to hold constant all variables but the one under consideration, to note the variation of the function with respect to this variable, and then to proceed with the others in detail in like manner. Mongrel conditions of testing should be abolished. As an instance, it is a well-known fact of rational dynamics that pure shearing stress under unequivocal conditions of loading may be obtained by twisting a circular shaft of the material. Special cylindrical moulds with slight enlargements of the ends to provide for local effects, and arranged to clamp together in halves, may be used to this end as easily as in the case of the compression tests. The few twisting strains conducted in this manner by Mörsch give definite helicoidal fractures, such as those obtained for cast iron. Little is known of stress-strain data for concrete in tension, and yet they are as easily determined as those for cement briquettes, steel, or earth specimens. As regards compression data, it was the writer's privilege to witness a number of the column tests of Wrentmore, Brodie, and Carey, cited by the authors.* These authorities used cast-iron "cups" filled with sand and caulked with waste to "secure a full even bearing on the sand." The restrained sand bearing is still used with good results for tests of sewer pipe under conditions simulating those existing in Nature.

Perhaps the most important testing device for ascertaining determinate straining in compression tests of concrete is the hydraulic bearing designed by Schuyler† in 1913, which still remains practically unnoticed by professional committees. It possesses the signal merit of being able to test a cylinder under absolutely pure compression with the elimination of the unknown restraints, end couples, etc., which, in a large measure, cause the variations from the means. A few hundred precise data of this kind will easily outweigh in value the many thousand of the so-called "practical" or "rough-and-ready" results obtained under ambiguous conditions of straining, but from which the definite information is relatively small. Having precise data under pure conditions of straining it will be an easy matter of course to proceed to definite determinations of failure under combined strains. For example, it is not a difficult problem to test in compression a concrete cylinder already exposed to a twisting couple. Hydraulic bearings with suitable ball races will suffice for applying the compressions. The methods of combining stresses given in dynamics are relatively much simpler than those given in engineering technology.

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 14.
† *Proceedings*, Am. Soc. for Testing Materials, 1913, p 1004, Fig. 14.

The pure compression test of Schuyler deserves more than passing comment. In the first place, the so-called Coulomb type of shear failure cited for so many years in textbooks disappears altogether, as do those of the Navier and Bouton type. The specimen fails in pure tension* in vertical planes through the axis usually 120° apart—for concrete having a tensile strength of only about one-tenth and one-half, respectively, of the compression and shearing values, it must fail in tension unless prevented from doing so by the methods adopted by investigators. Strictly speaking, one may usually call the compression strength of many materials infinity for most practical purposes as it is conditioned always, as in the case of concrete, by extraneous failures and not by compression proper. The apparent failure of timber in compression, for example, depends on the strength of the tracheid fibers; again, in a recent paper one author speaks of concrete sustaining 45 000 lb. per sq. in. under stresses from all sides by hydrostatic pressure, and it is doubtless capable of sustaining much higher pressures than this when suitably supported in the tests.

It has been known for many years, since the days of the scientists, Sedgwick, Lyell, and Sorby, that strain induces æolotropy of structure in tangible matter. The fact has already been commented on by Professor A. E. H. Love, the English elastician. The forces set up are akin to crystallization. The essential facts have been presented in an illuminating manner by the Irish scientist, John Tyndall, in a lecture before the Royal Institution of Great Britain, on June 6, 1856. Tyndall† remarks in connection with the discussion of a variety of other most interesting phenomena:

"There is no such thing in nature as a body of perfectly homogeneous structure. I break this clay which seems so uniform, and find that the fracture presents to my eyes innumerable surfaces along which it has given away, and it has yielded along those surfaces because in them the cohesion of the mass is less than elsewhere. I break this marble, and even this wax, and observe the same result; look at the mud at the bottom of a dried pond; look at some of the ungraveled walks in Kensington Gardens on drying after rain —they are cracked and split, and other circumstances being equal, they crack and split where the cohesion is a minimum. Take then a mass of partially consolidated mud. Such a mass is divided and subdivided by interior surfaces along which the cohesion is comparatively small. Penetrate the mass in idea, and you will see it is composed of numberless irregular polyhedra bounded by surfaces of weak cohesion. Imagine such a mass subjected to pressure—

* The writer has attributed these fractures on planes through the axis to failures in cohesion following the authority of John Tyndall as discussed later. That this is a gratuitous assumption, however, may be urged by a discerning mathematician; for, in dynamics, any stress distribution acting on a mechanical system must satisfy the conditions of rigid body equilibrium in accordance with the provisions of Newton's or d'Alembert's equations (see Love's "Theory of Elasticity," Second Edition, p. 166). Now, as the cylindrical face of the concrete specimen in a testing machine is free from stress, it is natural for the critical observer to raise the question (as did the writer) regarding the anomaly as to what balances the internal pull on the cylinder, assuming the stress distribution to be all tension and not a mutually balanced internal system?

In dynamics the radial and circumferential stresses on the sectors into which the cylinder breaks are rigorously zero. The writer can only state in the light of his own knowledge and experience that he believes this to be a legitimate field for the application of the hypothesis of "true stresses" of the late Mansfield Merriman, M. Am. Soc. C. E., and Prof. A. E. H. Love considers failure in tangible matter still to be a moot point. (See "Theory of Elasticity," Second Edition, p. 117). The hypotheses are: (*a*) That failure occurs under maximum stress; (*b*) under maximum shear; (*c*) under the difference of maximum and minimum principal stresses; and (*d*) under maximum strain.

† Fragments of Science, Chapter XII, "On Crystalline and Slaty Cleavage," pp. 241–243.

it yields and spreads out in the direction of least resistance; the little polyhedra become converted into laminæ, separated from each other by surfaces of weak cohesion, and the infallible result will be a tendency to cleave at right angles to the line of pressure."

He remarks further:

"Expanding our field of view, we find the selfsame law * * * extending into the domain of the pastry cook and iron founder; nay, a wheel cannot roll over the half-dried mud of our streets without revealing to us more or less of the features of this law."

The phenomena cited by Tyndall have been observed in a number of instances in research at Iowa State College. In the case of compression tests of virgin earth cylinders it has been possible to obtain a complete vertical network of cohesive failures—of which the Schuyler type of fracture may be regarded an elementary case, as perfect as those found in Iowa in depressions of the soil after water has evaporated and similar to those cited by Tyndall. In fact, if it were not known that these networks of fractures were produced by compression strains, they easily could be mistaken for the phenomena experienced in shrinkage of soils during the heat of summer. The writer has been informed by Dr. O. R. Sweeney, Professor of Chemical Engineering, who has studied the same class of phenomena, that this typically prismatic structure is observable in the product of the coke oven. On the other hand, in nature, the geologists, N. S. Shaler and J. B. Woodworth, have given an admirable illustration of the prismatic arrangement encountered in coke strata overlying the carboniferous beds of the Richmond, Va., Basin, north of Gayton on the James River.*

In the case of hollow building tile with cells running vertically the usual angular or shearing failures occur when the tests are conducted with plaster of Paris caps on the specimens. However, when the ends are ground smooth with a surfacing machine to realize partly the freedom from radical constraint of a hydraulic bearing, values as high as 12 000 lb. per sq. in. have been obtained by the writer. The specimens break simultaneously at the cell fillets into vertical "slats" without those "warnings" of early partial fractures which are so common in the tests with the plaster of Paris caps. The compression failures on soft iron and steel cited by Tetmajer clearly show the presence of vertical cracks. Rock cylinders tested by impact with the Page drop test give the same type of fractures as found by Schuyler for concrete. In the Michaelis "ball test", the balls of cement dropped on a slab from a given height to determine the relative consistency as a result of partial hydration of the colloidal particles show well-defined vertical cracks when they lack the requisite plasticity. In the case of brick pier tests, on the other hand, the influence of frictional constraint is in evidence as far as the quarter-points of the specimen. The exterior walls are "spalled off" by the wedging action of the pyramids in the same manner as the extraneous matter of a leaden bullet or a snowball slides off the cone of material which adheres to a wall or target. The cone must be caused by radial constraint at the platens. When there is no cone, there has been radial slippage on the platens; the

* 19th Annual Report, U. S. Geological Survey, 1899, pp. 498–499, Plate XIV.

material is free to expand circumferentially 2π times the radial strain until the Schuyler fracture occurs, a shear or compression fracture not being caused on account of the much greater resistances against these types of strain, except under conditions of platen supports.

In the case of helical reinforcement when present, as when absent, the question of flexure being neglected, there are two general types of matter to consider—the molar or continuous matter predicated in textbooks, and the actual granular material of the concrete. In the former case the construction of the helix regarded as a grating or series of rings—unless the winding cable effect is introduced to complicate vastly the analysis—will cause a radial constraint of h, using the authors' notation. This stress, h, may be obtained from the authors' rigorous dynamical Equation (16):*

$$e_v = \frac{v}{E_c} - \frac{2\,h}{m\,E_c} = \frac{1}{E_c}\left(v - \frac{2}{m}\,h\right)$$

in which, e_v = the uniform axial strain, v = the axial pressure, E_c = the modulus of the concrete, h = the lateral pressure, and m = the reciprocal of Poisson's ratio. It would be expected from theoretical considerations that such a column, if absolutely concentrically loaded, would fail in tension as in the previous cases after the yield point of the reinforcement had been passed, for the resistance to expansion offered by the platens tends to dissipate itself with the distance from the ends. As it is the rule, however, that most square-ended columns have inherent eccentricities, it will be safer perhaps to assume in practice that the failure is in shear, in lieu of any definite analytical data as to the strain measurements necessary in making an analytical decision. By resolution the relation between the stresses, h and v, is found to be:

$$h = \frac{\tan^2\theta}{\tan(\theta \pm \phi)}\,v$$

in which, θ is the angle between the plane of the stress, h, and the inclined interface on which slippage takes place, and ϕ is the obliquity of the resultant stress on this plane or angle of friction. The extrema, or maximum and minimum values of h, are found by platting or by taking the derivative to be, respectively, multiples of $\theta_1 = \frac{\pi}{4} \pm \frac{\phi}{2}$ and $\theta_2 = \frac{3}{4}\,\pi \pm \frac{\phi}{2}$, which is in agreement with the authors' Equation (24)† and Rankine's theory of earth pressure. The analysis from this point evidently will be similar to that of the authors or equivalent to it. They state† in this connection that "when f_s represents the stress in the steel as it reaches its yield point, failure of the columns will not occur even if the concrete within the helix is crushed by the imposed load." Therefore, it is natural again to ask, are not "tension" failures of the Tyndall or Schuyler type already present even before the shear failures occur? Indeed the records of tests show that frictional dilatancy alone is enormous. A porous sheet-steel, No. 16 gauge, container, 12 in. in diameter by 6 ft. long, filled merely with crushed rock, without cement, will carry

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 11.

† *Loc. cit.*, p. 13.

125 000 lb.* A tracing cloth container, 4 in. in diameter by 1 ft. 2 in., filled with pebbles, will carry about 2 cu. ft. of cast iron. The helix acts like the tracing cloth so that a column, even if it has become more or less fractured by tension or cleavage, has a considerable inherent compressive strength until the yield point of the helix is reached.

It is natural to raise the question of the uni-planar angular fractures of the type cited by Bouton†—why do these sometimes occur instead of the pyramidal or conical fractures? The methods of elementary analysis ordinarily used cannot discriminate here; for it is tacitly assumed that either type of failure may occur in the analysis. The interpretation of homogeneous stress is thus framed *ad hoc* to suit the case. The writer prefers to think of the Bouton failure as sporadic—a mere accidental failure, as one might say. To deal with such freak phenomena it is necessary to part with the conception of the molar matter theory and think of the concrete in its true status as inherently granular. It starts to fracture locally on some "plane of limited stability" in the language of Osborne Reynolds; and when a crack starts in nature it seems to possess almost human volition and persistence. A slight strain developed in matter causes an "infinity" of stress at the "point of divellence" or re-entrant angle of the crack. A crack in a plate girder, for instance, will tend to seek out the holes and re-entrant points within and on the edge of the plates. The writer has noted on concrete roadways on numerous occasions that a crack in a concrete roadbed passes into the adjacent earth beds beyond. Sir John Herschel‡ has made the apropos remark that,

"A crack once produced has a tendency to run for this plain reason, that at its momentary limit at the point at which it has just arrived, the divellent force on the molecules there situated is counteracted only by half of the cohesive force which acted when there was no crack, *viz.*, the cohesion of the uncracked force alone."

In the same manner it is entirely logical to suppose that the crack latent or incipient at a cleavage surface or plane of limited stability between the matrix and pebble is gradually developed under the strain by the tendency of the crack at the point of divellence to propagate itself by continuance of minute tears until the structure fails *en masse*.

In conclusion, the writer wishes to emphasize the need of distinguishing between the several types of fracture which have been discussed. He believes the phenomena of failure may be given other interpretations than those now current with technologists. The authors' paper is timely in that it raises many important points which should be considered. They are to be congratulated on their consistent mathematical treatment of a subject which needs careful investigation.

JOHN TUCKER, JR.,§ ESQ. (by letter).‖—The attack on the problem by a "rational analysis" is to be highly commended. Empirical solutions of en-

* *Proceedings*, Am. Soc. C. E., August, 1920, Papers and Discussions, p. 934.
† Johnson's "Materials," Fifth Edition, p. 118, Fig. 12.
‡ *Proceedings*, Royal Soc., Vol. XII, p. 678, quoted from Tyndall.
§ Pittsburgh, Pa.
‖ Received by the Secretary, March 26, 1926.

gineering problems are only to be tolerated until they can be placed on a rational theoretical basis of fundamental physical concepts and laws. Consistency demands a "rational analysis" to determine safe working stresses or factors of safety. The authors, arbitrarily selecting a factor of safety, do not utilize the rational method which has already been developed.* Their Equation (23)† gives promise of value as an aid in the analysis of the spirally reinforced column, but before the equation may be utilized, the constants, m and E_c, must be known, beyond the ultimate strain of concrete, and can not be "assumed * * * probable values." The other end equations, (21)† and (27),‡ are not valid, nor is the conclusion correct that the stress necessary to cause failure in the extreme column fiber is greater than the mean ultimate stress.

The authors have cited no proof, experimental or otherwise, that spiral§ reinforcement "tends to neutralize the effects of local imperfections in the concrete." Although the spiral "tends to neutralize the effects of local imperfections" yet "tests indicated that failure was due to flexure, but this was due, no doubt, to some local defect."

The attempted proof that "the unit compressive stress which will cause failure by shearing out a wedge in the extreme fiber [of a column] must be at least 1.75 times the unit compressive stress which will cause failure by shearing the entire section of the column," is erroneous. No hypothesis was offered limiting the size of the wedge, and, therefore, the analysis would be valid for a wedge the thickness of which was equal to the column diameter, with results contradictory to those obtained from Fig. 1.‖ This in itself is proof of error. Moreover, provided the proof presented were valid, it would have to be shown that the extreme column fiber could not fail in a manner requiring less stress than that required to shear out the wedge; for example, by the same type of failure that characterizes the failure of the column as a whole, a rupture in a plane or planes inclined to the column axis.

The analysis of the forces acting near the column surface may be simplified by considering one-half the small wedge shown in Fig. 2.¶ The plane, $a\,b$ (Fig. 6), perpendicular to the paper is tangent to the column surface along an element; $c\,b$ is perpendicular to the paper and to $a\,b$. The shear along $c\,b$ is zero, since it is perpendicular to a free boundary surface, $a\,b$, along which no shear exists. By the same reasoning, the shear on the plane parallel to the paper is zero. The summation of the horizontal forces gives (identically with the authors' analysis of Fig. 1):

$$f_s = f_c\,(\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi)$$

The compressive stress required to cause failure by shearing out a small wedge is the same as that required to cause a shearing rupture of the extreme fiber

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1127.

† *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 12.

‡ *Loc. cit.*, p. 13.

§ When such a precise and scholarly authority as A. E. H. Love uses the term "spiral," it may be accepted as correct, see "Mathematical Theory of Elasticity", Second Edition, p. 396.

‖ *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 6.

¶ *Loc. cit.*, p. 8.

in one plane and is the same as the compressive stress necessary to cause failure in the interior of the column.

The Synopsis* contains the conclusion: "(1) Plain concrete columns * * * fail in shear rather than in compression," and the authors also state† that "the cylinder * * * will fail in shear alone." Pure shear cannot be produced in a material by compression. It is true that concrete fails by separation and sliding of the new surfaces along a plane, or surface, such as that of a cone, the common form observed in concrete test cylinders. This plane of failure is not a plane of pure shear, but has compression forces acting across it as well as the shearing stresses. The authors' intent in using "failure in compression", evidently a form of failure in distinction to "shear failure," is obscure. "Crushing," the other method of failure mentioned, may be the result of several causes.

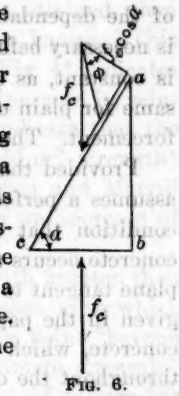

FIG. 6.

Compression failures may be classified as (*a*) plastic, exhibited in lead, short mild steel specimens, etc., a long specimen of the material failing by buckling (as a structural steel column); and (*b*) sliding or a shearing of the material, as in concrete, cast iron, terra cotta, etc. The so-called "splitting" failure is a secondary result of the action of wedges of the material originally produced by a shear failure, splitting the material between the wedge and the other surface producing the compression. Crushing may be a secondary effect of the shear failure in materials of low strength, the material being unable to remain uninjured beyond the point of original shearing, when the two surfaces are further compressed. The phenomena may also be due to a weakness or defect in the material throughout a volume of considerable magnitude. Thus, if a stratum of weaker material be included between two stronger strata, increase in stress will cause the "shearing" failure of the weaker material. If the stratum is not sufficiently thick to permit the two parts of the material to slide past one another, or if they are kept in the same axis, continued compression will break down the vertices of the shear cones originally produced, resulting in a series of additional "shearing" failures, until the weaker material is broken into small pieces, or "crushed." The phenomenon of "crushing" occurs in "lean" concrete, the failure being due to the wedge action of the large aggregate on the much weaker part of the concrete. No material is of absolutely uniform strength throughout. As the stress is increased, the concrete will rupture over small areas where the material is weakest, the areas of rupture increasing in number and magnitude with increasing stress. The greater the variation in strength of the material, the more pronounced and scattered will this network of ruptured areas be, and if sufficiently so, the result may possibly be the so-called crushing failure.

Great care must be exercised in the application of the friction concept. To one familiar with the irregular form of broken specimens of concrete, and also in comparison with the selection by the authors of values of "about

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 3.
† *Loc. cit.*, p. 6.

45°" and "about 50°", a value for α of 76° 43′ is absurdly accurate. Sound proof of the unquestionable experimental determination of ϕ, with the measure of the dependability of the result expressed by means of the probable error, is necessary before acceptance. It must be demonstrated that the friction angle is constant, as assumed in differentiating Equation (12),* and that it is the same for plain concrete and for identical concrete laterally restrained by reinforcement. The constancy of ϕ is at present purely hypothetical.

Provided that the characteristics of ϕ be known, the analysis in the paper assumes a perfectly homogeneous material of uniform strength throughout, a condition that concrete does not even loosely approximate. The failure of concrete occurs along an irregular surface, the angle between the axis and the plane tangent to the surface varying greatly from point to point. The analysis given in the paper, therefore, cannot hold, for the "shearing" strength of the concrete, which has been considered equal in magnitude to f_z, is a variable throughout the concrete.

The second conclusion, that "the imposed load must be sufficient to overcome the shearing resistance of the concrete and the frictional resistance of the material to incipient motion along the impending plane of rupture", is not a conclusion, nor a unique and sole possible interpretation of pertinent test phenomena, but is a hypothesis on which to build the analysis of column action. "The frictional resistance of the material is a more important factor than is commonly assumed"—in fact, it is an absolutely indispensable factor if the friction concept is used as the working hypothesis. If the hypothesis is not used, the frictional resistance is not used and is of no importance whatever. One weakness of the frictional hypothesis is in that the equations developed upon it are single-valued, and valid only at the point of failure of the material, whereas the ordinary stress-strain relations are valid for all stresses up to and including the maximum or ultimate.

The conclusion that "helical reinforcement is effective in resisting a shearing failure of the concrete core and, if sufficient reinforcement is used, it is possible to preclude a shearing failure of the concrete," is an apparent contradiction to the authors' statement, to the effect that for more than 1.6% reinforcement "the column will not fail, even after the concrete has crushed, until the helical reinforcement has been stressed considerably beyond its elastic limit," for any "crushing" of the concrete is a compound shearing. The columns, *T* and *U*, with 1% spiral, are experimental evidence that spiral reinforcement in itself, for one example at least, is impotent in preventing shear failure of the concrete, the column failure (maximum load) occurring at the approximate ultimate stress and strain of the plain concrete. (The theoretical column strength is to be based on the strength of the concrete in the test cylinders plus the strength added by the rods at 36 500 lb. per sq. in.)†

That point at which the integral state of the concrete is destroyed, either by a shear rupture, "crushing", or whatever method, marks the limiting strength up to which the column serves a useful purpose. This strength, pur-

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 10.

† *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1099, or by methods used by the authors.

ported to be given by Equation (27), is, as stated, of "academic interest only." If, however, as claimed by the authors, concrete reinforced with more than 1.6% spiral, crushes before the maximum column strength is attained, then it is imperative that the strength at which crushing occurs be known. This latter strength, supposedly given by Equation (21), plotted in Fig. 4* indicates more than a proportionate increase in column strength with increase in reinforcement. Contrary to this, the majority of column tests indicate that the maximum strength (which would be greater than the "crushing" strength) is proportional to the percentage of spiral, while the McKibben and Merrill tests indicate only a small increase in column strength with an increase of reinforcement beyond 0.5 per cent.

The statement of the mechanics of failure of a column with greater than 1.6% spiral reinforcement, is not clear: First, the concrete crushes, after which the "failure" occurs, when the reinforcement is stressed to some point "beyond its elastic limit and possibly, in some cases, almost to its ultimate strength." In exactly what phenomena the "failure" consists, is obscure. If failure has been used to connote the maximum column load sustained, the statement is then correct; the maximum load is sustained, however, at or shortly after the core concrete has begun its complete disintegration by "shear" or "crushing."

Experimental data fail to demonstrate that more than 1.6% of spiral reinforcement precludes a shearing failure. From the original source of the authors' contention, it appears that "the tests indicate, but can not be said to prove conclusively that * * * above 1.58% the ultimate strength is simply the amount which can be carried by the granular core supported by the spiral, with no cohesion of its own." This conclusion is mainly based on a unique interpretation of the plotted column strength test results. These points are represented by a most irregular empirical curve, whereas a straight line would do equally well, if not better. The test specimens, miniature columns with core diameters of about 3½ in., are poor means of obtaining information compared with the several columns 20 in. in diameter of the American Concrete Institute tests,† and the tests of columns 14 in. in diameter by Messrs. McKibben and Merrill,‡ neither of which have been included in the test data of Table 1.§ Professor Talbot states‖ that:

"The maximum load in some cases may not be reached until the hooping has been stressed to its elastic limit, and in other cases this load may be reached without developing the elastic limit of the hooping, especially when large lateral deflections of the column and general distortion from the general shape occur."

In the group thus referred to are five columns with more than 1.58% spiral reinforcement. This is a contradiction of the authors' statements.¶ Neither Professors Withey and McKibben nor Mr. Merrill have remarked on any

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 18.
† *Proceedings*, Am. Concrete Inst., February, 1916.
‡ *Loc. cit.*, 1916, p. 200.
§ *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 21, *et seq.*
‖ *Bulletin No. 20*, Eng. Experiment Station, Univ. of Illinois.
¶ *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, pp. 9 and 19.

special phenomena occurring in columns with more than 1.58% spiral reinforcement.

The strain history of the laterally reinforced column has not been given. This is essential in the analysis of the column, for maximum column strength is attained at a large strain, many times the ultimate strain of unreinforced concrete (approximately 0.0015 in. per in.). Experimental proof has been cited* that:

1.—For a 0.7% spiral reinforced concrete, strained to at least 0.006 in. per lin. in., the core concrete, except at the point in the column in which the failure occurred, was practically uninjured, and required severe blows from a heavy maul to break it.

2.—A spiral reinforced column, approximately of 80 kg. per sq. cm. unreinforced strength, was stressed to 666.7 kg. per sq. cm. The spiral was then stripped for a length equal to three diameters, and broken under a load of 61.2 kg. per square cm.

3.—Four columns (*T*-1, *T*-2, *U*-1, *U*-2), with 1% spiral and 6.11% rods, failed to have their strength appreciably increased by the spiral reinforcement, and the ultimate strain was not appreciably different from that of the unreinforced concrete.

Therefore, since concrete that has its strength increased by reinforcement may be strained unbroken and integral far beyond the unreinforced ultimate strain, and since the ultimate column strength occurs at the ultimate strain of the unreinforced concrete, in the one case where the concrete strength is not appreciably increased by the spiral, it is apparent that the maximum strength of the column is developed when the core fails by a shear rupture, or crumbling. A column reinforced with more than 1.58% spiral will have an ultimate strain much beyond the ultimate strain of the unreinforced concrete, and the core will remain sound (that is, unbroken and integral) much beyond the corresponding strain of the unreinforced concrete. It is absurd to consider that the addition of more spiral reinforcement will cause the concrete to fail by shear or crushing at a smaller strain; that would be contrary to the conclusions to be drawn from Equation (12), for f_z decreases with an increase in h.

Equation (23) gives with close approximation† the relation between the stresses in plain and reinforced concrete for equal strains, (e_v). This, of course, becomes meaningless for stresses in plain concrete beyond the ultimate stress. If, however, $E_c\ e_v$ is substituted for v_c in the equation, then for any value of e_v (the longitudinal strain in the spirally reinforced column), and E_c (the modulus of elasticity at that strain) v will be the compressive stress in the column. The value of e_v increases far beyond the ultimate strain of plain concrete, and it has been shown‡ that the product, $e_v\ E_c$, increases greatly beyond its value at the ultimate strain of plain concrete. The authors' interpretation of the equation as giving the "crushing" strength of spirally

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1227.
† E_c is not constant.
‡ *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1103, *et seq.*

reinforced concrete, in terms of the ultimate strength of plain concrete, is plainly in error, the interpretation connoting the "crushing" of the concrete at the ultimate strain of the unreinforced material, a phenomenon that holds only for concrete of high ultimate strength. In the form given in the paper, the equation is valid for one value of the variables only, and indicates the stress in a reinforced column which is compressed to the ultimate strain of the unreinforced concrete. No information of the ultimate strength of the reinforced concrete can be gotten from the equation.

Equation (27) does not give the relation between spiral stress and longitudinal column stress for a column with a crushed core. Moreover, because of the inconsistency of the several operations in developing, the equation is devoid of physical significance. A maximum and a minimum value cannot occur at the same point, as the authors seem to believe.

The desired equation may be developed thus: For the plane in the crushed mass in which equilibrium obtains, the forces tending to cause sliding are equal to the resisting forces of friction. This is expressed by Equation (12) when f_z is placed equal to zero, for the equation expresses, by f_z, the difference between the tangential components and the friction force. The equation reduces to:

$$(v - h) \tan \alpha - v \mu - \mu h \tan^2 \alpha = 0$$

in which,

$$\mu = \tan \phi.$$

Solving the equation as a quadratic:

$$\alpha = \tan^{-1} \left(\frac{v - h \pm \sqrt{(v - h)^2 - 4 h \mu^2}}{2 \mu h} \right)$$

In this case, α is not independent of the ratio of vertical to horizontal column stresses.

Equation (21) is apparently correct, provided ϕ has the same value for reinforced concrete as for unreinforced concrete. The equation of the curve well represents the 268 to 440-day observations. The straight line representative of these points indicates an increase in strength of 0.56 p, a close agreement with the value which has been empirically determined* as 0.573 p' from an entirely different group of columns. The points representative of the 31 to 77-day tests are so scattered that no law can be determined from them. The strength of concrete varies so considerably within this period and the curing conditions moreover in this particular case are so variable that consistency between the test results can not be expected.

In Table 1 are given values of m, the reciprocal of Poisson's ratio. As the value of m cannot be less than 2 without contradiction of the law of the conservation of energy,† the many values of less than 2 which are given in the table indicate the lack of reliance to be placed in this constant as experimentally determined.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1096.
† *Loc. cit.*, p. 1108.

Equation (21) is a function of m, and also of n, another constant difficult of determination for stresses near the ultimate of spiral reinforced concrete. E_c varies greatly for strains beyond the ultimate strain of plain concrete, decreasing with increasing strain, and at an increasing rapidity. It is not known that n and m are constant at the ultimate strength of reinforced concrete, independent of the percentage of spiral reinforcement, hence, for one-strength concrete, the curve representing Equation (21) in Fig. 4, for $f_c = 2\,833$, would probably have m and n variable with a variation in p', instead of constant values of $n = 21$ and $m = 3$, throughout, and would result in an altogether different form of curve.

JACOB FELD,* ASSOC. M. AM. SOC. C. E. (by letter).†—The authors are perfectly justified in their contention that the addition of helical reinforcement to a concrete column should not give it the name of "spiral" reinforced column". Neither the suggested nor the usual term is short enough to meet with general approval and the writer would be glad to see the development of a simple nomenclature in this as well as other branches of reinforced concrete. The expression "reinforced concrete" is also far from logical or truly descriptive.

The separation of the types of failure in concrete columns as given by the authors is quite logical, and may be described generally, using the same division as in the paper, as follows:

(*a*).—Some columns fail in accordance with the usual method of compression failure of solids (or granular materials) that are low in tensile strength, and, therefore, fail easily in diagonal shear. The equations for this type of failure are identical with those defining the limiting resistances of granular solids to displacement along a plane within the solid. In this theory, it must be assumed that the unit shearing stress, f_z, in the paper, is uniform over the entire area along which failure occurs. This assumption cannot be justified by experimental data on shear failure in concrete. The "friction angle" of the paper is the value now universally called the angle of internal friction of a solid body.

(*b*).—When sufficient steel is placed as a jacket around a granular body, failure by compressive load is a separation of the granular constituents of the core, that is, crushing. Separation of the internal structure necessarily requires a certain elastic yield of the steel helix. It is not possible, therefore, to admit the previous conclusion that helical reinforcement has little effect in preventing crushing of the concrete core.

(*c*).—Sufficient steel section can be placed around the concrete so that the elastic stretch under increasing loads is not sufficient to allow the internal structure to separate, or crush, and thus failure cannot occur, until, at some point, the steel helix is stressed beyond its elastic limit.

The authors fail to define the value $\frac{rl}{r}$; evidently, r is a radius of gyration, which expression is not easily defined in reinforced concrete theory. It is far

* Cons. Engr., New York, N. Y.

† Received by the Secretary, April 14, 1926.

simpler, and involves no greater (if as large) errors if the ratio of length to minimum width or diameter is used.

It is interesting to compare the equations for maximum stresses in axially and eccentrically loaded columns. One is surprised to find that these equations give the same possibility of failure for a given load placed axially or with an eccentricity of one-eighth the diameter. In addition, these results do not agree with the theoretical solution* of L. J. Mensch, M. Am. Soc. C. E., which was also based on a large collection of experimental data. The graphical solution of Mr. Mensch's formula requires that an eccentricity of one-eighth reduce the ultimate load to 70% of the ultimate axial load.

Based on the authors' analysis, it follows that the percentage of helical reinforcement should be limited between 1% and 1.6% for the economical use of the steel. The analysis, however, is not complete in disregarding such factors as the relative contraction of concrete core, steel helix, and concrete shell, during the period of setting. Taking such factors into consideration, it seems not unlikely that the usual building code restriction of from 1% to 2% is not only theoretically justifiable, but also marks the most economical use of the reinforcement.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1195, Fig. 18.

EVAPORATION ON UNITED STATES RECLAMATION PROJECTS

Discussion*

By Messrs. Reuben B. Sleight, Lynn Perry, Ralph L. Parshall, S. T. Harding, Charles H. Lee, C. M. O'Neil, and J. B. Lippincott.

Reuben B. Sleight,† Assoc. M. Am. Soc. C. E. (by letter).‡—This paper should be welcomed by all members of the profession interested in irrigation and hydraulics. The writer's contribution will be confined mainly to the summaries of part of the data collected by himself in a two-year period during which practically his entire time was spent on evaporation research.

In the summer of 1915 he went to Denver, Colo., as Engineer for the U. S. Department of Agriculture to build and operate an Irrigation Field Laboratory. This project was conceived by, and the work was done under the direction of, Samuel Fortier, M. Am. Soc. C. E., then Chief of Irrigation Investigations, Office of Public Road and Rural Engineering. The major investigations at the Laboratory for the two years of its operation before the World War were evaporation studies. The report§ on the first year's observations did not give to that eminent authority in irrigation engineering, Dr. Fortier, due credit for the conception of the evaporation research project and guidance of the work throughout the period of operation of the Laboratory.

One of the problems set, and the one in partial answer to which the figures presented in this discussion were obtained, was the determination of ratios between evaporation amounts from the surfaces of water held in containers of different sizes, shapes, and positions or setting.

The Laboratory,‖ its floating pans, etc., are shown in Figs. 6, 7, and 8. It was located within the city limits of Denver, but on the open prairie south of the residential districts. Methods, accuracy of work, and full description of all water-surface evaporation investigations have been presented elsewhere.§

The Laboratory was completed in time to make the first evaporation measurements during the third week of November, 1915. These measurements were carried through until September 30, 1917, and summaries of data obtained from them are given herein. These summaries combine the data published in 1917 with one additional year's observations. Many evaporation tanks

* Discussion on the paper by Ivan E. Houk, M. Am. Soc. C. E., continued from May, 1926, *Proceedings*.

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‡ Received by the Secretary, March 1, 1926.

§ "Evaporation from the Surfaces of Water and River-Bed Materials," by R. B. Sleight, *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

‖ See, also, *Engineering News*, Vol. 76, No. 23, p. 1080.

were used. In only two cases are the records continuous for the entire period, and as will be seen from Table 15 only two or three months' observations were possible from some of the types of tanks used.

The first item listed in Table 15 is Ground Tank No. 7, 12 ft. in diameter, 3 ft. deep, set 2.75 ft. deep, and with a water depth as near 2.75 as it could be kept; this form has been selected as a base upon which to compute ratios of evaporation from tanks of different sizes and shapes. The evaporation from Tank No. 7—85.47 in. during the period, November 15, 1915, to September 30, 1917, or whatever other amount was evaporated from Tank No. 7 during the period of operation of the tank with which a comparison is made—is taken as 100 per cent.

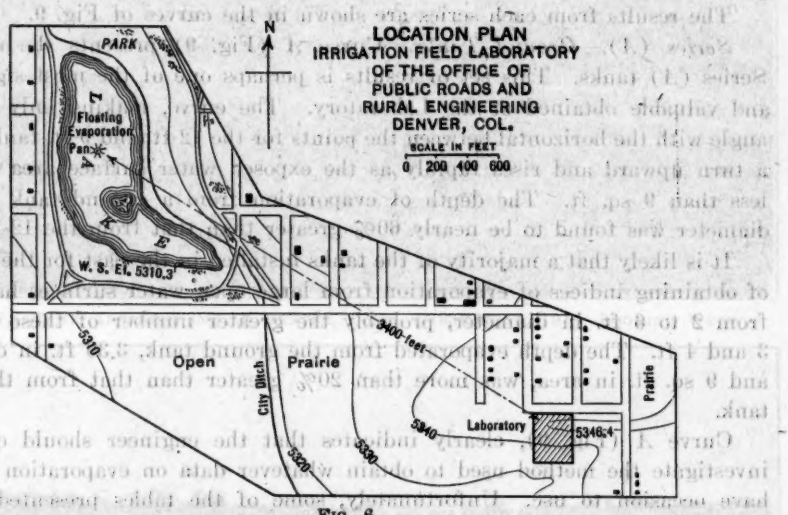


FIG. 6.

Tank descriptions, periods of observation, amounts evaporated, and the ratios between the amount evaporated from Tank No. 7 and that from other tanks are given in Table 15.

No attempt will be made here to explain the variations and ratios found between evaporation depths from tanks of different types. Some of them may be expressed quite readily by formulas, but it is believed that for practical use the curves of Fig. 9 will serve as well or better. It cannot be said that all the ratios are exactly comparable as it was not possible to operate each tank continuously, or all for equal lengths of time. However, the ratios between Ground Tank No. 7 and the others give factors for comparison that are believed to be the best bases for study.

It will be seen that, arranged according to type of container, there are five classifications or series of comparisons:

(*A*).—Cylindrical ground tanks, 3 ft. deep; set 2.75 ft. deep; water, 2.75 ft.; with diameters from 1 ft. to 12 ft. Tanks Nos. 1, 2, 4, 5, 6, and 7.

(*B*).—Ground tanks with square horizontal cross-section, 3 ft. deep; set 2.75 ft.; water depth, 2.75 ft., with water surface areas of 4 sq. ft. and 9 sq. ft.

Tanks Nos. 3 and 15. For comparison between square and circular exposed water surface there are Tanks Nos. 3 and 4, each with an area of 9 sq. ft.

(*C*).—Cylindrical ground surface tanks of the U. S. Weather Bureau type for Class A Stations; 10 in. deep; setting on wood foundations above ground. The standard 4 ft. diameter tank is compared with tanks 2 ft. in diameter and 6 ft. in diameter. Tanks Nos. 8, 69, and 70 give the relations.

(*D*).—Ground tanks cylindrical in form, having different depths. A series with diameters of 2 ft. and depths from 0.5 ft. to 6 ft. is furnished by Tanks Nos. 18, 19, 21, 22, and 2. Another series with diameters of 6 ft. and depths from 1 ft. to 3 ft., is given by Tanks Nos. 17, 5, and 16.

(*E*).—Floating cylindrical pans, 1 ft. deep, with diameters of 0.83 ft. to 6 ft., and a floating pan, 3 ft. by 3 ft. square and 1.5 ft. deep. Tanks Nos. 9, 92, 71, 72 and 73.

The results from each series are shown in the curves of Fig. 9.

Series (*A*).—*Ground Tanks.*—Curve *A* (Fig. 9) presents the data for Series (*A*) tanks. This set of results is perhaps one of the most significant and valuable obtained at the Laboratory. The curve, making only a small angle with the horizontal between the points for the 12-ft. and 9-ft. tanks, takes a turn upward and rises rapidly as the exposed water surface area becomes less than 9 sq. ft. The depth of evaporation from a ground tank 1 ft. in diameter was found to be nearly 60% greater than that from the 12-ft. tank.

It is likely that a majority of the tanks installed in the past for the purpose of obtaining indices of evaporation from large open water surfaces have been from 2 to 6 ft. in diameter, probably the greater number of these between 3 and 4 ft. The depth evaporated from the ground tank, 3.39 ft. in diameter and 9 sq. ft. in area, was more than 20% greater than that from the 12-ft. tank.

Curve *A* (Fig. 9), clearly indicates that the engineer should carefully investigate the method used to obtain whatever data on evaporation he may have occasion to use. Unfortunately, some of the tables presented in the past lack sufficient identification to justify their use.

Series (*B*).—*Ground Tanks.*—Only two square tanks, Nos. 3 and 15, were used for these observations. On a percentage basis, with the cylindrical Tank No. 7 as 100%, results are shown as Curve *B* (Fig. 9). The general slope of the line for the two sizes—2.0 and 3.0 ft. square—is about the same as for the cylindrical ground tanks. With equal exposed areas the evaporation is slightly greater from the square surface. Its ratio of perimeter to area is greater and there is a greater wetted area of metal. It follows, therefore, that evaporation is taking place from a slightly greater surface when a tank presenting a square water surface is used, than with a cylindrical one of equal cross-sectional area.

From Table 15 the relation between Tanks No. 3 (square surface, 3 ft. by 3 ft.) and No. 4 (circular, 3.39 ft. in diameter) based on Tank No. 7 is: $125.9 \div 120.2 = 1.047$. There is no cylindrical tank of the surface area of Tank No. 15, but Curve *B* (Fig. 9), shows that the relation would be practically the same.

Thus, within the limits of experimental work done at the Denver Laboratory, evaporation from a small ground tank with square exposed area was

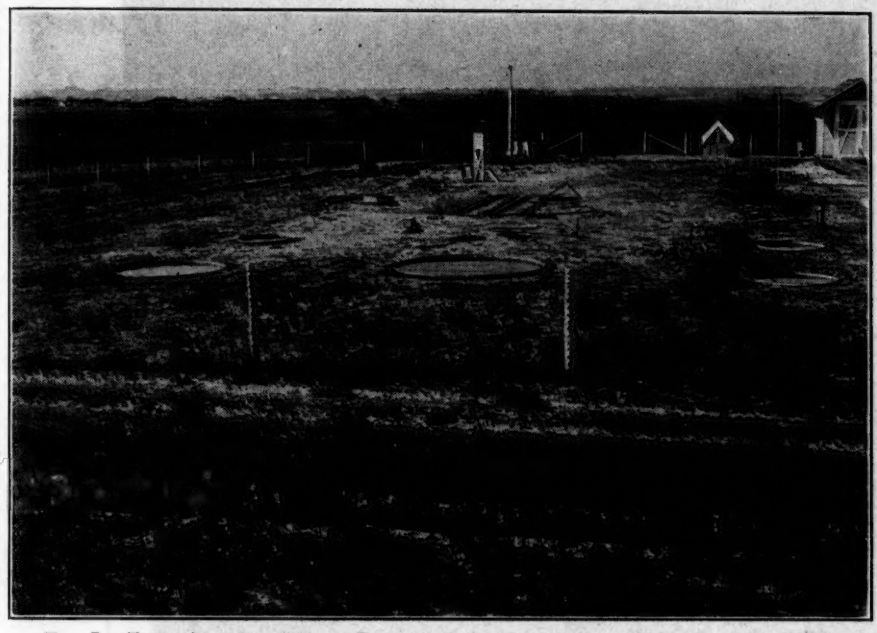


FIG. 7.—TANK ARRANGEMENT AT DENVER, COLO., LABORATORY, 1916, LOOKING WEST.

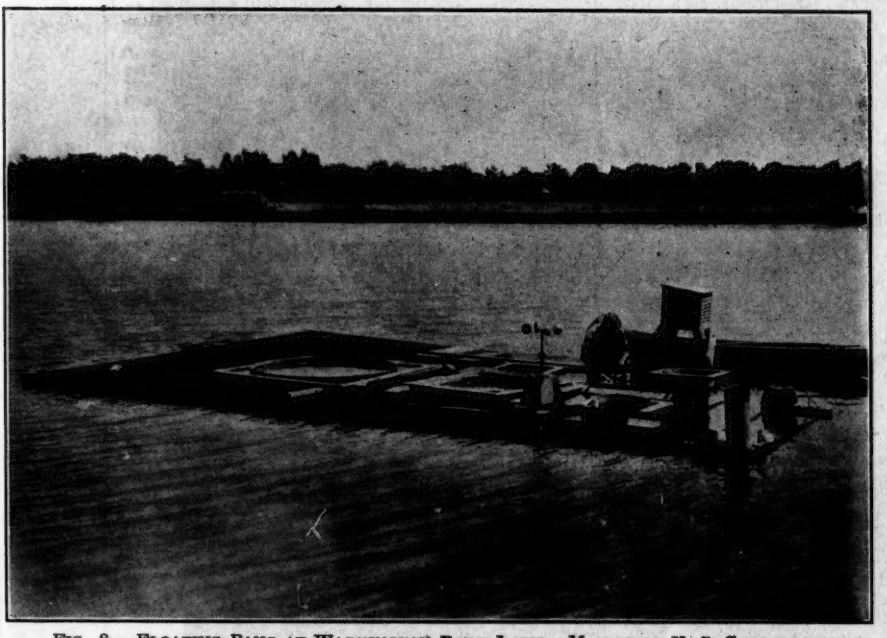
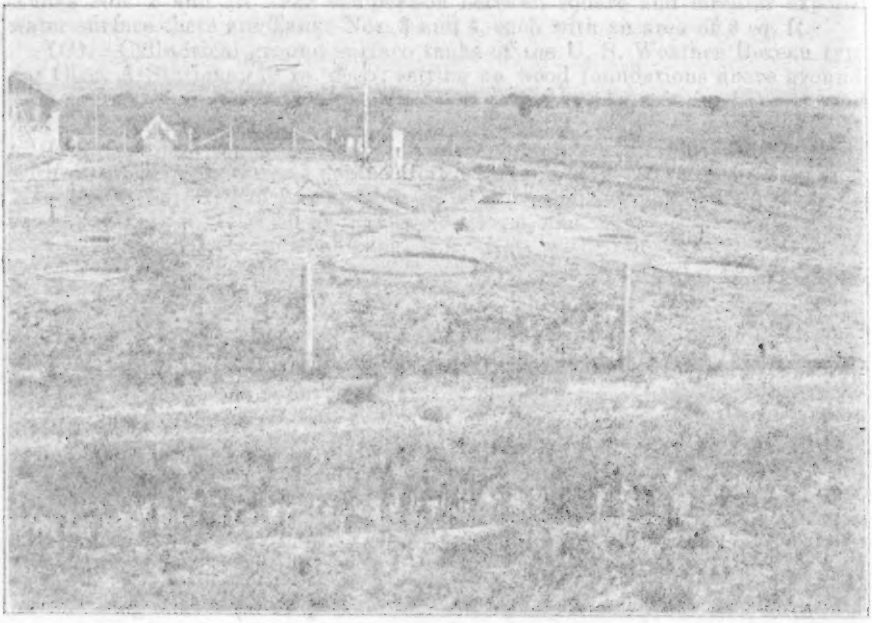


FIG. 8.—FLOATING PANS AT WASHINGTON PARK LAKE. MAN IS AT U. S. GEOLOGICAL TYPE NO. 9.



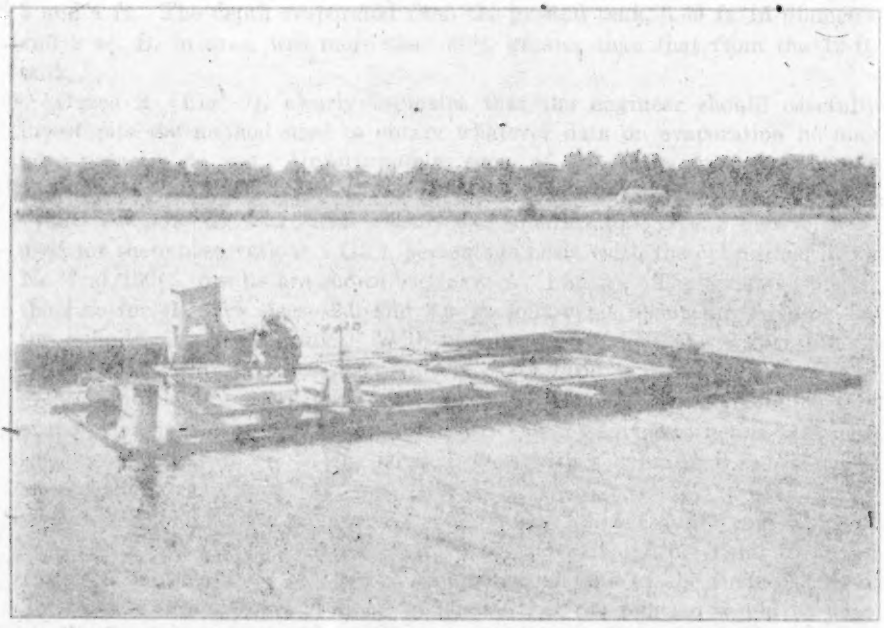


TABLE 15.—SUMMARY OF EVAPORATION MEASUREMENTS, DENVER, COLO., IRRIGATION FIELD LABORATORY.

Tank No.	Description.	Period of observations.	Evaporation During Period.		
			For given tank, in inches.	For Tank No. 7, in inches.	Percentage of that from Tank No. 7.
		Tanks on Land at Laboratory			
7	12 ft. in diameter; 3 ft. deep; set in ground, 2.75 ft.; water depth, 2.75 ft.	Nov. 15, 1915-Sept. 30, 1917.	85.47	85.47	100
1	1 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915-Sept. 30, 1917, except Dec. 4, 1916, to May 7, 1917.	126.59	79.68	158.9
2	2 ft. in diameter, otherwise same as Tank No. 7	Nov. 15, 1915-Nov. 13, 1916	63.12	49.16	128.4
4	3.89 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915-Sept. 17, 1917, except Nov. 21, 1916, to Mar. 11, 1917.	96.74	80.51	120.2
5	6 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915-Sept. 30, 1917.	92.87	85.47	108.9
6	9 ft. in diameter, otherwise same as Tank No. 7.	Nov. 15, 1915-Nov. 13, 1916.	49.63	49.16	100.9
15	2 by 2 ft. square by 3 ft. deep; set in ground, 2.75 ft.; water depth, 2.75 ft.	Apr 17-Nov. 13, 1916, and from May 29-Sept. 24, 1917	89.49	68.18	131.3
3	3 by 3 ft. square, otherwise same as Tank No. 15.	Nov. 15, 1915-Nov. 13, 1916, and May 8-Sept. 24, 1917.	98.63	78.33	125.9
8	4 ft. in diameter; 10 in. deep; setting on wood foundation; Weather Bureau Class A type.	Nov. 15-Dec. 13, 1915; Mar. 6-Nov. 13, 1916; and Apr. 11-Sept. 30, 1917.	116.09	81.69	142.1
69	2 ft. in diameter, otherwise same as Tank No. 8.	Apr. 11-Sept. 30, 1917.	54.91	32.53	168.8
70	6 ft. in diameter, otherwise same as Tank No. 8.	Apr. 11-July 30, 1917.	26.66	20.05	132.8
22	2 ft. in diameter; 0.5 ft. deep; set in ground, 0.25 ft.; and with water 0.25 ft. deep.	June 5-Oct. 9, 1916, and May 28-July 30, 1917.	55.62	43.47	127.9
21	2 ft. in diameter; 1.0 ft. deep; set in ground, 0.75 ft.; and with water, 0.75 ft. deep.	June 5-Oct. 9, 1916 and May 28-Sept. 24, 1917.	70.16	55.24	126.8
20	2 ft. in diameter; 1.5 ft. deep; set in ground, 1.25 ft.; and with water, 1.25 ft. deep.	June 5-Oct. 9, 1916, and May 28-Sept. 24, 1917.	70.69	55.24	127.9
19	2 ft. in diameter; 2 ft. deep; set in ground, 1.75 ft.; and with water, 1.75 ft. deep.	June 5-Oct. 9, 1916, and May 28, 1917.	55.30	43.47	127.9
18	2 ft. in diameter; 6 ft. deep; set in ground, 5.75 ft.; and with water, 5.75 ft. deep.	June 5-Oct. 9, 1916 and May 28-Sept. 24, 1917.	70.39	55.24	127.4
17	6 ft. in diameter; 1 ft. deep; set in ground, 0.75 ft.; and with water, 0.75 ft. deep.	June 5-Oct. 9, 1916.	30.48	28.49	106.9
16	6 ft. in diameter; 2 ft. deep; set in ground 1.75 ft.; and with water 1.75 ft. deep.	June 5-Oct. 9, 1916.	30.36	28.49	106.5

TABLE 15.—(*Continued*).

Tank No.	Description.	Period of observations.	Evaporation During Period.		
			For given tank, in inches.	For Tank No. 7, in inches.	Percentage of that from Tank No. 7.
Tanks Floating in Lake at Washington Park; Water Level in Tank Kept as Near as Possible to That in Lake					
9	3 by 3 ft. square; 1.5 ft. deep, with two supporting cylinders, U. S. Geological Survey type.....	Apr. 17–Nov. 6, 1916 and thirteen periods, ranging from 3 days to 1 week, from May 10–Sept. 17, 1917	72.04	64.06	112.5
92	0.83 ft. in diameter; 0.83 ft. deep.....	July 31–Sept. 17, 1917.....	13.42	10.27	130.7
71	2 ft. in diameter; 1 ft. deep.	May 10–19, May 23–28, June 1–4, June 7–12, July 31–Sept. 17, 1917.....	16.04	14.29	112.2
72	4 ft. in diameter; 1 ft. deep.	July 31–Sept. 17, 1917.....	10.63	10.27	103.5
73	6 ft. in diameter; 1 ft. deep.	May 10–19, June 1–4, June 7–12, Aug. 14–27, 1917.....	6.39	6.57	97.1

slightly less than 5% greater than that from a ground tank with a circular exposed area of the same size, both tanks being 3.0 ft. deep and having settings and water depths of 2.75 ft.

Series (C).—Ground Surface Pans.—This series compares the U. S. Weather Bureau type for Class A stations (No. 8) with others of the same design, but with different diameters. Curve *C* (Fig. 9) shows graphically the evaporations. Taking Tank No. 8 as 100%, the ratios are as given in Table 16.

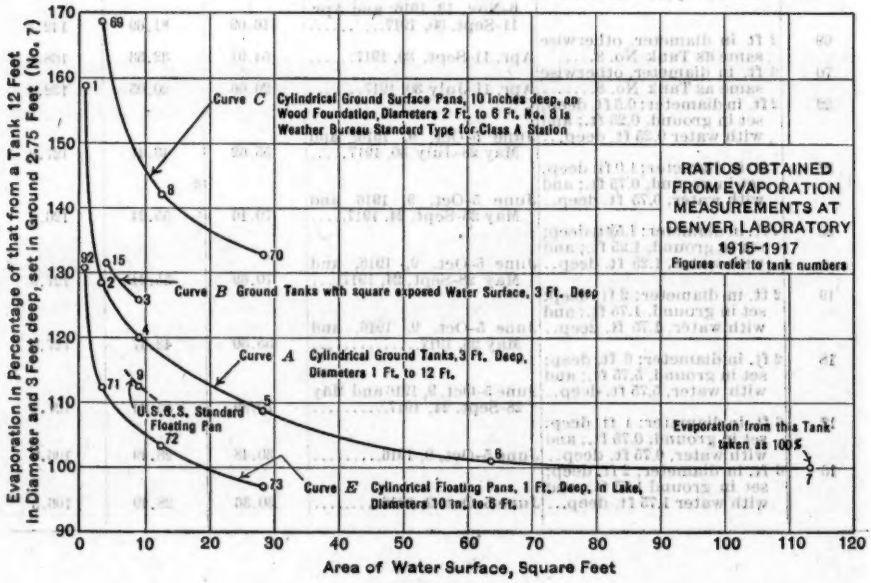

Evaporation in Percentage of that from a Tank 12 Feet In Diameter and 3 Feet deep, set in Ground 2.75 Feet (No. 7)
Area of Water Surface, Square Feet
Curve C Cylindrical Ground Surface Pans, 10 inches deep, on Wood Foundation, Diameters 2 Ft. to 8 Ft. No. 8 is Weather Bureau Standard Type for Class A Station
Curve B Ground Tanks with square exposed Water Surface, 3 Ft. Deep
Curve A Cylindrical Ground Tanks, 3 Ft. Deep, Diameters 1 Ft. to 12 Ft.
Curve E Cylindrical Floating Pans, 1 Ft. Deep, in Lake, Diameters 10 in. to 8 Ft.
U.S.G.S. Standard Floating Pan
RATIOS OBTAINED FROM EVAPORATION MEASUREMENTS AT DENVER LABORATORY 1915-1917
Figures refer to tank numbers
Evaporation from this Tank taken as 100%

FIG. 9.

Curve *C* (Fig. 9) has about the same form as that for the ground tanks.

Series (D).—Ground Tanks.—Series (*D*) can be split into two sub-series, one, comprising Tanks Nos. 5, 16, and 17, with tanks, 6 ft. in diameter, and the other with 2-ft. tanks. From Table 15 it is seen that the ratios, based on Tank No. 7 as 100%, are as given in Table 17.

TABLE 16.—COMPARISON OF EVAPORATION FROM GROUND SURFACE PANS.

Tank No.	Diameter, in feet.	Evaporation, in percentage of Tank No. 8.
69	2.0	108.8
8	4.0	100.0
70	6.0	93.5

The results given in Table 17 do not indicate any material variation in evaporation amounts. One questionable point arises from the data for Tanks Nos. 5 and 2. In each case the tank was used for a longer period than the others of the series and shows a slightly greater relative amount of evaporation. It is believed that under conditions similar to those at the Denver Laboratory slight differences in evaporation would be recorded over long periods of observation if tanks of different depths are used. However, it is probably not advisable to install any less than 2.0 ft. deep, if the results are to be extended to lake or reservoir computations.

TABLE 17.—COMPARISON OF EVAPORATION FROM LAND PANS.

Tank No.	Diameter, in feet.	Depth of setting and water depth, in feet.	Evaporation, in percentage of that from Tank No. 7.
17	6.0	0.75	106.9
18	6.0	1.75	106.5
5	6.0	2.75	108.9
22	2.0	0.25	127.9
21	2.0	0.75	126.8
20	2.0	1.25	127.9
19	2.0	1.75	127.9
2	2.0	2.75	128.4
18	2.0	5.75	127.4

Series (E).—Floating Pans.—Storms interrupted the observations on several floating pans; the results possibly are not trustworthy and, therefore, have been omitted. There is some doubt about the results from Tank No. 73—a pan, 6.0 ft. in diameter. It was operated successfully for only a short time. One pan, of the U. S. Geological Survey type, was operated for two seasons; the others, for part of the second season only. Curve *E* (Fig. 9) presents the results graphically.

Although there was no exactly corresponding size, the evaporation from the floating square pan appears from the curve to be about 5% greater than that from a cylindrical one of the same exposed area. This is based on a size of 9 sq. ft. and is practically the same as the relation found between evapora-

tion from ground tanks with square and circular water surfaces. It will be noted that Curve *E* (Fig. 9), follows the same general form as Curve *A*, with a possible tendency to flatten out more quickly as the area increases.

The ratios of evaporation from ground tanks at the laboratory divided by that from floating pans with the same exposed water area both in size and shape, are as follows:

Diameter of exposed water surface.	Area, in square feet.	Ratio.
2.0 ft.	4.0	128.4 ÷ 112.2 = 1.144
3.39 ft.	9.0	120.2 ÷ 106* = 1.134
4.0 in.	12.57	116.5* ÷ 103.5 = 1.125
6.0 ft.	28.27	108.9 ÷ 97.1 = 1.121
3.0 by 3.0 ft. square	9.0	125.9 ÷ 112.5 = 1.119

In the case of the cylindrical tanks, the ratio decreases as the area increases. This must necessarily be so; the condition of a large open water area is being approached.

As to convenience and reliability of observations, a large floating pan for practical use is not believed advisable. The 6.0-ft. size as used was not satisfactory. The use of the honeycomb arrangement described in the paper would perhaps have eliminated part of the trouble. The U. S. Geological Survey standard was found to be more satisfactory than any other tested.

Comparison of All Observations on a Common Basis.—For comparative purposes, Table 18, showing the results previously discussed, except the depth variation series, has been compiled.

TABLE 18.—Evaporation in Percentage of That from Ground Tank No. 7.

	Percentage Evaporation.				
	Circular Water Surface.			Square Water Surface.	
Exposed surface area, in square feet.	Floating pans, water about 0.75 ft. deep and at same level as lake.	Ground tanks, 3 ft. deep; 2.75 ft. water depth; 2.75 ft. setting.	Ground surface pans, Weather Bureau type, 10 in. deep.	Floating pan, U. S. Geological Survey type.	Ground tank, 3 ft. deep; 2.75 ft. water depth; 2.75 ft. setting.
113.09	.....	100	.....	.....	.....
63.62	.....	100.9	.....	.....	.....
28.27	97.1	108.9	132.8	.....	.....
12.56	103.5	117.5*	142.1	.....	.....
9.0	106.0*	120.2	145.0*	112.5	125.9
4.0	110.5*	126.0*	164.0*	.....	131.3
3.14	112.2	128.4	168.8	.....	.....
0.78	.....	158.9	.....	.....	.....
0.54	130.7	.....	.....	.....	.....

* Interpolated from curves, Fig. 9.

Discussions of evaporation may be found in handbooks, in texts on hydraulics, and in technical papers. Some present formulas, others give tables of evaporation by districts or localities. Not all of them describe the method of evaporation measurement, or the size and kind of container used as a gauge.

* Interpolated.

One glance at Table 18 shows the great necessity for a complete knowledge of all conditions when attempting to make practical use of any figures on evaporation. There is a great chance of being led astray.

The few floating pans used show a variation in depth of water evaporated of nearly 35%, depending on the size. Similarly, the tanks on land show a 60% range. Then come the other differences that apparently depend on the shape and type of tank. It is true that the ratios to be used for the extension of any tank measurements to a large open water surface are still to some extent doubtful; on the other hand, if the bases for the tank figures are not fully understood with all their attendant conditions a guess may as well be made at the beginning.

The writer fully agrees with Mr. Houk regarding the necessity of considering meteorological conditions. At the Denver Laboratory were kept records of air and water temperatures, relative humidity, wind movement and direction, and, of course, precipitation. In addition to the maximum and minimum and two other daily temperature readings by standard thermometers and two daily observations of relative humidity by wet and dry bulbs, continuous records were made by graphically recording thermometers and hygrometers. A graphic recorder was attached to one of the anemometers and a Hoff evaporimeter to one of the evaporation tanks. U. S. Weather Bureau standards were maintained throughout.

The results of the first year's meteorological observations, with other details, has been published.* Records of the second year's observations made at the Denver Laboratory are on file at the office of the U. S. Bureau of Public Roads, Berkeley, Calif. The summary of the first year's observations is given in Table 19.

In only one case can ratios obtained at the Denver Laboratory be directly compared with those presented by Mr. Houk, that of the U. S. Weather Bureau pan for Class A stations and the 3 ft. square ground tank. The depth of the 3 ft. square tank at Denver was 3 ft., while the one at Jackson Lake was 14 in. However, Denver observations indicated that the depths of ground tanks probably had little effect on evaporation except when the tanks are shallow.

Over the entire two-year period of operation at Denver, the depth evaporated from the 3 ft. square ground tank was 88% of that from the U. S. Weather Bureau Class A ground-surface pan, while the Jackson Lake ratio is 72 per cent. For the summer periods the Denver figures are as given in Table 20.

There is a slight difference in pan and tank settings. The circular pan at Jackson Lake was set on the ground, the one at Denver on a wooden foundation as specified by the U. S. Weather Bureau; the square tank at Jackson Lake was set in the ground (depth of setting not given) with earth banked around the sides to the water level, while the one at Denver was set in the ground to the water level. It seems, however, that these circumstances would tend to make a greater difference between the evaporation depths recorded at Denver than between those at Jackson Lake.

* *Journal of Agricultural Research*, Vol. X, No. 5, July 30, 1917.

The results of the comparison emphasize the necessity for complete descriptions of all methods and equipment in connection with records of evaporation measurements. They further indicate the value of meteorological data. The mean air temperature at Jackson Lake during the three months of 1924 in which the comparative records were made was 53°; during the 5-month summer period at Denver, it was 64° in 1916 and 63° in 1917. The relative humidity was 59% in 1916 and 65% in 1917, as determined by the hygrometer. The total wind movement for the period in 1916 was 14 600 miles as compared with 14 000 miles in 1917.

TABLE 19.—METEOROLOGICAL RECORDS AT DENVER, COLO., LABORATORY, 1915–1916.

Week ended:	Mean air temperature, in degrees Fahrenheit.	Percentage of mean relative humidity.	*Total wind movement, in miles.	Evaporation from Tank No. 7, in inches.
1915:				
November 22	40.6	..	1 112	0.69
" 29	38.8	..	1 015	0.70
December 6	38.8	..	633	0.25
" 13	39.5	..	518	0.43
1916:				
March 6	39.3	..	853	0.67
" 13	46.3	..	1 273	0.73
" 20	49.5	..	731	1.00
" 27	46.4	..	1 022	0.63
April 3	42.6	..	807	0.77
" 10	36.4	..	657	0.49
" 17	48.3	..	897	0.87
" 24	48.7	..	785	1.04
May 1	46.4	..	870	1.09
" 8	53.9	..	860	1.19
" 15	50.5	60	1 189	1·65
" 22	48.2	71	819	1.01
" 29	58.1	48	1 190	2.06
June 5	61.7	56	634	1.71
" 12	60.9	62	737	1.61
" 19	65.0	54	613	1.48
" 26	67.0	45	884	2.21
July 3	73.6	47	774	2.18
" 10	73.6	58	941	2.18
" 17	69.3	66	500	1.58
" 24	72.8	61	590	1.88
" 31	72.7	69	642	1.76
August 7	68.8	78	532	1.40
" 14	68.9	71	634	1.43
" 21	67.2	60	755	1.70
" 28	62.8	61	620	1.47
September 4	65.5	44	622	1.57
" 11	66.9	51	682	1.82
" 18	53.2	65	571	0.98
" 25	58.5	58	555	1.03
October 2	50.5	56	588	1.15
" 9	51.0	64	849	1.11
" 16	44.7	70	824	.63
" 23	39.6	73	712	.24
" 30	42.8	67	555	.33
November 6	51.2	48	700	.90
" 13	27.7	70	820	.38

* Anemometer 2 ft. above ground.

This one comparison of the Denver and Jackson Lake results raises the question whether the U. S. Weather Bureau pan for Class A stations is the best index for attempting an estimate of evaporation from large open water surfaces. It is easily installed, easily maintained, and the extent of its use is probably greater than that of any other. The ground tank more nearly approximates the reservoir. The temperature changes of the water in the ground-surface pan follow air temperature changes more closely than those of water in the reservoir or the ground tank. The temperature records of the water in the Class A pan at Denver show a maximum day temperature greater than, a minimum night temperature lower than, and an average temperature lower than, was recorded for the water of any ground tank. The high day temperature at the time of low relative humidity probably caused the rapid evaporation. It is to be expected that in districts where cool nights and rather high midday air temperatures prevail in summer the ratio between depths of evaporation from a ground tank or a reservoir and from the Class A pan will not be the same as in districts where daily variations in air temperatures are not so great. This may be the explanation of the difference in the ratios at Denver and Jackson Lake.

TABLE 20.—SUMMER EVAPORATION, DENVER, COLO., LABORATORY.

Period.	Evaporation from 3 ft. square ground tank, in inches.	Evaporation from Class A type pan, in inches.	Ratio, in percentage.
May 8-September 24, 1916	42.08	49.77	85
May 9-September 25, 1917	36.56	41.59	88

At the end of one season of its use the writer had formed the opinion that he could not recommend the U. S. Weather Bureau Class A pan (No. 8) in preference to a ground tank as an evaporation index. He believes that a ground tank with a square or circular cross-section not less than 3 ft. across and at least 2 ft. deep, with the water level about even with the ground level, should be used.

There are, however, many records from Class A stations. It is hoped that the discussion of Mr. Houk's paper will bring out reliable data indicating how these records can be used to the best advantage.

For a number of years the writer's Fig. 10 has been locked away awaiting the accumulation of data by which its value may be proven. It is presented as a means by which evaporation from large open water surfaces may be estimated from U. S. Weather Bureau meteorological data. It is based largely on the Denver research work, and meteorological conditions of the Denver region and involves the writer's conclusions regarding the extension of evaporation tank observations to large open water surfaces. It comprehends the main variables or influencing factors except barometric pressure. At an elevation lower than that of Denver, which is approximately 1 mile, under the same

conditions as to wind movement, temperature, and relative humidity, the evaporation is less than, and at higher elevations it is greater than, at Denver. As near as the writer can determine from the meager data available, three points on this curve are about as follows:

Elevation, in feet.	Evaporation in percentage of that at Denver under the same conditions of temperature, relative humidity, and wind movement.
127 (Yuma, Ariz.)	70
5345 (Denver, Colo.)	100
8180 (Nederland, Colo.)	120

The elements for Fig. 10 are to be found as follows: (1) Air temperature—from maximum and minimum, preferably two observations daily; (2) relative humidity—preferably from hygrograph records, approximated by the

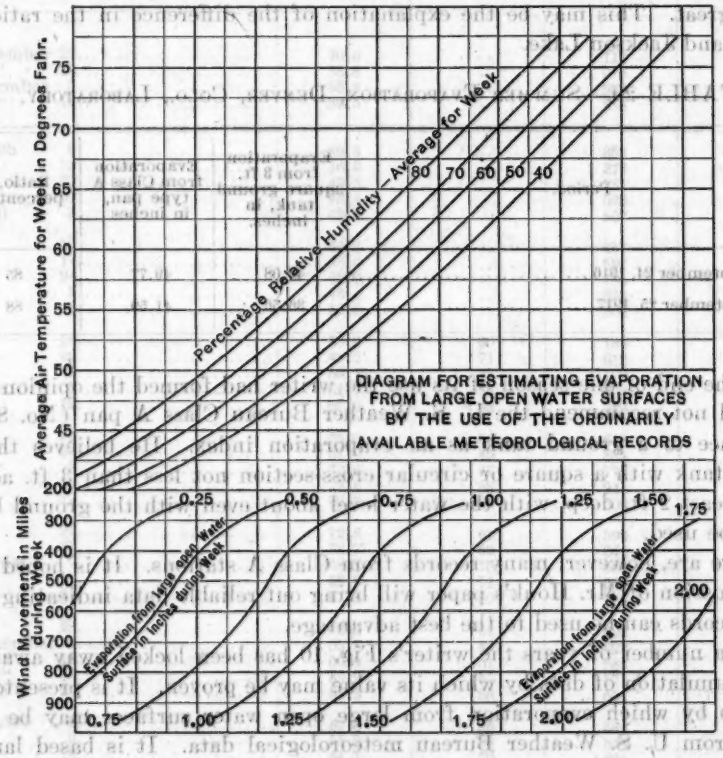


FIG. 10.

usual two observations per day; and (3) wind movement—from an anemometer 2 ft. above the water surface. For higher elevations, H ft. above the water or ground surface (up to 50 ft.), divide the wind movement by $(0.033\ H. + 0.93)$ to get the movement at the 2-ft. elevation.

In using Fig. 10, enter the diagram with the air temperature; at the intersection of the horizontal temperature line with the relative humidity curve, drop vertically to the horizontal wind movement line and read the evaporation from the curve.

Fig. 10 and the data as to the evaporations at various lengths should be checked further before applying them to the solution of a specific problem. The diagram is presented as a beginning of an analysis which the writer desires to develop further, and he earnestly hopes that the paper of Mr. Houk will bring out data of a character suitable for this development.

LYNN PERRY,* M. AM. SOC. C. E. (by letter).†—This paper, which presents much data in condensed form, should be of considerable value to the profession for this information is not readily obtained except from the files of the U. S. Reclamation Bureau. Although frequently discussed, reliable and accurate data on the subject of evaporation, rainfall, wind velocity, temperature, and humidity, are difficult to obtain for any given locality.

TABLE 21.

Station: Christiansted.
State: Virgin Islands.
Project: Municipal Water Supply.
Exposure: Ground.
Elevation: About 20.
Records: Precipitation since 1852.
Evaporation since 1919.
Wind velocity (see text) since 1909.
Maintained by: U. S. Navy Department.

MEAN RECORDS.

Month.	Evaporation, in inches.	Temperature, in degrees Fahrenheit.	Wind velocity (see text), in miles per hour.	Rainfall, in inches.
January	4.76	76.6	9.0	2.32
February	5.19	76.3	9.1	1.87
March	6.88	76.9	9.5	1.69
April	7.13	78.2	9.1	2.71
May	8.25	80.2	8.9	4.35
June	7.65	81.1	10.1	3.94
July	8.00	81.4	10.6	3.56
August	7.58	81.8	9.8	4.35
September	6.70	81.8	8.0	5.75
October	6.02	81.0	6.0	6.63
November	5.02	79.7	7.4	5.30
December	4.50	77.8	7.2	3.50
Totals and means	77.61	79.3	8.6	46.00

In this connection, the writer presents some data that were accumulated in connection with the studies leading to the design of the water-works at Christiansted, Virgin Islands. Rainfall has been observed at Christiansted, Kingshill, and Fredericksted, since 1852 and the records are in the files at the General Office. The temperatures on file at the office of the U. S. Weather Bureau cover a period of 17 years. The evaporation has been observed at

* Asst. Prof., Hydr. and San. Eng., Lafayette Coll., Easton, Pa.
† Received by the Secretary, April 8, 1926.

Christiansted from 1919 to 1923. For this purpose, a land pan was constructed and placed in operation in April, 1919; with the exception of June and July, 1919, the records are continuous. No wind velocity or humidity records for Christiansted were available. The nearest available wind-velocity records are for St. Johns, Antigen, and Basseterre, St. Kilts. At St. Johns, the average wind velocity is 8 miles per hour (11-year average) and at Basseterre, it is 9.5 miles. At San Juan, Porto Rico, observations over a long period average 10.6 miles per hour, which is probably higher than at Christiansted.

TABLE 22.—RANGE IN ANNUAL EVAPORATION AT CHRISTIANSTED, VIRGIN ISLANDS.

Full years record	5
Mean annual evaporation, in inches	77.61
Maximum Annual Evaporation:	
Year	1923
In inches	83.63
Percentage of mean	107.7
Total precipitation, in inches	32.04
Minimum Annual Evaporation:	
Year	1921
In inches	73.57
Percentage of mean	94.8
Total precipitation, in inches	25.34

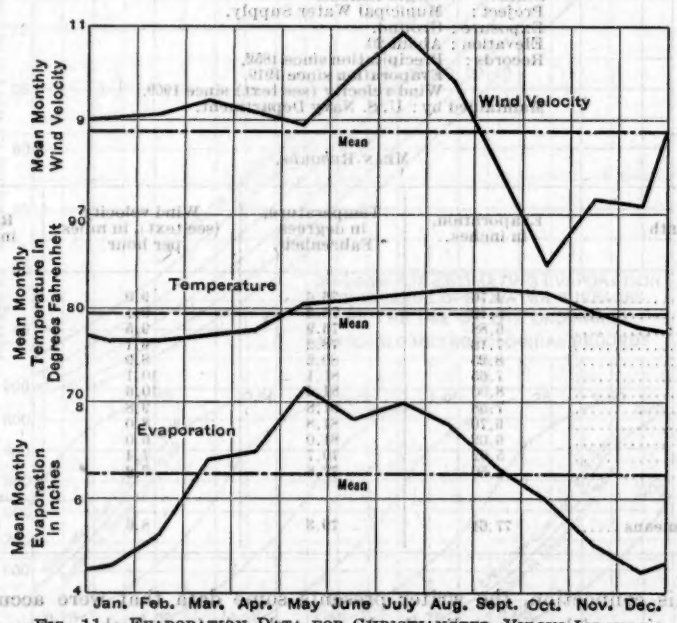


FIG. 11.—EVAPORATION DATA FOR CHRISTIANSTED, VIRGIN ISLANDS.

For the purpose of comparison the writer has used these data to construct Tables 21 and 22, and Fig. 11 to correspond with Tables 1* and 2† and Fig. 4‡ of Mr. Houk's paper.

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 44.
† *Loc. cit.*, p. 56.
‡ *Loc. cit.*, p. 60.

A relatively few years ago no method was available for estimating run-off. As data became available, various observers suggested methods of calculating run-off based on their own observations, taking into account the information published by others. With carefully made measurements, such as those given by the author, being made public at frequent intervals, it is possible that some method of estimating more accurately the probable evaporation at a given place will be suggested.

RALPH L. PARSHALL,* ASSOC. M. AM. SOC. C. E. (by letter).†—The phenomenon of evaporation has engaged the attention of scientific investigators for a long time. Because of the relatively small quantity of water, as measured by depth, evaporated from an open water surface within a reasonably short time, the problem of accurately measuring the loss has involved some difficulty. Where sufficient time has been given between observations to allow for a measurable loss caused by evaporation the influencing factors have changed to such an extent that correlation becomes hard to effect and may not be possible.

The writer has given more or less attention to a study of evaporation from free water surfaces. In endeavoring to promote a greater degree of accuracy in measuring the depth loss and at the same time decreasing the interval between observations, he has devised special apparatus by use of which hourly readings disclose this loss and accumulate the necessary contributing meteorological data. The most accurate method of determining the loss is by weighing, but when the total weight of the mass of water involved is considerable the errors in weighing become so appreciable as to render uncertain the relation of this loss to other factors.

An optical evaporimeter (Fig. 12) was perfected to determine accurately the decrease in depth of the water surface, and at the same time to show a measurable loss within a relatively short period. This apparatus includes a cylindrical brass float, 6 in. long and 2 in. in diameter. Reaching from the top of the float are two short horizontal, parallel arms that extend to a common bar. On the under side of this bar are two delicate pivot bearings supported by a rigid concrete pedestal placed close to the evaporation pan. Attached to the top of the pivot bar, at its middle point, is a small high-grade plane speculum mirror. At a distance of about 20 ft. from the mirror a high-powered telescope, with cross-hairs, is mounted on a concrete pedestal at a convenient height and focused on the image of a scale in the mirror. This scale is placed approximately 50 ft. from and about 20 ft. higher than the mirror. It is 6 ft. long and graduated accurately to 0.1 in. By adjusting the mirror and telescope in proper position, a clear and distinct image of the scale divisions may be observed. By careful calibration of this instrument, the constant or value of a scale division in difference in elevation of water surface was determined. In all observations the scale was read by estimation to one-tenth of the least scale division, or 0.01 in.

* Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.
† Received by the Secretary, April 10, 1926.

Calibration on this basis shows variations of from 3 to 5% from the mean constant. Different lengths of arms (float to pivot bar) were used, giving constants for the evaporimeter ranging from 0.000016 to 0.000060, that is, 0.01 in. on the scale representing, say, 0.000016-in., or 0.000060-in. difference in elevation of the water surface in the evaporation pan, depending on the length of arm used.

To determine the existing vapor pressure, an aspiration psychrometer (Fig. 12) was mounted so as to take air at a point 1 in. above the water surface in the evaporation pan. This instrument consisted of a thin-walled glass tube about 1⅜ in. in diameter and 24 in. long. Two thermometers with 14-in. stems, graduated to 0.1° Fahr. were fixed inside this tube. A single-ply linen sheath covered the bulb of one of these thermometers and was situated so that the bulb was immediately beneath a small aperture in the glass tube. A small pipette was used to moisten the wet bulb through this hole in the glass tube. When the psychrometer was in operation this aperture was stopped with a small cork. An aspirator connected to this instrument drew the air through the glass tube past the thermometers with a pressure of about 1 in. of water. The wet and dry bulb readings, taken through the wall of the glass tube, were recorded when the minimum temperature of the wet bulb occurred.

The air and water temperatures were taken using thermometers identical with those mounted in the psychrometer. The temperature of the air was observed at a point 1 in. above the water surface, and the temperature of the water surface at a point directly below the bulb of the air thermometer.

In taking the evaporation records a definite order was followed: First, the air and water temperatures were taken; second, the psychrometer readings; third, the barometric pressure; and, fourth, the optical evaporimeter reading, which included the time to 0.1 min. An hour later the optical evaporimeter was again read and the time noted, and the other readings were repeated in the foregoing order. The mean values of water and vapor pressures, temperatures, etc., were determined and assumed to be effective for the period between the initial and final reading of the evaporimeter. The evaporimeter reading, as well as the time, was taken subsequent to the temperature and psychrometer readings as the initial observations for the beginning of the next test. This double reading of the evaporimeter scale, with the time interval necessary to observe the other data, gave a reliable check on scale readings as well as evidence as to the action of the float.

Two complete sets of apparatus were maintained, one within the laboratory (Fig. 12) and the other outside under fully exposed conditions (Fig. 13). Within the laboratory the observations were taken for still air conditions, and by means of a special air duct (Figs. 14 and 15), the effect of wind on the rate of evaporation was observed. The study under wind conditions covered velocities ranging from about 1.5 to 12 miles per hour.

Fig. 16 shows a series of observations taken in the laboratory under still air conditions. One of the striking facts observed in studying the evaporation loss, under still air, is the maximum rate occurring during the early hours of the morning. It is to be noted that the rate of evaporation bears a singular relation to the temperatures of the air and water. During the morning and

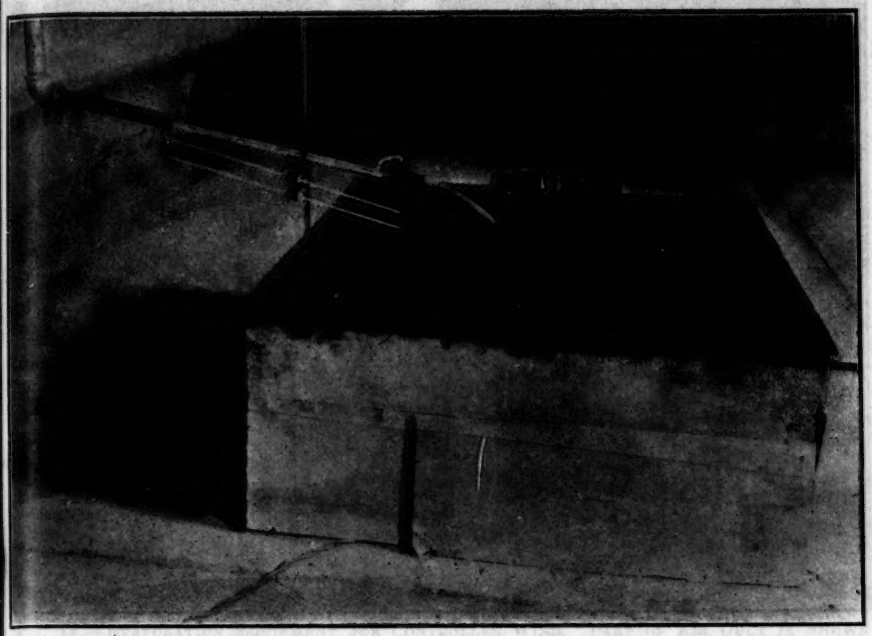
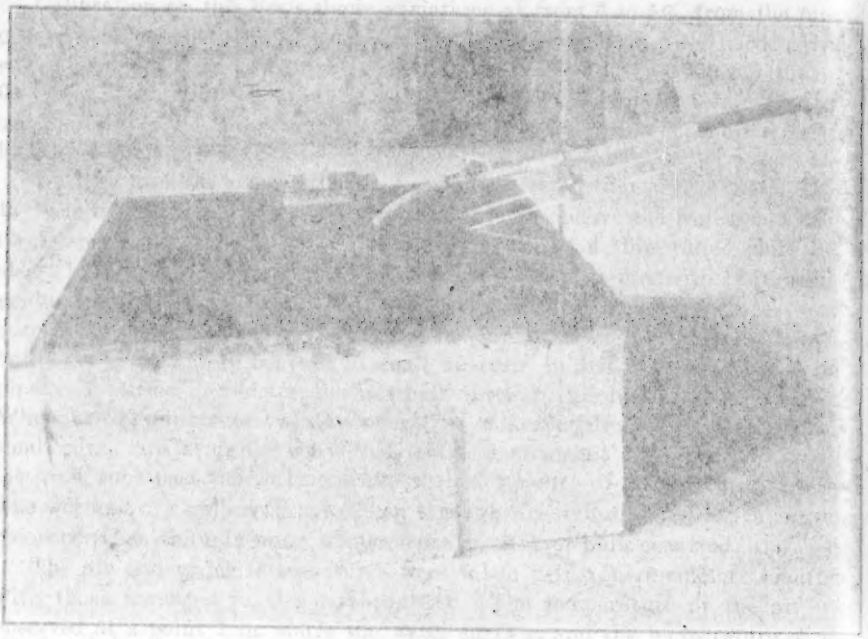
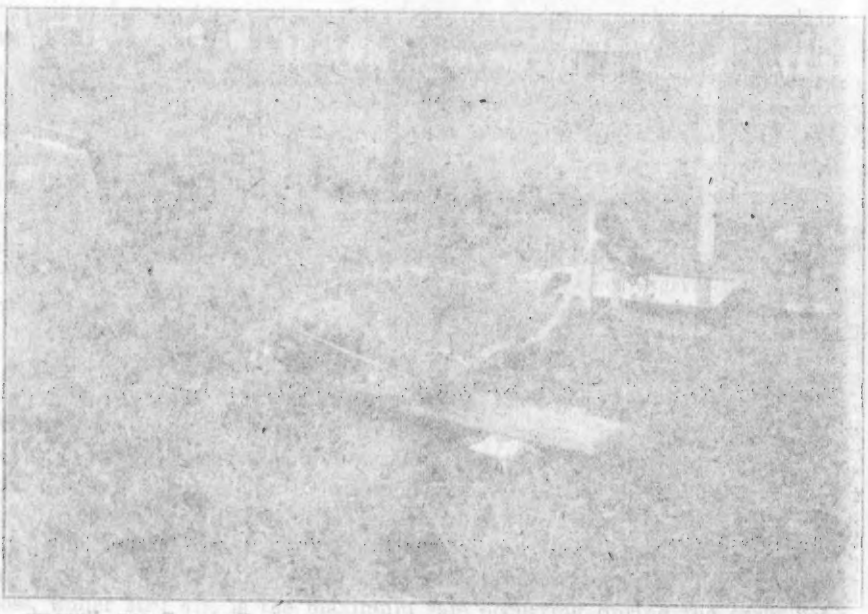


FIG. 12.—EVAPORATION EQUIPMENT FOR STILL AIR CONDITION, SHOWING ARRANGEMENT OF PSYCHROMETER, OPTICAL EVAPORIMETER, AND AIR AND WATER THERMOMETERS.

FIG. 13.—ARRANGEMENT OF APPARATUS FOR OBSERVATIONS UNDER FULLY EXPOSED CONDITIONS. ASPIRATOR FOR PSYCHROMETER IS SHOWN IN FOREGROUND.





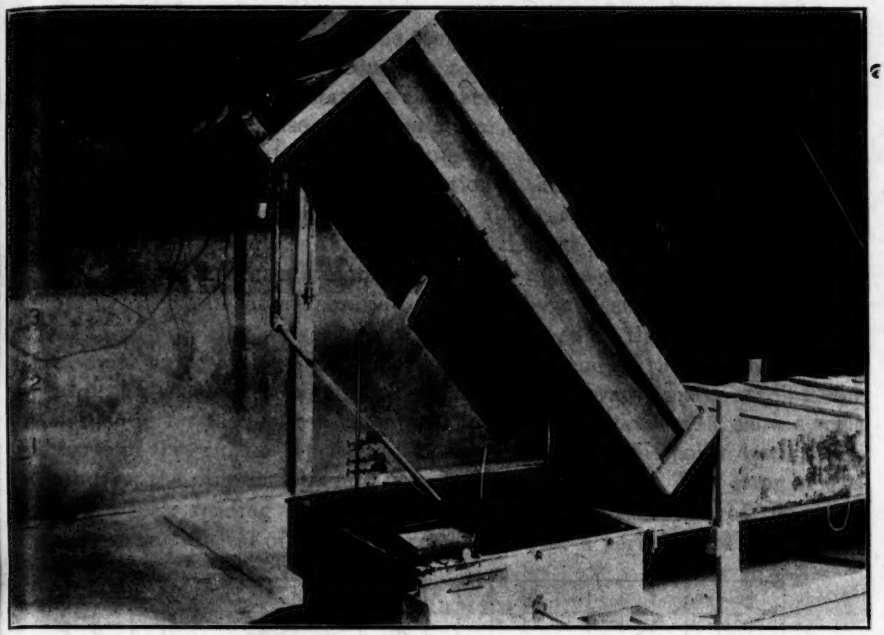


FIG. 14.—EVAPORATION EQUIPMENT FOR CONTROLLED WIND. PART OF AIR DUCT IS SHOWN RAISED TO PERMIT MAKING OF TEMPERATURE OBSERVATIONS.

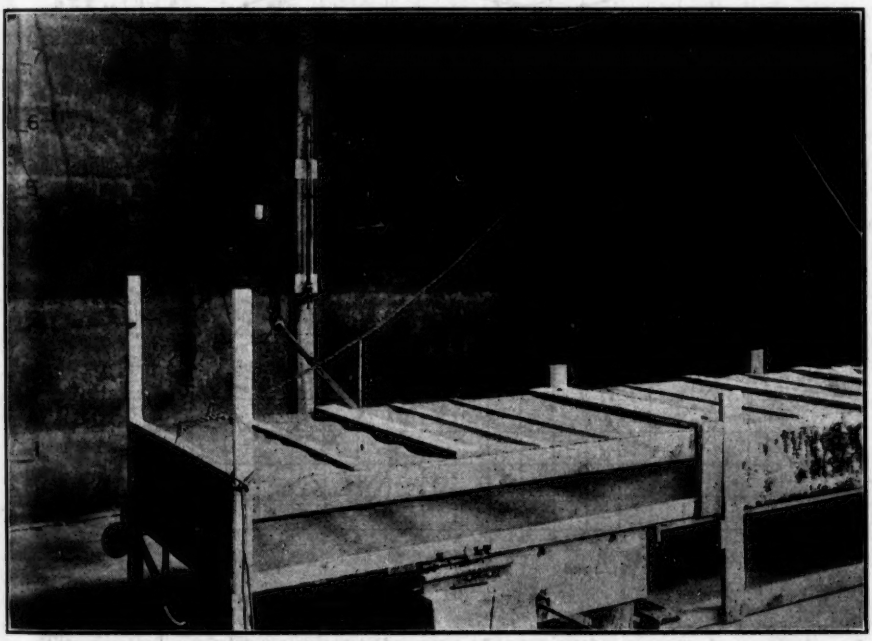


FIG. 15.—EVAPORATION EQUIPMENT FOR CONTROLLED WIND, WITH AIR DUCT LOWERED AND ANEMOMETER IN PLACE.

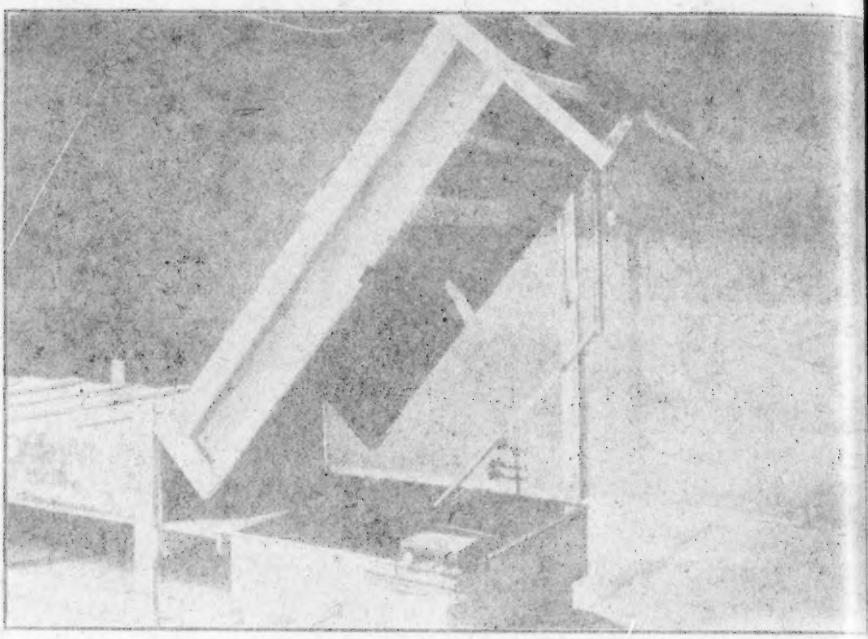


FIG. 14.—EVAPORATION EQUIPMENT FOR CONTROLLED WIND. PART OF AIR DUCT IS SHOWN RAISED TO PERMIT MAKING OF TEMPERATURE OBSERVATIONS.

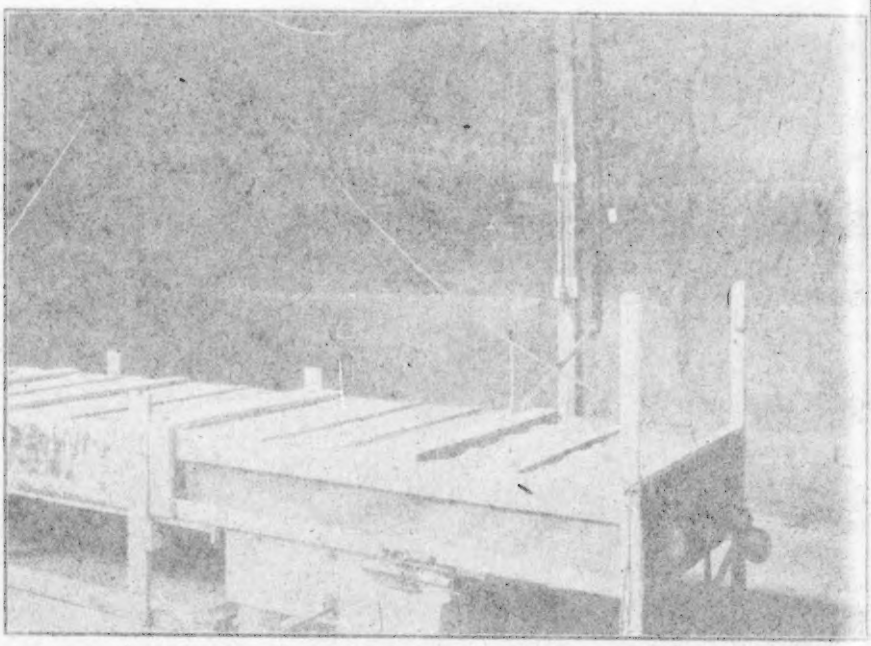


FIG. 15.—EVAPORATION EQUIPMENT FOR CONTROLLED WIND, WITH AIR DUCT LOWERED AND ANEMOMETER IN PLACE.

early afternoon of September 15 the temperatures are rising while the rate of evaporation shows a marked reduction. This is apparently true for the following day. As these temperatures rise their difference increases, and as they fall the difference decreases. The relation, difference in vapor pressure, or vapor pressure of the water surface minus the vapor pressure of the dew point, according to Dalton, shows a direct variation with the rate of evaporation. The reciprocal of the relative humidity, $R\,H$, also varies directly with the evaporation, but its variation is through a more limited range than the difference in vapor pressure. Near midnight on September 15, a decided increase appears in the observed rate of evaporation, followed by a decrease. It would seem logical to suspect that an error in reading occurred at this point, as a mean value would agree quite nicely with the general trend of the loss during this interval. To account for this apparent error in the evaporimeter reading, the original notes for the point observed at 11:24 P. M., September 15, show a scale difference of 0.24 in. for a period of 61 min. To move this point down to the mean curve, the difference should have been about 0.20 in. This error of 0.04 in. in the scale reading amounts to 0.0001 in. in the elevation of the water surface for an interval of 61 min., or 0.0024 in. for a period of 24 hours.

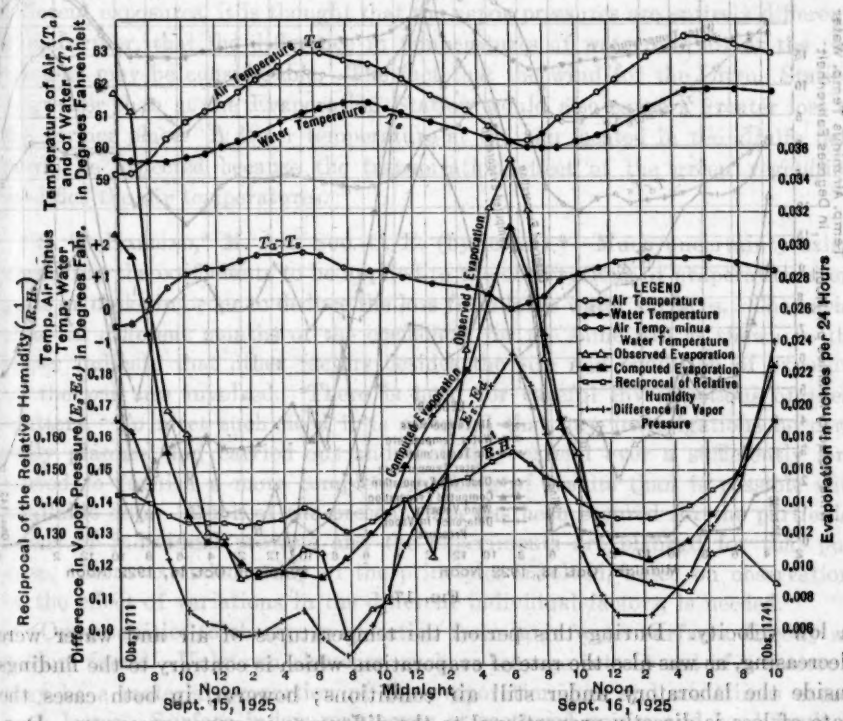


FIG. 16.

The formula used to compute the rate of evaporation per 24 hours is based on a great number of observations, and inasmuch as the study of this subject has not been fully developed, certain minor alterations or combinations may

later permit closer agreement with the observed rate of loss. The computed loss shown in Fig. 16 is based on the formula:

$$E = 0.08\,[3 - (t\,a - t\,s)]^{\frac{2}{3}}\,(E\,s - E\,d)$$

in which,

E = evaporation, in inches per 24 hours.

$(t\,a - t\,s)$ = temperature of air minus temperature of water, in degrees Fahrenheit.

$(E\,s - E\,d)$ = vapor pressure at water surface minus vapor pressure of dew point, in inches of mercury.

In Fig. 17 is shown a series of observations taken outside the laboratory under fully exposed conditions. The arrangement of the apparatus is shown in Fig. 15. Under these conditions the rate of evaporation appears to be at variance with the general trend as observed within the laboratory. For the period 6:00 P. M., October 12, to 6:00 A. M., October 13, the wind, as recorded by a standard Robinson anemometer set near the evaporation pan, indicated

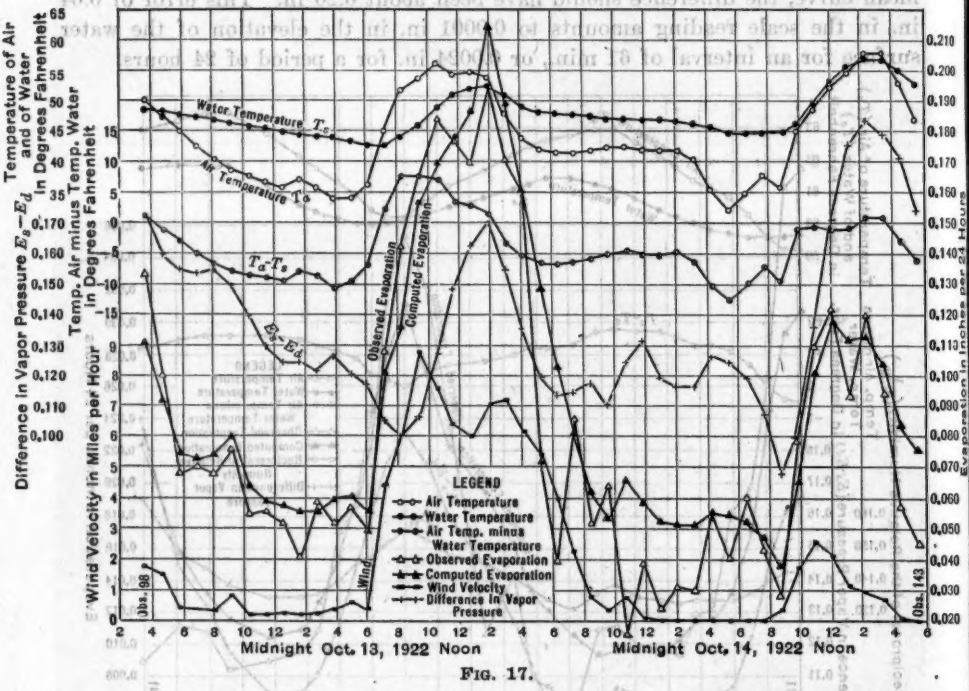


FIG. 17.

a low velocity. During this period the temperatures of air and water were decreasing, as was also the rate of evaporation, which is contrary to the findings inside the laboratory under still air conditions; however, in both cases the rate of loss is directly proportional to the difference in vapor pressures. During the day of October 13, there was considerable wind; also for this period the rate of evaporation was greatly increased. About 10:00 A. M. the rate of wind decreased but the rate of evaporation still increased, following the constantly rising difference in vapor pressure. As would be expected, the maxi-

mum rate of evaporation occurred at the time of maximum temperatures. The formula applied to compute the evaporation loss in this case differs somewhat from that developed to apply under still air conditions in that the factor for wind is introduced. This formula is:

$$E = (0.44 + 0.118\ W)\ (E\,s - E\,d)$$

in which,

E = evaporation, in inches per 24 hours.
W = wind velocity, in miles per hour.
$(E\,s - E\,d)$ = vapor pressure at water surface minus vapor pressure of dew point, in inches of mercury.

In applying this formula, with the wind factor considered, to an extended series of evaporation records of mean monthly values, promising agreement is shown.

In the matter of comparing the evaporation losses of the two stations at Yuma, Ariz., as mentioned by the author, it is believed that this difference may be explained in part by the fact that one station is situated on a desert mesa while the other is located in an irrigated alfalfa field. Because of these different exposures, it is thought that the vapor pressures are entirely different, and, further, that the difference in temperatures of water and air at the two stations may be considerable. The fact that the wind for the Citrus Station is greater than at the Evaporation Station would also cause a greater loss at the former place. A lower temperature at the pan located in the alfalfa field would be expected because the transpiration effect of the green vegetation modifies the air temperatures.

S. T. HARDING,* M. AM. SOC. C. E. (by letter).†—Much uncertainty exists regarding the coefficients to be applied to the observations of evaporation from pans or tanks in order to deduce the loss from large water surfaces. The variations in different months of the coefficient for the same pan as shown in the paper indicate that other factors besides the size and condition of exposure of the pan are involved. There is need for careful investigations of these matters. To meet such need it is essential that any investigations be carefully planned and carried out and that they extend over a sufficiently long period to furnish a more complete checking of results than is possible with available data. Much of the present data has been secured for the particular needs of individual projects and the experiments are planned for that purpose. More extended study of the principles involved, based on observations of the effect of variations in the different individual factors, is needed.

Opportunities to observe evaporation from large water surfaces directly are seldom found. Unknown elements, such as seepage into or out of a reservoir, may be a material proportion of the evaporation loss and sometimes exceed such losses. Surface inflow may occur at scattered points and be difficult of measurement. It is not usual to have periods without inflow or outflow of sufficient length to enable the evaporation alone to be observed.

* Prof. of Irrig., Univ. of California, Berkeley, Calif.
† Received by the Secretary, April 12, 1926.

There are certain types of lakes in California in which opportunity is afforded to observe evaporation directly. These are the basins receiving the surplus flow of streams in which (1) the inflow is limited to a part of the year; (2) the outflow, if any, may be measured; and (3) soil and ground-water conditions are such that seepage inflow and outflow into the lake does not occur. Among such lakes are Tulare Lake, Buena Vista Lake, and Lake Elsinore.

Tulare Lake.—Gauge height records on Tulare Lake were begun in 1906 by the U. S. Geological Survey. Pumping from the lake began in 1916. During the intervening period the fluctuations of the lake at times of no surface inflow show the evaporation loss.

Tulare Lake is a shallow depression having an area at maximum stages of about 200 000 acres. It receives the surplus flow of the Kern, Tule, and Kaweah Rivers and part of the surplus flow of Kings River. The lake bed consists of fine, impervious material. In periods of small surface inflow the lake becomes entirely dry, with no indications of ground-water seepage into it. Until recently wells in the lake bed flowed, showing the ground-water gradient to be above the bed. There is no indication of absorption by the clay sub-soil of the lake.

The records of fluctuation at periods when the flow in the contributing streams was too small to result in inflow to the lake, from 1906 to 1916, have been assembled, together with the mean monthly evaporation, in Table 23.*

The nearest station for which climatic records are available is Hanford, about 20 miles northeast of the lake. The rainfall at Hanford is probably somewhat higher than the average rainfall on the lake area. The temperatures may be generally similar.

TABLE 23.—EVAPORATION DATA FOR TULARE LAKE.

Month.	Mean evaporation, in inches.	Mean precipitation at Hanford, Calif., in inches.	Mean temperature at Hanford, Calif., in degrees Fahrenheit.
January	1.4	1.98	45.7
February	1.6	1.51	50.2
March	3.0	1.70	54.0
April	3.6	0.57	59.4
May	6.0	0.48	66.1
June	8.4	0.09	74.6
July	9.6	0.00	79.8
August	7.2	0.01	78.0
September	7.2	0.32	72.0
October	3.6	0.26	61.9
November	2.4	0.65	52.2
December	1.2	1.07	45.5
Year	55.2	8.64	61.6

Buena Vista Lake.—Buena Vista Lake is a broad shallow basin, which is now used as a reservoir, a levee having been built along its eastern side. Part of the flow of Kern River is stored in the lake for irrigation use. Some

* See, also, *Bulletin 9*, California State Dept. of Eng., entitled "Water Resources of Kern River and Adjacent Streams and Their Utilization."

observations in 1920 of the fluctuation of the lake at times of no inflow and of only limited outflow have been secured (Table 24).*

TABLE 24.—EVAPORATION FROM BUENA VISTA LAKE.*

Month.	Total lowering of lake during month, in inches.	Lowering of lake due to outflow, in inches.	Indicated lowering due to evaporation, in inches.
July	7.3	....	7.3
August	8.4	1.4	7.0
September	8.8	1.7	7.1
October	5.4	0.8	4.6
November	2.2	0.2	2.0

* No rainfall during period.

The lake was only a few feet in depth during these months. Ground-water conditions are similar to those in Tulare Lake.

Lake Elsinore.—Lake Elsinore receives the surplus run-off of the San Jacinto River. In periods of excessive run-off the lake may overflow; its outlet channel is tributary to the Santa Ana River. The area westerly from the lake rises steeply, the valley being narrow on that side. The remainder of the surrounding area is flat with some low hills. The lake has gone dry in periods of deficient rainfall.

The lake bed appears to be impervious. It is stated that no inflow from adjacent ground-water was apparent when the lake was dry. There is no indication of seepage from the lake, adjacent wells with water below lake level showing no indication of mixture with the salty water of the lake. The lake has been at relatively high stages since 1916. It has an area of about 6 000 acres and a depth of about 40 ft. at the present overflow level. There is no use for the lake water due to its high salt content.

Since December, 1915, records of elevation have been secured by the U. S. Geological Survey. These records to September, 1922, were used by the writer in a report on the San Jacinto River Hydrographic Investigation made for the Division of Water Rights, California State Department of Public Works, as the basis of an estimate of the evaporation loss. A direct record of inflow and outflow was also available. These occurred only during a part of the period covered. Data on rainfall at Elsinore were also at hand.

The inflow, outflow, rainfall, and fluctuations were used with the known area and capacity curves of the lake to estimate the evaporation by months (Table 25). These are considered to include all elements of supply and draft. The records for different months were generally consistent; in some cases variations in the reading of the lake elevation, due probably to rough water at the time of reading, gave inconsistent results between adjacent months.

Observations with Pans.—Observations of evaporation were made under the writer's direction in 1920 on Buena Vista Lake, a natural depression now used as a reservoir.†

* *Bulletin 9*, California State Dept. of Eng.

† For detail results, see *Bulletin 9*, California State Dept. of Eng.

In 1920, Buena Vista Lake received little inflow and remained at depths of only a few feet. Its area was about 10 000 acres.

Both a floating and land pan were installed. These were of the type used by the U. S. Geological Survey, 3 ft. square and 2 ft. deep. The land pan was placed about 300 ft. from the edge of the lake, and the floating pan from 100 to 600 ft. from the shore, being moved with the fluctuations of the lake. Owing to the flat slopes of the lake bed the floating pan was at times set in an excavated area in order that it should be submerged to less than 6 in. from the top. The shallowness of the surrounding water may have introduced temperature factors not applicable in deeper areas.

TABLE 25.—Estimated Mean Evaporation from Lake Elsinore.

Month.	Mean evaporation, in inches.	Mean precipitation, in inches.	Mean temperature, in degrees Fahrenheit.
January	1.8	3.43	50.0
February	0.6	2.86	52.4
March	3.0	2.72	55.3
April	4.8	0.51	60.2
May	5.4	0.39	64.7
June	6.0	0.02	72.1
July	7.2	0.10	78.1
August	7.2	0.15	77.2
September	6.6	0.20	73.2
October	5.4	0.49	65.2
November	3.6	0.95	56.9
December	2.4	2.07	51.5
Year	54.0	13.39	63.0

The usual form of raft was placed around the floating pan to prevent wave action. Some washing into the pan occurred in June and July; corrections were made by comparison with the land pan. An unexpected source of error occurred in February and March due to the use of the pan for bathing purposes by pelicans. A repetition of this was prevented by placing chicken-wire screens above the pan. The results of these observations are given in Table 26.

TABLE 26.—Evaporation from Pans, Buena Vista Lake.

Month.	Land pan, in inches.	Floating pan, in inches.	Ratio of evaporation, floating pan to land pan.
January	1.82	1.82	1.00
February	2.45	2.50*	1.30†
March	2.83	2.89*	1.88†
April	5.99	6.78	1.13
May	9.05	10.58	1.17
June	10.00	11.72	1.17
July	10.14	12.61	1.24
August	9.62	11.27	1.17
September	7.21	8.46	1.16
October	4.88	5.42	1.11
November	2.29	2.44	1.06
December	1.56	1.50	0.96
Total	67.84	77.99	1.15

* Errors in floating-pan record; figures given are estimated.

These results indicate that for the particular conditions the evaporation from the floating pan exceeded that from the land pan and that the ratio varied during the season. These observations probably present an illustration of the extent to which local factors may affect the results rather than a basis for what would be expected under more normal conditions.

The direct observations of the lowering of the lake from July to November give the coefficients for the floating and land pans shown in Table 27.

TABLE 27.—COEFFICIENTS TO BE APPLIED TO PANS TO GIVE CORRECT EVAPORATION.

Month.	Floating pan.	Land pan.
July	0.58	0.73
August	0.62	0.73
September	0.84	0.98
October	0.84	0.93
November	0.80	0.85

These coefficients may be affected by local conditions. The variations in the coefficients for different months, however, illustrate the uncertainties involved in applying pan records to the evaporation from large water surfaces.

CHARLES H. LEE,* M. AM. SOC. C. E. (by letter).†—The author has performed a valuable service to the profession in collecting and compiling the numerous records of evaporation presented in his paper. These records cover a wide geographic range, and represent varied conditions of exposure and meteorology. The data have been assembled in convenient form for reference. Such efforts will advance the present deficient knowledge of this subject.

The paper does not call for detailed comment. Before entering on a general discussion, however, the writer desires to point out that apparently there is no statement regarding precipitation, and particularly as to whether in all cases the reported depths of evaporation are gross or net. For comparative purposes it is desirable that the evaporation from rainfall caught by the pan be included in the monthly totals as well as from water added by the observer.

A precipitation record is essential in connection with evaporation observations if the record is to have any but local use. The next most important auxiliary record is the temperature of the water surface. This can easily be obtained by means of a maximum and minimum recording thermometer floating on a small wooden raft, with the bulb just submerged. Although not definitely stated in the paper, it is assumed that the temperatures reported are for the air and not for the water surface. It is believed that if a comparative record of water surface temperatures at the two Yuma stations were available, it would be found that that at the Yuma Evaporation Station would be much less than the one at the Yuma Citrus Station, and would thus show a third important factor not referred to by the author. Temperatures at low levels in an alfalfa field are usually considerably less than on the adjacent bare ground.

* Cons. Hydr. Engr., San Francisco, Calif.
† Received by the Secretary, May 8, 1926.

A complete meteorological record should also include air temperature, wind movement, and relative humidity, as stated by the author.

Controlling Factors.—The process of evaporation consists essentially of the gradual "bombardment" into the adjacent atmosphere of the rapidly moving molecules which compose a liquid. The effect of various factors on this process may be illustrated as follows: If the temperature of the liquid at and near its surface is increased, the rate of molecular vibration is correspondingly increased, and more molecules break from the liquid surface in a given time; if the degree of saturation of the atmosphere immediately adjacent to the surface is decreased, either by a lowered relative humidity of the surrounding atmosphere, or by an increased rate of wind movement, the vapor pressure on the surface is diminished and it becomes easier for molecules to break from the surface, and hence more of them leave in a given time (variations in atmospheric pressure have a similar result); finally, if the liquid is heavily charged with mineral substances in solution the motion of the liquid molecules is retarded and fewer escape in a given time. If the specific gravity of the liquid advances to such a point that crystallization occurs and a film or crust forms over the surface, a marked decrease in the rate of loss occurs.

In brief, the important factors controlling evaporation are the relative temperatures of the liquid surface and of the adjacent atmosphere, the vapor and barometric pressure immediately above the liquid surface, and the specific gravity of the liquid. For ordinary conditions, temperature and vapor pressure are the most important factors, although in special cases the specific gravity of the water must be considered. Evaporation from capillary films, such as form on snow and ice, or from moist soil or crystal surfaces, involve changed conditions with respect to the form and continued existence of the liquid body and hence introduce modifications in the rate. Otherwise, this process corresponds with that of evaporation from a free water surface.

Evaporation from Water Surface.—The author presents data in Tables 2* and 3† giving the range in annual and monthly evaporation at selected stations. For those who are interested, similar variations for 3-day periods during nearly 2½ years near Independence, Calif., have been published.‡ The data are assembled in diagrammatic form (Plate X), the maximum and minimum daily air temperature, relative humidity at 5:00 A. M. and 5:00 P. M., and average daily wind movement being shown in addition to evaporation. The diagram was prepared by the writer from submerged floating-pan evaporation observations made under his direction on Owens River, California, and meteorological observations by the U. S. Weather Bureau, 4 miles distant at Independence, Calif. The data show clearly the predominating influence that temperature exercises on the evaporation rate throughout the annual cycle, both directly, and indirectly through its effect on relative humidity. Short-period rates of evaporation show the temporary effect of high wind or abnormal humidity conditions, but these represent only minor irregularities on the curve.

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 56.
† *Loc. cit.*, p. 59.
‡ *Water Supply, Paper 294*, U. S. Geological Survey, Plate X.

The quantitative relation between the changes in the temperature of the water surface and the rate of evaporation has been carefully investigated by R. B. Sleight, Assoc. M. Am. Soc. C. E., at Denver, Colo.* All other factors being similar, he found that for each degree Fahrenheit within the natural range of large water bodies there occurred a change in the rate of evaporation averaging 6 per cent. This result agrees closely with the writer's experience at Independence, and elsewhere.

The effect of atmospheric pressure on evaporation at any one station is probably negligible. At Denver, Mr. Sleight could not detect any variation in evaporation for a range of 1 in. on the barometer. The maximum elevation at which reservoirs are constructed seldom exceeds 10 000 ft. As the normal barometric pressure at sea level is 30 in. and at an elevation of 10 000 ft., 21 in., the effect of atmospheric pressure is relatively small even when comparing evaporation rates from water at different altitudes. The observed differences of rates at different altitudes, it is believed, are due almost entirely to temperature differences.

The effect of specific gravity on the rate of evaporation has received little attention from investigators in the past as there has been no general demand for information. For the solution of problems arising in the writer's practice, he found it necessary, a number of years ago, to undertake an experimental study of the subject. The details and results of this study are presented herewith. The procedure adopted was as follows: Two samples of water, one distilled and the other a brine with a specific gravity of 1.11, taken from Owens Lake, California—a mineralized lake typical of the Great Basin—were evaporated under exactly similar conditions except as to specific gravity. The samples were placed in circular, flat-bottomed pans, 4½ in. deep and 12 in. in diameter, and filled to the same depth. The pans were left open at the top, immersed for 4 in. of their depth in an electrically heated air bath and placed in a room free from drafts. A constant temperature was maintained in the air bath by means of a thermostat. The successive depths of water in the pans were calculated from the observed weight and specific gravity of the samples and the known dimensions of the pans. The results of the test are shown on Fig. 18. It was found that the rate of evaporation from Owens Lake brine decreased as the specific gravity increased, and was 27% less than distilled water when crusting commenced at a specific gravity of 1.275 (Reading 13, Fig. 18). When the crust began to form it was broken up at short intervals. The rate as then observed was 75% less than distilled water for a specific gravity of 1.32 (Reading 17, Fig. 18) and continued to decrease with increasing densities.

As a practical check on Fig. 18 there was available for the period, January 1, 1908, to December 31, 1914, detailed data of the inflow into Owens Lake and the fluctuations of the lake level. The lake basin has no surface outlet, and its bed is completely lined with clay deposits that preclude the existence of a subsurface outlet. The average area of the exposed water surface during this period was 58.173 acres and the specific gravity of the water, 1.11. The

* "Evaporation from the Surfaces of Water and River-Bed Materials," *Journal of Agricultural Research*, Vol. X, 1917, pp. 229–230.

average annual depth of evaporation from the lake surface during the 7-year period of observation, as determined from the hydrographic data, was 60.8 in. (Table 28). The observed rate from fresh water at Owens River, near Independence, for the three years, 1908 to 1911, as observed in a floating submerged pan, was 67 in. annually.* Reducing this by 10% as indicated by Fig. 18, there results 60.3 in. for the annual loss from Owens Lake as compared with 60.8 in. as determined from the hydrographic data.

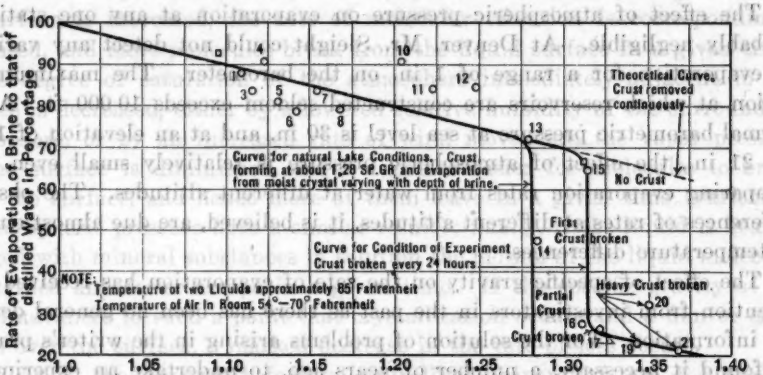


FIG. 18.—COMPARISON OF EVAPORATION FROM OWENS LAKE BRINE AND DISTILLED WATER.

It is believed that Fig. 18 is generally applicable to bodies of highly mineralized water. It should be noted, however, that crusting will probably begin at slightly differing specific gravities, depending on the chemical composition of the salts in solution.

Evaporation from Capillary Films.—The practical application of hydrology to water supply problems in Western America involves a knowledge of evaporation not only from free water surfaces, but also from capillary films surrounding soil grains, crystals of precipitated alkaline salts, and ice crystals, including snow. In addition, there should be included the evaporation from moist soil grains as modified by the transpiration losses from vegetation, the surface crusting due to the accumulation of salts precipitated from evaporating soil moisture, and the presence of clay in a highly colloidal state resulting from concentrated alkali solutions.

Less is known of the process of evaporation from capillary films than from free water surfaces. For example, it is probable that the modifying effect of concentrated alkaline salt solutions on clayey soils is unknown, except by a few soil physicists, and yet in the Great Basin region of Western America as well as in similar inland basins throughout the world, there are large areas of moist soils of this type to which some rate of evaporation must be applied by the hydrologist in estimating the ground-water supply. In this discussion it will be impossible to go into the details of these subjects. It is desired, however, to outline the problems in the hope that systematic investigation will be stimulated and ultimately lead to useful results.

* *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 177.

In general, evaporation from capillary films surrounding soil or crystal grains is controlled by the same factors as that from free water surfaces. Evaporation may take place from moist grains either at the surface of the granular body in contact with the atmosphere, or in the upper zone of the body if it is sufficiently open for atmospheric circulation. In the latter case it is termed interior evaporation. One complicating factor involved is the degree to which the capillary film is replenished with water as losses occur. If the source of replenishment is precipitation, surface flooding, or irrigation, the film has no permanent and constant source of supply, and passes through successive stages of fatness and leanness and may even disappear entirely. If, on the other hand, the source of supply is a permanent ground-water table within capillary distance the film is replenished constantly by capillarity and suffers little variation, except as the water rises or falls. In the event that the water-table drops far enough below the surface to exceed the capillary limit the upper portions of the film might be withdrawn into the material so far that evaporation in measurable quantities would cease. A condition of leanness or complete dessication of the film results in the diminution or complete cessation of evaporation. The availability of moisture controls directly the rate and total amount of evaporation from soil or crystal bodies; it has been termed the "evaporation opportunity."

TABLE 28.—DEPTH OF ANNUAL EVAPORATION FROM OWENS LAKE, CALIFORNIA, BASED ON MEASURED INFLOW AND LAKE FLUCTUATION.

(Period of Time Covered, January 1, 1906, to December 31, 1914.)

Elevation of lake surface, January 1, 1906, in feet	3 565.32
Elevation of lake surface, December 31, 1914, in feet	3 576.60
Rise in lake level, in feet	11.28
Area of lake surface, January 1, 1906, in acres	44 800.00
Area of lake surface, December 31, 1914, in acres	61 500.00
Water accumulated in lake, January 1, 1906, to December 31, 1914, in acre-feet	599 699.00
Inflow from all sources, except rainfall, January 1, 1906, to December 31, 1914, in acre-feet	3 120 125.00
Water lost by evaporation, January 1, 1906, to December 31, 1914, in acre-feet	2 520 426.00
Average area exposed water surface, January 1, 1906, to December 31, 1914, in acres	58 173.00
Total depth evaporation, net, January 1, 1906, to December 31, 1914, in feet	43.3
Annual depth evaporation, net, January 1, 1906, to December 31, 1914, in feet	4.81
Annual precipitation on surface of lake (Keeler), in feet	0.26
Annual depth of evaporation, gross, in feet	5.07
Annual depth of evaporation, gross, in inches	60.8

Until recent years observations of evaporation losses from moist soil have been confined to conditions of widely varying evaporation opportunity, either for the purpose of determining in humid regions the proportion of rainfall returned to the atmosphere from a water-shed or, in arid regions, the atmospheric losses from water applied in irrigation. These observations are well known and need no discussion, as they constitute special cases. The drainage and ground-water problems of Western America during the past ten or fifteen years have led to observations of soil evaporation from areas of shallow ground-water where the evaporation opportunity is fairly constant. It is

information of this kind which is especially valuable to the Engineering Profession, but not accessible.

For moist river-bed materials, ranging from fine sand to fine gravel, without vegetation, the investigations of Mr. Sleight at Denver give fairly complete and reliable data.* He used water-jacketed tanks, 2 ft. in diameter and 4 ft. deep, maintaining permanent water-tables at different depths and determining losses by weighing the tanks. The greatest depth to water-table was 24 in. The observations ran for three months.

For finer materials including sandy clay and clay soils having a greatest depth to water-table of 5 ft. and supporting a growth of salt grass, the writer's experiments in Owens Valley, California, may be noted.† Tanks, 7.5 ft. in diameter and 10 ft. deep, were used, set permanently in the ground, water being introduced in measured quantities into covered tanks feeding into the soil tanks at the bottom. These experiments were continued for two years.

Both sets of experiments indicated that the rate of evaporation was greatest for a saturated soil surface and diminished as the water-table lowered, becoming zero when the water-table fell below the capillary limit. They also indicated the possibility of expressing soil evaporation as a percentage of that from a free water surface, such percentage varying for different materials, and, for each material, for different depths to the water-table and different seasons of the year. The writer's conclusion, based on his experience in Owens Valley, modified by subsequent work in other localities and a comprehensive analysis of the work of other observers both in America and Europe, is that the evaporation loss from a saturated moist sand or soil is less than that from a free water surface. The ratio for loam soil and fine sand has been found to vary from 0.83 to 0.97 by different investigators; in ordinary practice the writer uses 0.90 (Table 29). Mr. Sleight's work indicates that the loss from saturated coarse sands and fine gravel was less than from fine sand and soil. The ratio decreases for moisture content at the surface less than the optimum and becomes zero when the water-table drops to the capillary limit. The exact relation between the evaporation from the soil and the depth to the water-table has not been thoroughly determined, but the indications are that it is a direct proportion.

The growth of vegetation on the soil introduces a modifying factor of greatest importance. Although it may reduce the direct soil loss by shading, etc., it greatly multiplies the evaporating surface in foliage and often increases the loss as compared with that from bare soil. Transpiration depends on the plant species, light, soil moisture available to roots, soil texture, mineralization of soil moisture, temperature, wind movement, etc. The writer's experience and studies indicate that for fresh-water vegetation which thrives with a high water-table, or in swamps, the rate of loss during the active growing season may exceed the rate from a free water surface, but that when considered for the complete annual cycle the rate is less. For ordinary uses, therefore, the ratio of maximum loss by combined evaporation and transpira-

* "Evaporation from the Surfaces of Water and River-Bed Materials," *Journal of Agricultural Research*, Vol. X, 1917, No. 5, pp. 242–259.

† *Water Supply Paper* 294, U. S. Geological Survey. C. H. Lee, 1912 (Preliminary Results); *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), pp. 181–196 (Final Results).

TABLE 29.—SUMMARY OF EXPERIMENTAL DATA EVAPORATION FROM SATURATED BARE SOIL.

No.	Observer.	Location.	Experimental Equipment		Period of observation.	Depth of Evaporation, in Inches.		Ratio, evaporation from saturated soil to water.	Reference.
			Water evaporation.	Soil evaporation.		Water.	Soil.		
1	Ebermayer	Aschaffenberg, Bavaria	Lamont evaporometer	Tanks artificially fed at bottom	Two seasons: April to September	0.97		0.97	*Bulletin No. 7*, U. S. Dept. of Agriculture, Div. of Forestry.
2	Slichter	Deerfield, Kans.	Tank in soil	Tanks artificially fed at bottom	August 6 to September 8, 1905	12.11	10.0*	0.83	*Water Supply Paper 153*, U.S. Geological Survey, p. 44.
3	C. H. Lee	Owens Valley, California		Tanks artificially fed at bottom	September, 1909, and July, 1910	7.75 (Average)	6.5*	0.84	*Water Supply Paper 364*, U.S. Geological Survey, Plate XV and Table 51.
3	Sleight			Tanks artificially fed at bottom	July 31 to October 16, 1916	18.93	*	0.91	*Journal of Agricultural Research*, Vol. X, No. 5, pp. 243-259.
5	Fortier	Davis, Calif.		Tanks artificially fed at bottom	August 2 to October 3	21.59	18.33	0.85	"Use of Water in Irrigation," Fortier. (Revised Edition).
6	Fortier			Tanks artificially fed at bottom		1.88	1.75	0.93	"Use of Water in Irrigation," Fortier (First Edition), p. 131.
	Average							0.89	

* Value taken from extrapolated portion of curve.

tion lies between 0.90 and 1.00. For salt grass in Owens Valley the writer found it to be 0.95.

There have been numerous attempts to determine transpiration and evaporation-transpiration losses during the past seventy-five years both in Europe and America. European forestry experiments have been extensive, and in America the field of agriculture has been most active. Both in Europe and America the Engineering Profession has been vitally interested from the standpoint of water supply, but little fundamental work has been done. It is to be hoped that the recent impetus that Western irrigation and ground-water problems have given to the subject will produce results of practical value. The present interest among irrigation engineers in the determination of the so-called "consumptive use" of water in irrigation, is a most promising field of investigation, and with the existence of extensive areas of irrigated land, large-scale measurements covering projects and whole valleys are being found to produce important results. In the field of ground-water hydrology this method of measurement is also coming into general use for determining losses from both irrigated and natural vegetation. Two large-scale determinations of the latter made by the writer have given results as follows:

1.—Independence region of Owens Valley, California: 54 sq. miles of natural meadow land and moist soil varying from luxuriant fresh-water swamp growth to salt-grass pasture and bare crusted alkali flats:

Average annual evaporation from free water surface..5.58 ft.
Annual evaporation-transpiration (observed loss)....2.26 ft.*

2.—San Luis Rey Valley, San Diego County, California: 6 640 acres of moist bottom-land covered with willow groves, meadow grass, etc., and with bare moist sand along the river channel:

Average annual evaporation from free water surface..5.16 ft.
Annual evaporation-transpiration (observed loss)....2.19 ft.†

An intensive study of the consumptive use of water in irrigation is being made by the Irrigation Section of the Society, and much valuable data on this subject should be available at an early date.

The writer is not aware of any thorough study of the effect of surface crusting resulting from precipitation of alkali salts, but his experience leads to the belief that material reduction in the rate of evaporation results from such crusting. It is a subject that needs investigation. Of a similar nature is the effect of an excessive amount of deflocculated colloids. The writer has observed soil in desert playas so moist that a footprint in the mud would fill with brine within a few minutes, but from which little if any evaporation was taking place, even under the great heat of summer. There is practically no data available for water losses from such materials.

Evaporation from Moist Crystal Deposits.—There are several saline and alkaline deposits in the Southwest that are free from mud, the crystal body being exposed directly to the atmosphere. Brine occupies the voids in the crystal, the upper surface of the brine during the summer period being at

* *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 204.
† *Water Supply Paper 446*, U. S. Geological Survey, 1919, p. 105.

a depth of 1 in. to 1 ft., or more, below the surface, and flooding the surface in winter. The rate of evaporation from the brine that is drawn up around the surface crystals by capillarity is a matter about which little data exist, but one which is of sufficient importance in ground-water studies to need investigation. The writer has conducted some experimental work on this subject, but the results, beyond indicating that the rate of evaporation is small, are unsatisfactory.

Evaporation from Ice and Snow.—This phenomenon is important in hydraulic studies in connection with the determination of stream run-off. It is possible that loss takes place directly from ice crystal surfaces at low temperatures, although the greatest loss occurs when the temperature is sufficiently high to permit of melting. The source of water available for evaporation is thus not drawn up from below by capillarity, but is furnished directly by the crystal surface. Some experimental work has been done in recent years, particularly in the water-sheds of the Wasatch Mountains east of Salt Lake City, Utah, the results of which have been published by the U. S. Weather Bureau.* Further study of this subject is needed.

Methods of Measurement.—Although there is general recognition of the individual factors that control evaporation and the relative importance of these factors is becoming understood, yet, judged by the diversity in methods of observation and the frequent failure of observers to state the observational conditions, there is not full appreciation of the practical bearing of these factors. As a result there has been a lack of standardization in methods for measurement, the absence of which has resulted in much inaccuracy and confusion. It is believed that much of the difficulty could be obviated if comparable experimental conditions were secured in making the observations. It is common experience to find pans, established for the purpose of determining evaporation from a reservoir surface, placed so that the temperature of the water in them has no relation to that of the surface of a large body of water. Pans are sometimes completely surrounded by air; they are often placed on the ground; and, in some cases, they are set on a float anchored on the surface of a body of water. In these "unnatural" positions they are exposed on all sides to the air and direct sunlight and are subject to abnormal increase in temperature by the heat absorption and radiation effects of the metal pan.

The most disconcerting feature of these "unnatural" exposures is that no general relation can be established that will enable results to be converted into usable data. For example, the depth of evaporation from water in a land pan set on the surface of the ground or on a platform may vary from 10% less to 70% greater than that from an adjacent lake or reservoir surface, depending on the local combination of meteorological factors and the heat-absorptive character of the container and its surroundings. The writer has personally observed this range of variation in different localities by running parallel series of observations using the Class A land pan adopted by the U. S. Weather Bureau and standard submerged floating pans. The con-

* "Some Field Experiments on Evaporation from Snow Surfaces," by F. S. Baker, *Monthly Weather Review*, July, 1917, p. 363.

trolling factor is the relative speed with which the temperature of small and large bodies of water changes. The contents of the exposed pan heat rapidly in the day and cool just as rapidly at night. The large body, on the other hand, holds to a fairly constant temperature day and night. In a region of warm nights the temperature of the pan holds up at night and the average for the day exceeds the average for the large body. In a region of cold nights the reverse occurs. The latter condition is well illustrated by observations made under the writer's supervision at Lake Almanor, California, where the nights are characteristically cold (Fig. 19). On this diagram the pan designated "Sutter Butte Pan No. 1", was a Class A Weather Bureau pan while the lake pan was of the submerged floating type. As a result, in the first case the rate of evaporation from the exposed pan exceeds that from the large body of water, and in the second case it is less. The writer's experience has led to the conclusion that in order to use data obtained from the Class A land pan it is necessary to determine a correction factor, which will differ for every locality or meteorological condition.

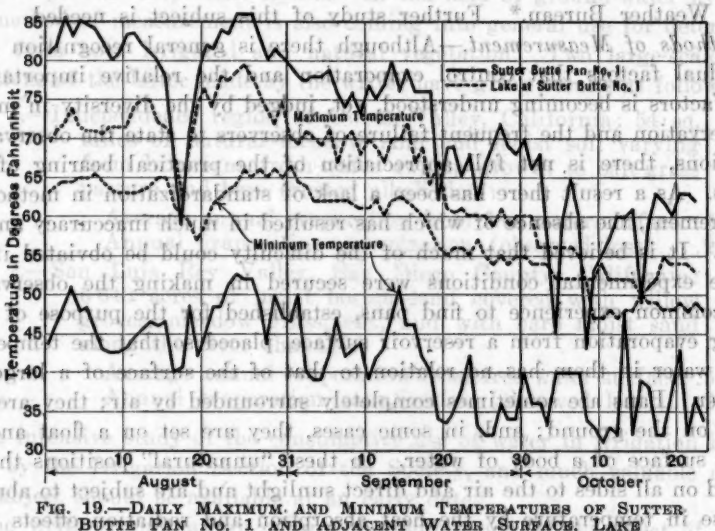


FIG. 19.—DAILY MAXIMUM AND MINIMUM TEMPERATURES OF SUTTER BUTTE PAN NO. 1 AND ADJACENT WATER SURFACE, LAKE ALMANOR, AUGUST TO OCTOBER, 1924.

Land pans, placed below the surface of the ground 2 or 3 ft., or still better, floating pans submerged in a large body of water, approach much closer to the actual temperature conditions of a lake or reservoir than does any type of pan the sides of which are exposed. Experience has shown that the temperature of water in submerged floating pans that are made of light-colored metal and kept clean, does not ordinarily vary more than 1° or 2° Fahr., from that of the surrounding water. This is illustrated by Fig. 20 presenting the results of maximum and minimum thermometer readings for Lake Almanor, at Prattville, Calif. Thermometers were fastened to floats with the bulbs just submerged and the observations represented surface temperatures.

In the matter of vapor pressure there is an even greater departure from natural conditions. Often, attention is not given to insuring the same degree of free air movement. Pans are placed near the vertical faces or objects that break the wind, or are closely surrounded by protecting fences. The writer has even observed land pans about which the grass has been permitted to grow, so as almost to prevent any air movement across the water surface.

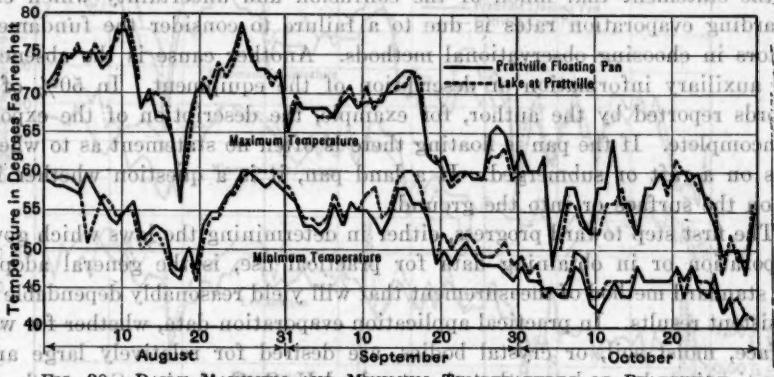


FIG. 20.—DAILY MAXIMUM AND MINIMUM TEMPERATURES OF PRATTVILLE FLOATING PAN AND ADJACENT WATER SURFACE, LAKE ALMANOR, CALIFORNIA, AUGUST TO OCTOBER, 1924.

Then, again, the size of the pan has an important influence, which often is not given consideration. The ease with which the saturated vapor from a water surface escapes into the atmosphere, is affected by the size of the surface. A lake or reservoir, considered as a whole, has not as great an opportunity for dissipation of its vapor as a pan, as escape is practically limited to the vertical direction. The perimeter of a small pan, however, is relatively large, compared to its area, as it varies directly with the diameter, while the area varies as the square of the diameter. The vapor dissipating horizontally from a small pan thus bears an appreciable ratio to the total, whereas from a large body of water it is practically negligible. The ratios of area and perimeter also have a bearing on marginal losses through wave action and capillary movement.

Another important relation, often lost sight of, is that of the ratio of rates of wind movement to the distance across a body of water. A wind moving at the rate of 10 miles per hour, for example, would cross a 3-ft. pan in 0.2 sec., whereas to cross a lake 1 mile in diameter would require 6 min. These examples are sufficient to show that marked differences in the rate of evaporation may occur from water surfaces of different sizes. In so far as the vapor pressure is involved, a submerged floating pan, if fully exposed to the wind, would seem to give results most comparable to the large body of water, as it is covered by the same vapor blanket and therefore its size does not introduce the element of horizontal vapor dissipation.

The specific gravity of the water is sometimes overlooked in making comparisons of evaporation rates. Well-placed pans are sometimes not emptied at regular intervals and the water concentrates until its density considerably

exceeds that of fresh water. An error of considerable magnitude thus may be introduced. A similar error would result from the use of a correctly observed evaporation rate from fresh water in determining losses from mineralized bodies of water, such as Salton Sea, or Great Salt Lake, and *vice versa*.

A review of these examples, it is believed, will fully bear out the writer in the statement that much of the confusion and uncertainty which exists regarding evaporation rates is due to a failure to consider the fundamental factors in choosing observational methods. Another cause is the absence of full auxiliary information or description of the equipment. In 50% of the records reported by the author, for example, the description of the exposure is incomplete. If the pan is floating there is often no statement as to whether it is on a raft or submerged. If a land pan, it is a question whether it is set on the surface or into the ground.

The first step toward progress, either in determining the laws which govern evaporation or in obtaining data for practical use, is the general adoption of a standard method of measurement that will yield reasonably dependable and consistent results. In practical application evaporation data, whether for water surface, moist soil, or crystal bodies, are desired for relatively large areas. Investigations have shown that the standard U. S. Geological Survey floating pan submerged in water gives results that closely approximate the actual loss from a large body of water and that for this pan successive observations for short periods depart less from the mean than for any other measuring device in use. These considerations, and the fact that the floating submerged type of pan seems to reproduce most nearly the natural conditions with regard to factors controlling evaporation, weigh heavily in favor of the adoption of this method of measurement as a standard for water surfaces.

In certain localities where strong winds are prevalent great difficulty has been experienced in overcoming splash. The land pan or tank set into the ground as nearly to full depth as possible, and near the margin of the water body, is believed to be the best alternative method for use under such conditions. Neither the depth nor diameter of the tank should be less than 3 ft. Usually it is possible to obtain satisfactory observations from floating pans for occasional short periods even in windy districts. By establishing both land and floating pans the continuous record obtained from the land pan can be used as a basis for determining what the floating pan evaporation would have been by use of the factors obtained when records of both are at hand. These factors should be available for different times of the year as they vary considerably.

To illustrate the relations of evaporation for a pair of land and floating pans Fig. 21 is presented. The observations were made under the writer's direction at Lake Almanor, near Prattville, Calif. The land pan was located within 100 ft. of the water's edge in moist soil, and was set in the ground so that the top rim was almost flush with the surface. The differences in depth of evaporation as observed in the two pans correspond closely with the observed temperature differences. During August and September the loss from the land pan was about 15% greater than from the floating pan. In

October the two were practically the same. The writer's experience has been that for the full annual cycle there is little difference between the land pan set in the ground and the submerged floating pan, but that during the summer the rate of loss from the former is greater than and in winter is less than the latter. It is necessary, therefore, to obtain factors at intervals throughout the annual cycle in order to get complete records.

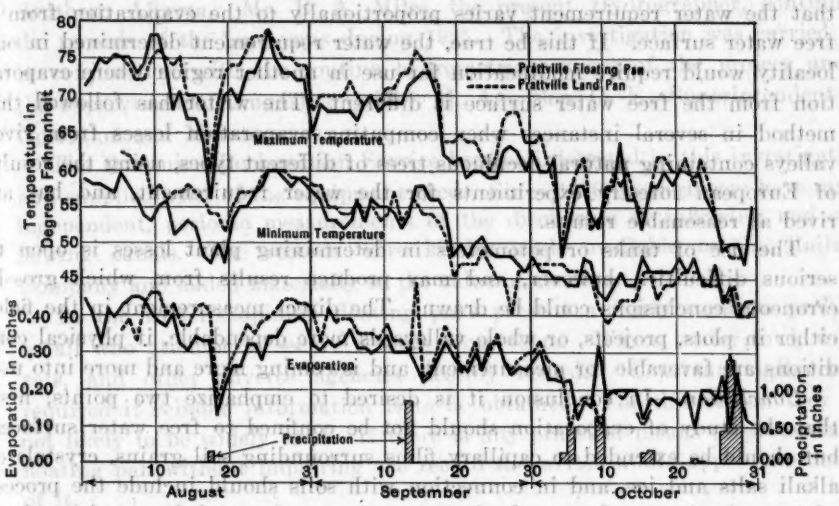


FIG. 21.—DAILY MAXIMUM AND MINIMUM TEMPERATURES OF WATER SURFACE, PRECIPITATION, AND EVAPORATION FROM PRATTVILLE FLOATING AND LAND PANS, AUGUST TO OCTOBER, 1924.

The measurement of evaporation from soil and the evaporation-transpiration loss from cultivated crops and natural vegetation is more in need of standardization even than that of evaporation from water surfaces. To be of use to engineers such data should be obtained by uniform methods and be expressed in terms of a generally accepted standard. Evaporation from a large free water surface as measured by the submerged floating-pan method is probably the best basis for reference. If a full annual temperature cycle is covered by the data it is believed that either the land or floating pan would be satisfactory for measuring water evaporation. In view of the wider availability of floating pan records it is believed that this form of apparatus is preferable, although for shorter periods the land tank may be more desirable as temperature conditions are more comparable.

For moist soil and close growing vegetation, such as grass, the loss may be determined direct by means of soil tanks. For other types of low growing vegetation, such as shrubs and trees, the potometer is used. In the case of perennial plants and trees the dry leaves are weighed, while annuals are cut at the root crown, and then stalk, leaves, and seed are weighed. The ratio of the weight of dry matter produced to water absorbed is then obtained. A great deal of similar data have been observed by soil physicists and agricultural and forestry investigators. Such data give relative values for various types of vegetation but must be converted to obtain equivalent depth of

water over the land area covered by the growth. For this purpose the weight of dry material produced by a unit of land area must be known. In the case of agricultural field crops the acre yield should be sufficient. For trees, shrubs, and natural vegetation, typical areas must be selected and the leaves stripped, dried, and weighed. It is probable that transpiration from the same variety of plant differs according to the locality. Certain investigators state that the water requirement varies proportionally to the evaporation from a free water surface. If this be true, the water requirement determined in one locality would require modification for use in another region where evaporation from the free water surface is different. The writer has followed this method in several instances when computing evaporation losses from river valleys containing natural deciduous trees of different types, using the results of European forestry experiments for the water requirement, and has arrived at reasonable results.

The use of tanks or potometers in determining plant losses is open to serious difficulties, however, and may produce results from which grossly erroneous conclusions could be drawn. The direct measurement in the field, either in plots, projects, or whole valleys, is more dependable, if physical conditions are favorable for measurement, and is coming more and more into use.

Conclusion.—In conclusion it is desired to emphasize two points, first, that the study of evaporation should not be confined to free water surfaces, but should be extended to capillary films surrounding soil grains, crystals of alkali salts and ice, and in connection with soils should include the process of transpiration; and, second, that there is greatly needed a study and an authoritative adoption of standard methods of measuring the various types of evaporation. It is believed that the activities of the Irrigation Section of the Society in the investigation of the subject of evaporation could well include the promotion of research in these fields.

C. M. O'Neil,* Esq. (by letter).†—The author has brought together in a concise and interesting paper much valuable data on the loss by evaporation from pans installed on United States Reclamation Projects.

It is to be hoped that in the not too distant future records of these losses obtained by the use of standard equipment and standard methods of installation may be available from many more points on the American Continent. Not only the Engineering Profession but the public is beginning to feel the need of a better knowledge of losses from evaporation. A better general knowledge of the application of collected data to large bodies of water is also desirable. However, so many factors influence the rate of evaporation that any relevant formula can only tend toward increased accuracy in estimating or computing this loss.

The rate of evaporation varies locally from day to day and from year to year, particularly in latitudes subject to ice conditions during the winter season. It may also vary widely in different locations depending on climatic, topographic, and surface conditions as to cultivation, vegetation, etc. It also

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† Received by the Secretary, May 3, 1926.

apparently varies considerably with the type of pan used and the method of installing and operating it.

During the years 1919 to 1924, inclusive, the writer conducted investigations, hoping to obtain data that would enable him to segregate seepage from evaporation losses occurring at the Lake Newell Reservoir on the Eastern Section of the Canadian Pacific Railway Company's Irrigation Block in Southern Alberta. Mr. B. A. Miles, the present Hydrographer, continued the records of the land pans during 1925. The investigation was carried on in conjunction with the routine hydrometric surveys of the project under the supervision of Augustus Griffin, M. Am. Soc. C. E., Superintendent of Operation and Maintenance.

The Dominion Reclamation Service became interested in this investigation and supplied most of the equipment used. Since 1920 its engineers have made independent, periodic measurements of the discharges of inflowing and outflowing canals. The records were kindly made available to the Railway Company and added materially to the value of its data.

In the writer's opinion floating pans present a fairly common problem to all who attempt to secure records from them. Waves, water fowl, water rats, and other adverse agencies destroy records. Constant vigilance is required if reliable information is to be obtained. For this reason they are not likely to be widely used. If there is any practical means of protecting a floating pan without impairing the record the writer would appreciate details of it.

The Lake Newell Reservoir occupies a natural depression in an area of uncultivated, rolling prairie land. Its approximate location is 100 miles east of Calgary, Alberta. At full supply level it covers an area of 16 430 acres, is 8 miles in length, and averages 3½ miles in width. Water is conveyed to it through two large canals and it, in turn, supplies two canals capable of diverting an aggregate of 1 280 cu. ft. per sec. Daily discharges of the canals are obtained from the continuous records of Steven's water-stage recorders. Its range of stage above the full supply level of the outflowing canals is from Elevation 2 471 to Elevation 2 485 and the available storage is 189 000 acre-ft. at full supply level. The maximum water depth is about 60 ft., the lower portion of the storage not being available. Corresponding to fluctuations in stage the drainage area of virgin prairie contiguous to the water surface varies roughly from 3 000 to 9 000 acres. Little is known of the nature of the subsoil of this particular area. The surface soil of adjacent areas may be described as varying from a sandy to a silty loam, to 2 ft. in depth. During the summer months a dense growth of aquatic plants flourishes along the margin and in bays in water less than 12 ft. deep. Water was first admitted to the basin in the summer of 1914 and the storage was gradually built up. There were 12 000 acres under water in the fall of 1918.

The first complete season's record of evaporation from pans was obtained in 1919. Since then two land pans have been continually in operation during the open-water period of each year. A floating pan was operated from 1919 to 1924, inclusive.

One pan was set on the north shore of Lake Newell approximately 40 ft. back from the foreshore at a height above the lake surface varying from 5 to 10 ft., according to the lake elevation. As this point was 5.5 miles south of the project headquarters at Brooks it was seldom that observations could be taken more frequently than once a week until 1923 when an observer became available. This resulted at times in losses of as much as 2 in. before refilling.

As a check on this pan and to admit of a closer study, a similar land pan was installed 40 ft. from the shore of a reservoir covering about 40 acres in the immediate vicinity of Brooks. Here the water surface is about 35 ft. below the full supply level at Lake Newell and is 500 ft. wide in front of the land pan. The exposure and topographical features of both locations corresponded closely in 1919, but since then the area around Brooks has come under cultivation and irrigation with thousands of trees making a vigorous growth. As the prevalent winds are from the northwest and southwest and the parkland lies to the northeast across the reservoir from the pans, it is doubtful whether the trees have had any appreciable influence on the records.

On this latter body, centrally located on 6 ft. of water, a floating pan, 18 in. deep and 3 ft. in diameter, was suspended in the water from a raft at a point approximately 300 ft. from the land pan.

Both land pans were made from No. 22 gauge, galvanized metal, 4.0 ft. in diameter and 20 in. deep (Canadian Government standards). Wells for the pans were cut in the soil 20 in. deep and 4.0 ft. in diameter. A wooden post 4 in. square was driven into the center of the bottom leaving slightly more than 2 in. projecting, on which the bottoms of the pans had a bearing to prevent buckling. Two wooden breathers with openings 2 in. square were embedded vertically on opposite sides of the pans to provide for the free circulation of air through the 2-in. space beneath the pans. This arrangement left from 1½ to 2 in. of the pans projecting above the ground surface. Water in the pans was set at ground level, or about 2 in. below the rim. This proved ample to accommodate the heaviest average rainfall. Rain gauges were placed 18 in. above the ground and 3 ft. from the pans. Both were protected by a 2-in., wire-mesh fence, 12 ft. square and 5 ft. high. An anemometer was placed near the pan at Brooks with the cups 2 ft. above ground.

The floating pan (No. 22 gauge, galvanized metal) was floated in much the same manner as those used on U. S. Reclamation projects except that no honeycomb baffles were placed within. The water level was maintained as far as possible 4 in. below the rim or slightly below the surface of the surrounding water in the reservoir. The 4 in. were deemed necessary to prevent sloshing during light winds. As records from this floating pan, 3 ft. square, and from land pans, 4 ft. in diameter, were not considered properly comparable, a pan identical with the land pans was floated from a separate raft a few hundred feet from the square pan during the summer of 1923. The circular pan was immersed somewhat deeper than the square pan to equalize the reflecting areas of metal of each pan above the water, both inside and outside. The season's record showed that the amount evaporated from the

smaller pan was 1% greater than that from the larger. The square floating pan was then discontinued.

Gauge readings were made once or twice daily for the land pan at Brooks Reservoir. Accurate readings of the gauges in the floating pans could only be made during calm weather when the pans were refilled. The water in the pans was changed often enough to prevent mossy formations but light silt adhering to the sides or deposited on the bottom was not removed as it was thought that this inside coating approached reservoir bed conditions. Little difficulty was experienced in the operation of land pans but floating-pan records were occasionally destroyed or rendered doubtful. All doubtful records were discarded and no attempt was made to interpolate the daily evaporation for periods of lost or doubtful records extending over two days. Since 1922 the average fluctuation of the water level in the land pan at Lake Newell has not exceeded ¼ in.

It is noteworthy that the records have consistently shown the average rate of evaporation from a floating pan to be greater than that from a land pan, while records collected on the U. S. Reclamation projects show the reverse ratio. This variation is possibly due chiefly to the difference in dimensions and to the method of installing the land pans.

In the aggregate these records have yielded the following results:

1.—The depth of water lost from a land pan 20 in. deep and 4 ft. in diameter set in soil is 80% of the loss from a similar pan suspended from a wooden raft.

2.—The loss from a floating pan 18 in. deep and 3 ft. square is 1% greater than from a floating pan 20 in. deep and 4 ft. in diameter.

3.—The loss from a land pan installed on the windward side of the 40-acre reservoir near cropped and irrigated areas varied from 93 to 101% (averaged 95%) of the loss from a similar pan, similarly installed, on the leeward side of a 12 000-acre reservoir 5.5 miles south in an uncultivated area of rolling prairie.

Mr. Houk gives a brief summary of the more important features of the investigations of the U. S. Reclamation Bureau to date. The following relations are extracted from this summary:

"4.—The evaporation from a land pan, 14 in. deep and 3 ft. square, set in the ground with earth banked around the sides, was found to be only 72% of the evaporation from a Class A land pan, 10 in. deep and 4 ft. in diameter, set on the ground with no earth around the sides."

* * * * * * *

"6.—Evaporation from a floating Class A pan mounted on a raft was found to be 67% of the evaporation from a similar pan on land."

Assuming that this Class A floating pan and the Canadian Reclamation Service standard circular floating pan used on the Brooks Reservoir may be considered as forming a common basis for comparison, the following relation between the two records may be established:

(*a*).—The U. S. Reclamation Bureau pan, 14 in. deep and 3 ft. square, set in soil and banked gives a loss by evaporation of 108% of that from the Class A floating pan.

(*b*).—The Class A land pan set on land gives a loss of 149% of that from the Class A floating pan.

(*c*).—The Canadian Reclamation Service standard land pan as installed at Brooks gives a loss of 80% of that from the similar floating pan.

By deduction it is found that the amount of evaporation from the Canadian standard pan is 74% of the loss from the U. S. Reclamation Bureau square land pan and 54% of the loss from the Class A land pan.

It will be seen that the loss by evaporation from the Canadian Reclamation Service standard land pan is much less than from the U. S. Reclamation Bureau land pans when compared with the loss from floating pans.

It is quite probable that the distance below the rim of the pan at which the water surface within was maintained had a measurable influence on the rate of evaporation. However, considering the conditions under which it was necessary to operate it was not deemed advisable to maintain the water level any higher.

It has not been possible to obtain data from which to make comparisons of relative humidity and wind movement at the Lake Newell and Brooks Reservoirs. No doubt the larger size of the body of water at Lake Newell is conducive to a greater wind movement. It may be due to the fact that the land pan at Lake Newell was on the leeward side of the reservoir that the average rate of evaporation there has been found to exceed that of the land pan at Brooks, which was on the windward side, by 5 per cent.

TABLE 30.—TOTAL LOSSES AS RECORDED FOR WINTER SEASONS.

Season.	Change in stage: depth, in feet.	Precipitation: depth, in feet.	Total water lost: depth, in feet.
1918–19	0.04 decrease	0.16	0.20
1919–20	0.07 increase	0.14	0.07
1920–21	0.05 increase	0.13	0.08
1921–22	0.07 decrease	0.09	0.16
1922–23	0.04 increase	0.08	0.04
1923–24	0.07 decrease	0.03	0.10
1924–25	0.01 increase	0.13	0.12

The average depth of water lost from the Lake Newell land pan for the 7-year period is 108% of the total loss of water from the reservoir due to all causes. A study of Table 35 will show that the seepage loss, or loss other than that measured by the evaporation pan, when taken by the season is only in evidence for seasons when storage has been increased and an area not previously saturated was flooded. The figures given for run-off under balancing factors to satisfy the excessive evaporation loss are not compatible as they are in every case greatly in excess of the probable maximum run-off from the tributary drainage basin. It will further be observed that the land-pan record should be reduced by as much as 30% in certain seasons to correspond with the actual loss from the reservoir. That the correction varies widely for different seasons will also be observed. The problem that still presents itself is: What corrections can safely be applied to the evaporation measured in land pans that will reveal with some measure of accuracy the quantity of water lost from this reservoir to adjacent areas by seepage?

Assuming that the seepage loss is negligible the corrections are indicated in Table 35; but what weight should be given to the various factors causing the seasonal differences? The writer is not able to make any suggestions that would be of value in this direction.

TABLE 31.—RELATION BETWEEN THE RATES OF EVAPORATION AS MEASURED FROM A LAND PAN AND A FLOATING PAN AT BROOKS RESERVOIR, ALBERTA, CANADA.*

Month.	Ratio: Floating Pan / Land Pan.						Averages.	
	1919 May 9-September 30.	1920 June 1-October 5.	1921 May 15-October 31.	1922 June 1-October 17.	1923 July 1-October 18.	1924 June 1-October 7.	Floating pan / Land pan	Land pan / Floating pan
May	1.13	....	1.19	....	....	....	1.16	0.86
June	1.18	0.96	1.34	1.29	....	1.14	1.18	0.85
July	1.23	1.13	1.18	....	1.30	1.18	1.20	0.83
August	1.51	1.11	1.28	....	1.40	1.29	1.32	0.76
September	1.47	1.10	1.45	1.49	1.39	1.40	1.38	0.72
October	....	1.10	1.42	1.42	1.49	1.19	1.32	0.76
....	1.30	1.08	1.31	1.40	1.39	1.24	1.28	0.80

* Floating pan used from 1919 to 1922 was 3 ft. square and 18 in. deep; that used from 1923 to 1925 was 4 ft. in diameter and 20 in. deep.

TABLE 32.—SHOWING DIFFERENCE BETWEEN EVAPORATION AS OBSERVED FROM LAND PANS SET 40 FEET FROM THE MARGINS OF LAKE NEWELL AND BROOKS RESERVOIRS.*

Year.	Lake Newell.		Brooks.		Brooks / Lake Newell
	Period of record.	Evaporation, in feet.	Period of record.	Evaporation, in feet.	
1919	May 20-October 23	2.573	May 9-October 23	2.549	....
1920	May 26-October 16	2.439	May 26-October 16	2.200	0.90
1921	May 16-October 31	2.640	May 16-October 31	2.477	0.94
1922	May 16-October 11	2.033	May 16-October 3	1.773	....
1923	May 9-October 31	2.229	May 5-October 18	2.030	....
1924	May 1-October 7	2.299	May 1-October 7	2.229	0.97
1925	May 4-September 30	2.007	May 4-September 30	2.031	1.01

* Brooks land pan is located on the windward side of the reservoir near young park land and area under cultivation and irrigation. Lake Newell land pan is on the leeward side of the reservoir in virgin prairie.

That loss by seepage from this particular reservoir is indeed small is fairly well indicated by records taken during the winter seasons when it was under complete ice cover and the canals were not in operation (Table 30). Winter precipitation is generally in the form of snow. The drifting over the reservoir surface and from adjacent areas doubtless introduces an error. To

what extent evaporation from the surface of ice and snow may account for the losses here given is problematical. Condensation from the atmosphere may also be an influencing factor.

TABLE 33.—MONTHLY AVERAGES OF RECORDS FOR LAKE NEWELL RESERVOIR.*

Month	Years of record.	LAKE NEWELL RESERVOIR: TOTAL EVAPORATION, IN INCHES.				Wind, in miles per hour.	Average temperature, in degrees Fahrenheit.	Average precipitation, in inches.
		Mean monthly.	Maximum.	Minimum.	Range.			
January	7	Ice conditions				2.3	12.83	0.47
February	7	" "				2.2	13.63	0.36
March	7	" "				2.9	22.33	0.64
April	7	" "				3.4	40.84	1.16
May	7	4.13†	....	....	....	2.9	52.23	1.21
June	7	6.14	7.13	5.25	1.88	2.1	61.31	1.56
July	7	7.04	8.78	5.42	3.36	2.0	67.97	1.73
August	7	5.69	7.61	4.14	3.47	2.0	64.82	1.58
September	7	3.98	5.10	2.05	3.05	1.9	54.21	1.37
October	6	2.12‡	....	....	....	2.1	41.72	0.62
November	7	Ice conditions				1.5	25.15	0.42
December	7	" "				1.8	14.35	0.50
Yearly average		29.11	(Open-water period)			2.26	39.28	11.52

* Temperatures and precipitation are complete for 7-year period, 1919 to 1925, inclusive. Anemometer records cover years 1923, 1924, and 1925.

† Average for last 18 days.

‡ Average for first 20 days.

TABLE 34.—EVAPORATION RECORDED AT LAKE NEWELL RESERVOIR FROM A LAND PAN.

(Depths are given in inches.)

Month.	1919 May 19 to October 23.	1920 May 26 to October 16.	1921 May 16 to October 31.	1922 May 16 to October 11.	1923 May 9 to October 31.	1924 May 1 to October 7.	1925 May 4 to September 30.	Average.	Remarks.
May	2.94	1.06	2.75	2.71	3.94	5.53	5.33	4.18	Average of last 18 days.
June	7.13	6.65	6.46	5.88	5.25	5.65	5.60	5.14	.....
July	8.56	7.00	8.78	5.42	5.77	7.68	6.11	7.01	.....
August	5.68	7.61	7.35	5.46	4.14	4.58	4.99	5.69	.....
September	4.94	5.10	3.49	4.14	4.43	3.72	2.05	3.98	.....
October	1.63	1.86	2.80	0.78	2.82	0.42	....	2.12	Average of first 20 days.
Total	30.88	29.27	31.63	24.40	26.35	27.59	24.08	29.10	.....

In Table 30 the total losses in every case cover the months of December, January, and February, and while they may appear small there is no reason to believe that subterranean water reaches and augments the reservoir storage.

TABLE 35.—SYNOPTIC CHART SHOWING RELATION OF TOTAL LOSS OF WATER FROM LAKE NEWELL RESERVOIR TO THE LOSS BY EVAPORATION BASED ON THE RECORD FROM A LAND PAN 40 FEET FROM ITS SHORE LINE.*

Year.	Period of record.	Change in surface elevation.	Change in quantity stored, in acre-feet.	Delivered to reservoir canals and precipitation, in acre-feet.	Withdrawn by canals, in acre-feet.	Total loss from reservoir, in acre-feet.	Evaporation Loss.		Ratio of land pan evaporation to total reservoir loss, percentage.	Balancing factors + run-off etc. — seepage, in acre-feet.
							Depth from land pan, in feet.	Equivalent from reservoir, in acre-feet.		
1919	June 2..... September 30....	2 474.42 74.32	— 1 198	37 257	18 631	19 824	2.192	25 871	130	+ 6 047
1920	May 26..... October 16.....	74.95 74.28	— 8 027	52 822	36 863	23 986	2.439	28 850	120	+ 6 699 May 25-July 31. — 2 229 August 1-September 30. + 394 October 1-16.
1921	May 16 ..... October 31.....	74.24 75.78	+ 18 764	111 616	59 962	32 890	2.640	301 78	92	+ 4 473 May 16-August 31. — 7 185 September 1-October 31.
1922	May 16..... October 11.....	75.86 73.52	— 28 179	58 118	65 728	20 569	2.083	24 181	118	+ 4 198 May 16-August 31. — 586 September 1-October 11.
1923	May 9..... October 18.....	73.43 75.98	+ 31 053	119 424	64 883	23 988	2.229	25 401	106	+ 5 777 May 9-June 30. — 5 003 July 1-September 30. + 689 October 1-18.
1924	April 23..... September 30....	75 60 79.73	+ 55 038	173 702	82 622	36 042	2.359	31 056	86	+ 2 684 April 23-30. — 914 May. + 1 120 June and July. — 7 876 August and September.
1925	May 4..... September 30....	79.52 76.69	— 38 272	69 802	81 436	26 138	2.007	27 338	104	+ 4 276 May 4-July 31. — 3 076 August 1-September 30.
.....	Average.....	.....	.....	.....	.....	.....	.....	.....	108	.....

* Balancing factors have been copied from detailed annual reports. Yearly mean surface area of reservoir may be found by dividing depth of evaporation into the equivalent loss from the reservoir in acre-feet.

If the records from the floating pan used on this project are comparable with the records obtained from the Class A floating pan used by the U. S. Reclamation Bureau it appears that the evaporation loss measured in the Canadian type of land pan approaches actual reservoir loss more closely than does the loss from the land pans described in the paper.

In this discussion and in Tables 31 to 35, in an abridged form, an effort has made to point out some of the important factors that affect the records of evaporation from pans. The variable relation that these records may have to the loss from a large reservoir is shown by Table 35.

J. B. LIPPINCOTT,* M. AM. SOC. C. E. (by letter).†—The author has presented a valuable record of evaporation as measured by standard methods on the various projects of the U. S. Reclamation Service in arid America. The determination of evaporation losses in the dryer sections of the country is of importance because cycles of drought as great as seven years in succession are recorded, which call for large storage capacity to hold over from years of flood. For instance, the Henshaw Reservoir in San Diego County, California, is built to a capacity of 164 000 acre-ft., and the net safe yield therefrom is taken at only 24 000 acre-ft. With evaporation ranging from 4 to 5 ft. and hold-overs for such long periods, the loss of impounded water by evaporation is likely to be from 30 to 50% of the total. There is much difference of opinion on the part of competent engineers as to what ratio exists between the loss from pans and from large water surfaces. This is difficult to determine, not only because of the variation in the evaporation from year to year, but in the results obtained from the relatively small shifting of the location in which the pan records are kept. Variations in such records of 10 and 20% within 3 or 4 miles have been noted at practically the same elevation. It is difficult to determine the evaporation from large reservoir surfaces because of the fluctuations in inflow and withdrawals. The information which has been compiled is not consistent, although not necessarily erroneous. It is important, however, for the water supply engineer to form a judgment as intelligent as possible as to this relation. The paper‡ by Edwin Duryea and H. L. Haehl, Members, Am. Soc. C. E., brought out much discussion on this subject and widely different opinions were expressed. Since that time the writer has been gathering data of evaporation from lake or reservoir surfaces. In California there is little summer rain, and it is possible to find places where the run-offs into the lakes during the summer months are negligible and where the outflows are either nothing or may be measured with accuracy. It has been the writer's intention to present these data in a paper to the Society. However, as the author's collection of pan evaporation records suggests a further discussion of this question, these notes are submitted.

Evaporation Losses from Large Water Surfaces.—In the paper by Messrs. Duryea and Haehl, previously mentioned, conclusions are drawn to the effect

* Cons. Hydr. Engr., Los Angeles, Calif.

† Received by the Secretary, May 17, 1926.

‡ "A Study of the Depth of Annual Evaporation from Lake Conchos, Mexico," *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1829.

that the rate of evaporation from large water surfaces was substantially lower than that from a floating standard evaporation pan. It was stated that the evaporation loss from the water surface of Lake Conchos was certainly less than 67.5% (and probably as low as 62%) of that from an evaporation pan 3 ft. square, floating on the lake surface. Messrs. Duryea and Haehl state that the rate of evaporation from a small pan is greater than that from a large one, that is, the evaporation from a floating pan 2.0 ft. square is 108% of that from a pan 3.0 ft. square. Their conclusions have been accepted by many engineers, and two-thirds of the recorded rate of evaporation from a standard 3.0-ft. floating pan has frequently been used in estimating evaporation losses from reservoirs.

Relative to the subject, R. B. Sleight, Assoc. M. Am. Soc. C. E., has made the following statement:*

"A careful consideration of all factors mentioned, and the curve (Fig. 4) which is approaching the horizontal for the large areas, leads the writer to present the following conclusions:

"(*a*) Evaporation figures from tanks 2 feet or greater in depth, preferably circular, set in the ground so that but a narrow metal rim not over 3 inches wide projects above the ground and in which water is kept approximately at the ground level, are most applicable for extension to large open-water surfaces.

"(*b*) Data on such tanks may be quite safely extended to large open-water surfaces under exactly the same conditions of wind, air temperature, and relative humidity by multiplying the evaporation depth from *a*:

2-foot tank by 0.77	9-foot tank by 0.98
4-foot tank by 0.84	12-foot tank by 0.99
6-foot tank by 0.90	

"Factors for depths from other tanks used have been found:

3 by 3 by 3 feet, sitting 2.75 feet in ground (Fort Collins type)	0.80
U. S. Geological Survey floating standard 3 by 3 by 1.5 feet	0.91
U. S. Weather Bureau for class A station 4 ft. in diameter, 10 inches deep, above ground	0.66"

Comparative Evaporation from Tulare Lake and at Kingsburg, Calif.—Tulare Lake, in the San Joaquin Valley, in Central California, is a basin into which a number of rivers discharge when in flood. During the late summer and fall of ordinary and low years, little, if any, water now finds its way into the lake because of the irrigation diversions on tributary streams. Some pumping from the lake was begun in the fall of 1915, but since 1916 these pumping operations have been substantially increased for the irrigation of marginal lands. There were periods, however, prior to 1916, when the only fluctuation in the lake surface was caused by evaporation. The lake is without outlet, except when it is abnormally high, and it has not overflowed since 1878. Evaporation records from Tulare Lake are given in Table 36.

Thirty miles to the northeast of the lake, at Kingsburg, in the San Joaquin Valley, evaporation records were kept by the State Engineer of California,

* "Surfaces of Water and River-Bed Materials," *Journal of Agricultural Research*, Vol. 10, July 30, 1917, p. 287.

under conditions of elevation, air, temperature, and general exposure very similar to those at Tulare Lake.

The measurements of evaporation at Kingsburg were made during the period from November, 1881, to October, 1885, inclusive, (*a*) from a pan floating in Kings River; and (*b*) from a "pan in air".

The results by monthly averages for the period are given in Table 38, together with the evaporation figures as computed for Tulare Lake by the writer. The data for the Kingsburg Station were originally published by the California State Engineer, William Hammond Hall, M. Am. Soc. C. E., in 1886 under the title, "Physical Data and Statistics of California."

TABLE 36.—EVAPORATION, IN FEET, FROM TULARE LAKE.

Month.	1906.	1907.	1908.	1910.	1911.	1912.	1914.	1915.	Mean.
July	.....	.....	0.857	.....	.....	0.607	.....	0.748	0.737
August	0.845	0.690	.....	0.575	0.515	0.714	0.663	1.109	0.730
September	0.700	0.575	.....	0.489	0.607	0.874	.....	0.608	0.559
October	0.500	.....	.....	0.326	0.272	0.405	.....	0.816	0.464
November	0.310	.....	.....	0.120	0.219	0.151	.....	.....	0.200
December	.....	.....	.....	.....	0.146	0.335	.....	.....	.....

The floating pan near Kingsburg was 2 ft. square and 1 ft. deep. The pans used in making the observations in open ground at Kingsburg were 3 ft. square and 15 in. deep. The water surface was maintained about 5 in. below the rim of the pan. When floated in the river, the pans were protected from the splash of the waves. The contrivance used in Kings River, at Kingsburg, for this purpose was a large wooden frame, into which the pan was fitted so as to float horizontally, but was not protected from the sun or movement of the air. The water level in the pan was kept at uniform height by making daily additions, by measurement, into it during the summer months. Daily records of the temperatures of the air and the water were kept.

After the first three months, the "pan in air" referred to in Table 38 was set into the ground with earth banked around its sides to the elevation of the water in it, with its full surface exposed to the sun and wind. During the first three months, or from November, 1881, to January, 1882, inclusive, this pan was on a railroad bridge, exposed on all sides to sun and air.

Kings River flows from the western slope of the high Sierra Nevada range, which culminates in a crest having an elevation of from 12 000 to 14 000 ft. The river is fed mostly from melting snows. The distance from the crest to Kingsburg is approximately 70 miles. The elevation of Kingsburg is 297 ft. The river waters at Kingsburg, therefore, are cold, especially during the period of melting snow. After July 1 the river is at low stage and the water is relatively warmer. The low temperature of the river water would reduce its rate of evaporation as compared with warmer water. The water in the "pan in air" would be warm.

Tulare Lake receives its water from the Kings, Kaweah, Tule, and Kern Rivers. All these rivers increase in flow in the spring and early summer, due to the melting snows, culminating in discharge about June 1 and by July 1

reaching low stages which usually are maintained as such until winter. The low summer flow of these rivers is diverted for irrigation purposes before reaching the lake, and it is only during flood periods that the streams discharge into it. No direct measurements of these discharges into the lake prior to 1915 are available, so that a determination of the evaporation from the lake surface has to be restricted to periods during which there is no inflow into it, and also when the level of the lake is below its outlet. Hydrographs of these rivers indicate low flow periods to extend from July to December. They coincide with the months of little or no rainfall in this region, as seen in the records of precipitation at Hanford, Calif. (Table 37), the point nearest the lake for which rainfall records are available. These records have been used in connection with the computation of evaporation on the lake. Any variation in the rainfall on the lake from that at Hanford, will have little effect on the level of the lake. The seepage into the lake from surrounding lands is negligible. If either seepage water or stream water entered the lake during the periods for which comparisons are being made between lake and pan evaporation, it would decrease the indicated evaporation from the lake surface, and if adjustment were made on such an hypothesis, it would emphasize the general conclusions presented in Table 38.

TABLE 37.—RAINFALL, IN INCHES, AT HANFORD, CALIFORNIA.

Month.	1906.	1907.	1908.	1909.	1910.	1911.	1912.	1913.	1914.	1915.
January	1.59	3.88	1.86	4.69	2.40	3.79	0.52	1.26	4.36	2.69
February	1.92	0.65	2.84	0.49	0	1.46	0.02	1.55	1.25	3.44
March	4.05	2.39	0	2.31	1.06	2.94	3.24	0.34	0.37	0.30
April	0.62	0.29	1.05	0	0.24	0.58	1.52	0.78	0.11	1.37
May	2.06	0	0.50	0	0.05	0	0.27	0.76	0.00	0.91
June	0.02	0	0	0.42	0	0	0	0.06	1.06	0.00
July	0	0	0	0	0	0	0	0.08	0.00	0.00
August	0	0	0	0	0	0	0	0	0	0
September	0	0	0.95	0	1.51	0.02	0	0.08	0	0
October	0	0.64	0	0.36	0.31	0.02	0	0	0	0
November	0.12	0	0.72	0.95	0.24	0.23	0.61	1.88	0.02	0.00
December	3.43	1.58	0.31	1.95	0.28	0.55	0.21	1.35	2.50	1.71
Annual	13.81	9.43	8.23	11.17	6.69	9.54	6.39	8.09	9.67	10.42

The mean annual rainfall in the neighborhood of the lake is about 10 in., nearly all of which occurs during the winter months. There are occasional storms in September, October, and November. The rise of the lake in November, 1912, over the October levels, suggests an October rain.

Tulare Lake is broad and shallow and during the period of this study, it ranged from an area of 103 000 to 198 000 acres. Its depth varied from 0 to 12 ft. The bottom of the lake is 180 ft. above sea level. The temperatures of the San Joaquin Valley are high in summer and practically the same at Tulare Lake as those at Kingsburg where observations on the pans were made. The bed and shores of the lake are clay. Very little water would be lost by seepage into the soil or would return from the soil when the lake levels fell.

Systematic records of the stage of the lake are available since 1906, when the U. S. Geological Survey established a gauging station thereon. Measure-

ments have been made at weekly and monthly intervals to date, but since 1916, the stage of the lake has been affected by withdrawals, by pumping of water for irrigation, and the computation of the evaporation has been carried from 1906 to 1916 only.

The gauge was usually read to 0.01 ft. and the observer was instructed not to take readings during wind storms which might cause abnormal levels in parts of the lake. Although the observations for evaporation were not taken at the same time from both the pans at Kingsburg and the surface of Tulare Lake, they were of such duration that it is believed they will throw light on the relation between the actual records of evaporation from the lake surface and the rate of evaporation from a floating pan in this locality.

Table 38 shows the evaporation from Tulare Lake as compared with that at Kingsburg, as measured in the pans.

TABLE 38.—EVAPORATION FROM TULARE LAKE COMPARED WITH EVAPORATION AT KINGSBURG, CALIF.

Month.	At Tulare Lake.		At Kingsburg.		
	In feet per month.	Percentage of floating pan evaporation at Kingsburg.	Pan floating in river, in feet per month.	Pan in air, in feet per month.	Percentage of floating pan evaporation.
July	0.737	117	0.628	0.904	144
August	0.730	101	0.721	0.861	119
September	0.559	103	0.541	0.615	114
October	0.464	139	0.339	0.289	85
November	0.200	114	0.176	0.174	99
	2.690	112	2.405	2.843	118

From Table 38 it will be noted that the rate of evaporation observed from the surface of the lake was 112% of that from the floating pan in the river and 95% of that from the "pan in air" at Kingsburg. The evaporation from the "pan in air" at Kingsburg was 118% of that from the pan in the river at Kingsburg. The lake water probably was warmer than the river water, which would increase the rate of evaporation from the lake.

S. T. Harding and Harry Barnes, Members, Am. Soc. C. E., have made a study* of the evaporation losses in Kern County, California, with particular reference to their relation to Buena Vista Lake which much resembles Tulare Lake in position and exposure. For 1920 the evaporation pans were of the type used by the U. S. Geological Survey, 3 ft. square and 2 ft. deep. The floating pan was surrounded by a raft to baffle wave action. The water surface was maintained 3 in. from the top of the pan. Owing to the flat slopes of the bed of the lake and the shallowness of the water during a part of the season, the pan was set in an excavated hole so that the water at the outside did not get more than 6 in. below the top of the pan at any time. The floating pan

* "Water Resources of Kern River and Adjacent Streams and Their Utilization," *Bulletin 9*, California State Dept. of Eng., 1920.

was covered by a chicken wire screen so as to exclude water fowl. The evaporation from the land pan for the 12 months of 1920 was 67.84 in. The rate of evaporation from the floating pan is given as 115% of that of the land pan. During the months, July to November, inclusive, there was no inflow to the lake. The outflow was small, but was measured. The comparison of evaporation from the floating pan with the actual lowering of the water surface of Buena Vista Lake in 1920 was as given in Table 39.

TABLE 39.—COMPARISON OF EVAPORATION FROM FLOATING PAN AND FROM SURFACE OF BUENA VISTA LAKE.

Month.	Indicated lowering of Buena Vista Lake, due to evaporation, in feet.	Measured evaporation from floating pan, in feet.	Indicated coefficient for floating pan.
July	0.61	1.04	0.58
August	0.58	0.94	0.62
September	0.59	0.70	0.84
October	0.38	0.45	0.84
November	0.16	0.20	0.80
5 months	2.32	3.33	0.74

It has been estimated* that the evaporation for the floating pan for the full year was 77.99 in., that the estimated actual evaporation from the whole lake was 54.7 in., and that the estimated coefficient for the year was 0.70. The coefficient, however, for the months of actual observation was 74 per cent.

Messrs. Harding and Barnes also made a comparison between the evaporation from the surface of Tulare Lake and the pan observations made in previous years by the State Engineer at Kingsburg, referred to previously. It is stated† that the estimated evaporation measured from the pan at Kingsburg Bridge is 4.4 ft. per year and that from the lake itself the evaporation loss was 4.6 ft. It is concluded, as follows: "For this portion of the San Joaquin Valley the total evaporation for shallow lakes appears to be about 4½ ft. per year."

Comparative Evaporation at Lake Hodges.—Lake Hodges is in San Diego County, California, 10 miles from the coast. The high water surface is 315 ft. above sea level, and the maximum depth of the water at the dam is 115 ft. The sides of the reservoir are largely steep and rocky.

This study for Lake Hodges is based on the results obtained from a standard pan, 3 ft. square and 18 in. deep. The water in the pan was kept within 4 in. of the top, and the pan was buried flush in the ground on a slope with a southern exposure to the sun the whole day. The pan was protected by a screen of 2-in. chicken wire. This pan was at the west end of Lake Hodges.

There is a prevailing wind movement from the west over the pan from the spillway cut of the dam. This prevailing wind traversed a 12-mile stretch of uncultivated country, before it reached the lake surface. The lake is 7 miles long, about 2 000 ft. wide, and lies in the general direction of these pre-

* *Bulletin 9*, Dept. of Eng., State of California, 1920, p. 63.

† *Loc. cit.*, Table 25.

vailing westerly winds. The evaporation from a standard pan in the ground near the east end of the lake for 1922 was only 88% of that from the pan at the dam. The prevailing winds probably have absorbed some moisture from the lake before reaching the eastern pan and hence cause less evaporation.

There was a little rainfall during months of the observation period, but no resulting run-off as shown by the daily gauge readings at the reservoir. This rainfall was considered in the lake as well as in the pan evaporation record. There were no withdrawals from the lake during the period of comparison.

The lake evaporation was 96% of that from the pan at the dam at the west end of the lake. This is considered approximately the true relationship existing between the evaporation from the lake and the pan. The Hodges Reservoir, which was first filled in 1919, has few marginal swamps. The observer at Lake Hodges was an experienced engineer.

Comparative Evaporation at Cuyamaca Reservoir.—Cuyamaca Reservoir is in San Diego County, California, 40 miles from the sea, near the crest of the Coast Range, at an elevation of 4 600 ft.

These pan records were obtained from a standard pan 3 ft. square, 18 in. deep, the water being kept within 4 in. of the rim. For November, 1916, and January, 1919, the pan was floating in the water in the deep section of the lake near the dam, and was protected from wave motion by log booms. In November, 1921, and June, 1922, the pan was buried flush with the ground on the hillside north of the lake, fully exposed to the sunshine and wind movement, except for a 2-in. mesh screen.

The estimated evaporation from the lake based on the observations for these four months was 115% of that from the pans, which is 17% higher than the corresponding figures at Lake Hodges. This may be due to the rapid summer plant growth on the large marginal swamp area at Cuyamaca Lake. The evaporation figures from the lake for June, 1922, were 2 in. more than those from the pan. The lake has a mean depth of only 12 ft. and hence becomes warm during the summer months. A screen covering the pan may have somewhat reduced the evaporation therefrom. The wind movement over the Cuyamaca pan was practically the same as that over the lake. During the summer months the water temperature at Cuyamaca Lake was 73° Fahr. on the surface and 70° Fahr. at the bottom, as compared with 71° Fahr. on the surface at Lake Hodges and 55° Fahr. on the bottom. With due allowances for the plant growth and the screen over the pan, the evaporation from the surface of Cuyamaca Lake would probably be much the same as that from the pan.

The evaporation from a standard pan set in the ground at Cuyamaca Lake and observed from 1914 to 1917, inclusive, was 69.46 in. per year. From a floating pan, from 1914 to 1921, inclusive, it was 66.88 in. per year. The evaporation from the lake surface as expanded from fragmentary records, taken during the dry season of little run-off during two years* was at the rate of 57.6 in. per year. Based on these data, the lake surface evaporation was

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 2052.

86% of that of the floating pan. Table 40 shows the relative evaporation for Lake Hodges and Lake Cuyamaca.

TABLE 40.—RELATIVE EVAPORATION, LAKE SURFACE AND STANDARD PAN, AT LAKE HODGES AND CUYAMACA, FOR PERIODS OF NO RESERVOIR DRAFT AND LITTLE OR NO RAINFALL.*

LAKE HODGES, AREA, 1317 ACRES.

(Pan located about 35 ft. above lake surface and near spillway opening, where wind movement is greatest. Two-inch mesh screen over pan.)

	Lake evaporation, in feet.	Pan evaporation, in feet.
June, 1919	0.62	0.60 (pan in ground)
July, 1919	0.70	0.68 (" " ")
September, 1919	0.50	0.57 (" " ")
June, 1920	0.57	0.61 (" " ")
October, 1920	0.44	0.45 (" " ")
October, 1921	0.37	0.42 (" " ")
Totals	3.20	3.33

Lake evaporation = 96% of pan.

CUYAMACA LAKE, AREA, 978 ACRES.

	Lake evaporation, in inches.	Pan evaporation, in inches.
November, 1916	3.88	3.82 (pan in water)
January, 1919	2.50	2.07 (" " ")
November, 1921	3.25	3.21 (pan in ground)
June, 1922	9.25	7.28 (" " ")
Totals	18.88 (1.573 ft.)	16.38 (1.365 ft.)

Lake evaporation = 115% of pan.

* Comp. by F. E. Green, Hydrographer.

Evaporation from Upper Otay and Lower Otay Reservoirs Compared.—H. A. Whitney, M. Am. Soc. C. E., has given a statement* as to the relation between evaporation of the water surface of the Upper Otay Reservoir in San Diego County, California, to evaporation observed from floating pans 3 ft. square, at the Lower Otay Reservoir. The Upper Otay Reservoir is 13 miles southeast of San Diego, Calif., and about 550 ft. above sea level, and is approximately 1 mile long and ½ mile wide. The drainage area is small with steep rocky sides. The reservoir has an area of 160 acres and an average depth of 20.7 ft. Accessions to it and withdrawals from it are comparatively infrequent. Mr. Whitney shows that during the three months, October to December, 1915, inclusive, the evaporation from the water surface of this reservoir was 87% of that from the floating pans.

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1988.

In the paper by Messrs. Duryea and Haehl, previously mentioned, the probable mean annual evaporation from the Upper Otay Reservoir is given as 62.3 in., which is greater than the pan records at Sweetwater only 6 miles distant and with much the same exposure, the record of which is given subsequently.

Arthur J. Ellis and Charles H. Lee, Members, Am. Soc. C. E., have made a study* of the comparative evaporation from a pan floating in the La Mesa Reservoir in San Diego County, California, with that of the Otay Reservoir which is situated 11 miles to the southeast and which was neither drawn upon nor received accessions during the period January, 1913, to December, 1915. The rainfall was observed at the Lower Otay Dam about 2½ miles to the west of the Upper Otay. After making corrections and expanding the record Mr. Lee concludes that the evaporation from the Otay Reservoir was 94.2% of that shown by the pan floating in the La Mesa Reservoir.

Evaporation from Sweetwater Reservoir.—The evaporation† from the Sweetwater Reservoir for the period, 1889 to 1894, inclusive, was measured for 4 years in a standard pan, averaging 58.71 in. and, subsequently, for 2 years with a Pische evaporimeter which showed a yearly loss of only 45.77 in.

Mr. John F. Couvert, Engineer for the Sweetwater System, computed the gross evaporation of the Sweetwater Reservoir from 1910 to 1914, inclusive.‡ Based on his meter readings, W. S. Post, Assoc. M. Am. Soc. C. E., found that the evaporation losses from the water surface amounted to 56.98 in., indicating that from this series of readings the loss from the reservoir surface was 97% of the pan record. The Sweetwater Reservoir is 8 miles east of San Diego, at an elevation of about 250 ft.

Lake Elsinore Evaporation.—Lake Elsinore, which is shallow and large, is situated 26 miles from the coast in Riverside County, Southern California, in an inland valley where summer temperatures are relatively high. The bottom of the lake is 1 218 ft. above sea level, and the lake overflows at Elevation 1261 when its area is 5 500 acres. The water is quite saline except during overflow. The floods of the San Jacinto River discharge into the lake, and it overflows at high stages through an outlet at the Town of Elsinore. On rare occasions the lake is entirely dry, but it was high during 1916, and overflowed.

Evaporation records have been kept from a 3-ft. pan floating in water and the gauge height has been taken on the lake itself. This work was done for the San Diego County Water Company, under the direction of its Engineer, Mr. W. S. Post, from May to September, 1916.

The lake is surrounded by quite extensive areas of flat land which possibly may drain into it as the water levels recede. Several hundred acres are irrigated by pumping from these marginal areas, the water being spread on the surface thereof. In the opinion of the engineers of the California State Division of Water Rights these ground-waters do not originate from the lake. The result of comparative evaporation measurements between the

* *Water Supply Paper 436*, U. S. Geological Survey.
† *Water Supply Paper 81*, U. S. Geological Survey, p. 344.
‡ *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1914.

lake surface and the evaporation pan for 5 months during the summer of 1916 shows that the loss from the surface of the lake is 84% of that from the pan. (See Table 41.) During this period the inflow and outflow from the lake were known and the corrections made therefor as given in the table.

TABLE 41.—LAKE ELSINORE EVAPORATION.*

Date.	Evaporation from lake surface, in inches.	Evaporation from pan in water, in inches.
May, 1916	4.92	7.75
June, "	5.40	7.35
July, "	8.52	8.42
August, "	6.14	8.56
September, "	7.83	6.90
	32.81	38.98
Lake evaporation = 84% of pan.		

* As computed by W. S. Post, Assoc. M. Am. Soc. C. E.

Professor S. T. Harding has written a report* in which he has presented the estimated gross evaporation of Lake Elsinore. The inflow of the San Jacinto River into the lake and the outflow of the lake were measured, together with the local rainfall, and, by deduction, the record of evaporation from the lake surface as shown in Table 42 is determined. Professor Harding states, as follows:

"The resulting figures cannot be considered as being more than a general estimate of the probable run-off. * * * All of the local run-off is considered as reaching the local ground water, direct inflow to the lake being small and infrequent".

Again, it is stated:

"The available records are sufficient to enable the evaporation from Lake Elsinore since 1916 to be computed. This has been done in Table 17. The fluctuation of the lake for each month is caused by evaporation, direct rainfall and depth of inflow or outflow. The records of inflow and outflow, with the area and capacity of the lake, enable the depth of inflow or outflow to be computed. Fluctuations not accounted for by other items have been considered to represent evaporation".

Professor Harding estimates the evaporation from the surface of Elsinore Lake for the months, May to September, inclusive, to average 2.7 ft. for a period of six years. (See Table 42.) Mr. W. S. Post, during one season found the loss from a floating pan, as given in Table 41, for these months to be 3.25 ft. On this basis of comparison the lake surface loss was 83% of that of the floating pan.

Charles H. Lee, M. Am. Soc. C. E., has referred to certain evaporation observations that he has made.† A pan was floated on the surface of Owens

* "San Jacinto River Hydrographic Investigations, 1922," Div. of Water Rights, State of California, Table 17.

† *Water Supply Paper 446*, U. S. Geological Survey, p. 103.

TABLE 42.—ESTIMATED GROSS EVAPORATION OF LAKE ELSINORE.

(Based on fluctuations of lake, direct rainfall, and depth of inflow and outflow.)

	Evaporation Depth, in Feet.												
Year.	October.	November.	December.	January.	February.	March.	April.	May.	June.	July.	August.	September.	Year.
1915-16	....	....	....	2.34*	1.06*	0.30*	0.92*	0.50	0.50	0.70	0.50	0.60	....
1916-17	0.26	0.36	—0.3	0.08	0.18	0.23	0.33	0.59	0.30	0.28	0.40	0.70	3.68
1917-18	0.50	0.51	0.40	0.22	0.10	0.31	0.42	0.42	0.50	0.71	0.52	0.42	5.03
1918-19	0.57	0.26	0.28	0.12	0.01	0.14	0.42	0.63	0.40	0.40	0.70	0.54	4.47
1919-20	0.44	0.26	0.09	0.27	—0.17	—0.37*	0.47	1.21*	0.60	0.60	0.70	0.50	4.52
1920-21	0.48	0.21	0.14	0.18	0.06	0.23	0.40	0.29	0.50	0.61	0.81	0.71	4.62
1921-22	0.40	0.21	1.18*	—0.22*	0.23	0.69*	—0.16*	0.56	0.50	0.71	0.50	0.50	5.05
Mean except excluded months..	0.44	0.30	0.18	0.17	0.07	0.23	0.41	0.49	0.47	0.57	0.59	0.57	4.49
Adjusted estimated mean....	0.45	0.30	0.20	0.15	0.05	0.25	0.40	0.45	0.50	0.60	0.60	0.55	4.50

NOTE.—The large indicated evaporation in months of large inflow, such as January, 1916, to December, 1921, may be due to ground storage in lands newly submerged which would be lost by evaporation later after the lowering of the lake. The mean annual rainfall at Elsinore = 14.0 in.

*Months for which records are uncertain and which were omitted in computing the means.

TABLE 43.—COMPARATIVE EVAPORATION BETWEEN FLOATING EVAPORATION PAN AND LARGE WATER SURFACES IN CALIFORNIA.

Location.	Duration of Record.		Size of pan, in feet.	Evaporation, in Feet.		Ratio: Lake to pan.	Authority.	Remarks.
	Pan.	Lake.		Pan.	Lake.			
Tulare Lake.....	4 years	5 summers	2 (square)	2.405	2.690	112	State and U. S. Geological Survey.	Pan floating in river which was cooler than lake. Record for 5 summer months.
Tulare Lake.....	.....	.....	2 (square)	4.4	4.5	102	State Engineer.	Estimated for 12 months.
Buena Vista Lake.	5 months	5 months	3 (square) 2 (deep)	3.33	2.32	74	State Engineer.	.....
Lake Hodges.....	6 "	6 "	3 (square) 1½ (deep)	3.33	3.20	96	F. E. Green.	Pan in ground.
Cuyamaca Lake...	4 "	4 "	3 (square) 1½ (deep)	1.365	1.573	115	F. E. Green.	Pan, 2 months in water and 2 months in air.
Cuyamaca Lake...	4 years	Fragmentary	3 (square) 1½ (deep)	5.58	4.80	86	Duryea and Haehl.	Comparison with floating pan.
Upper Otay Lake.	3 months	3 months	3 (square)	1.05	0.917	87	Whitney.	Pan evaporation measured at Lower Otay Reservoir.
Upper Otay Lake.	4 years	Yearly mean	3 (square)	4.89	5.19	106	Duryea and Haehl.	Pan evaporation records at Sweetwater used in ratio. Lake evaporation at Upper Otay estimated by Duryea and Haehl.
Upper Otay Lake.	3 "	3 years	" "	5.51	5.19	94.2	Ellis and Lee.	Based on floating pan record at La Mesa Reservoir.
Sweetwater Reservoir.....	4 "	5 "	" "	4.89	4.75	97	Couvert.	Lake evaporation estimated by Couvert, adjusted by Post.
Lake Elsinore.....	5 months	5 months	" "	3.25	2.73	84	Post.	.....
Lake Elsinore.....	.....	6 years	" "	3.25	2.7	83	Post and Harding.	Lake evaporation losses estimated by Harding for 6 years and averages compared with pan records taken by Post for May to September, 1916, inclusive.
Owens Lake.....	.....	.....	.....	.....	.....	99	Lee.	Based on record of pan floating in river near lake probably in cooler water.

River 20 miles north of Owens Lake. The water of this river during high stages comes from the melting snows of the adjacent high Sierras and during the late summer largely consists of return water from irrigation. Owens Lake is broad, shallow, and saline, and has no outlet. The wind movement in this region is high. The inflow was measured and the fluctuations in the surface of the lake were observed. Mr. Lee finds the evaporation from the pan and from the lake to agree within 1 per cent. As in the case of Kings River, however, the water of the river probably was much cooler than that of the lake, which may cause the rate of evaporation from the pan floating in the river to be less rapid as compared to the lake surface evaporation.

Conclusions.—From the data available, the writer believes that the loss from a reservoir surface is not quite as great as that from a floating pan under similar conditions of exposure, the difference depending on the general character of the reservoir or lake, probably amounting to 90 to 95% of that from a standard floating pan similarly exposed. This is a subject that requires much further information and study. When it is remembered how uncertain it is to compute the net safe yield from any given water-shed through a group of prospective dry years, it would appear that conservatism would lead to the acceptance of floating pan evaporation records without deduction in determining the evaporation losses from lake water surfaces.

PIPE TUNNEL UNDER GOWANUS CANAL, BROOKLYN, NEW YORK

Discussion*

By Messrs. Frederick L. Cranford, Charles W. Staniford, and John F. O'Rourke.

Frederick L. Cranford,† M. Am. Soc. C. E.—The Gowanus Gas Tunnel is a perfect working model of what can be accomplished in the building of a subaqueous tunnel and demonstrates that a trench under water can be dug successfully and that the water pressure against the sides of the trench has great value in holding its banks even adjacent to important structures on the shore. The west shaft of the tunnel is right alongside a large sewer and concrete retaining wall holding several thousands of tons of coal and within about 40 ft. of important buildings of the Brooklyn Union Gas Company. There was no disturbance whatever to these structures, although this shaft was in no way braced, except by the cantilever value of the master piles.

The operation of sinking such a tunnel, by bulkheading and sinking it either with a permanent or a temporary shaft fastened to the ends, provides a control that is absolute and perfect. The buoyancy of the tunnel having been neutralized the sinking was exactly regulated by permitting water to enter each shaft, through valves provided for that purpose; one man on the top of each shaft could control the speed of sinking and maintain the tunnel perfectly level by the regulation of the valves. Conversely, should there be any slip or hitch in the operation, by pumping the water out of the shaft the tunnel can be raised to the surface again.

The speed of sinking such a tunnel in navigable waters, is controlled by the size of the intake valves. This particular tunnel, with only 4-in. valves, sank in the beginning at the rate of 1 ft. in 6 min. and, later, 1 ft. in 4 min. There is no reason why this speed could not be greatly increased if it were desirable.

As regards the economics of such a structure, a four-track tunnel would be vastly more desirable proportionally than a single track. On large tunnels the economic gain very nearly multiplies as the size. Again, on small work where the necessary air plant and its operation is a large factor in relation to the value of the project, there would be an advantage in this method of construction.

* Discussion on the paper by L. S. Stiles, M. Am. Soc. C. E., continued from May, 1926, *Proceedings*.

† Pres., Frederick L. Cranford, Inc., Brooklyn, N. Y.

CHARLES W. STANIFORD,* M. AM. SOC. C. E.—In common with the other engineers who have discussed this paper the speaker has often considered the possible adaptation of the method used in this pipe tunnel to deep river crossings and canal crossings adjacent to New York as well as to similar construction with the tunnel built in several sections.

It is certain that in shallow crossings this tunnel method will be widely recognized in the future. The difficulty of making land connections where the deep tunnel section proper ends may be obviated by continuing the tubes inshore up the approach grades, until a point is reached where the depth of portal cut may be assumed economically as an open braced excavation.

This again illustrates the advantage of the open trench method, in that the tunnel may be built very near the grade required by the navigation clearance. The elevations of the portal shafts, thus brought to a minimum depth, result in less grade to be overcome on the approaches, with a consequent lessening in the disturbance to property and streets, which is of prime importance.

The sinking of the structural tube of the Gowanus Tunnel was kept under perfect control by watering the shafts; after overcoming the air in the upper 18 in. near the roof, the tube sank symmetrically at each shaft, averaging about 2 in. per min. The sinking was halted about 1 ft. above the bottom of the trench for final alignment. There appears to be no reason why the operation should not be duplicated from shaft to shaft by proper bulkheading. On account of the perfect control over the sinking operation, connection may be made to adjoining sections by any of the various devices used at the St. Clair Tunnel, the Harlem River Tunnel, or by other mechanical means which may be improvised from the experiences gained with this particular undertaking.

In many river crossings, particularly in the approaches to the New York Metropolitan District, draw-bridges are becoming an impediment to the navigation and highway needs of even the present day. In these districts much of the marsh land is unused, but is rapidly being developed, each dock and terminal adding to the commerce on the streams, while the immense increase in the use of these highway crossings by motor trucks and pleasure cars renders the frequent opening of the draw-bridges a menace to both. Judging from present progress, the time is not far distant when these interferences will unquestionably prove a most serious drawback to the growing navigation needs; the new highways may not be able to function so as to permit of the intensive use estimated to meet their great cost to the States, and the railroads may fail to satisfy the demands of the quick suburban service required.

A structural steel section, built on land and sunk into a submerged trench built for it, offers at least one method that should henceforth receive the serious consideration of engineers as a means of eliminating some of these menaces from important navigable streams.

It was for this purpose that the speaker in collaboration with Victor Gelineau, M. Am. Soc. C. E., Director of the Board of Commerce and Navi-

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gation, State of New Jersey, has submitted to this Board a plan for proposed crossings of the Hackensack and Passaic Rivers to be built in an open trench, after the general plans used in the Gowanus Tunnel. These similar tunnels (Fig. 14) are designed to take the place of two proposed new draw-bridges, where the conditions even at the present time create a serious handicap to proper navigation and highway movement.

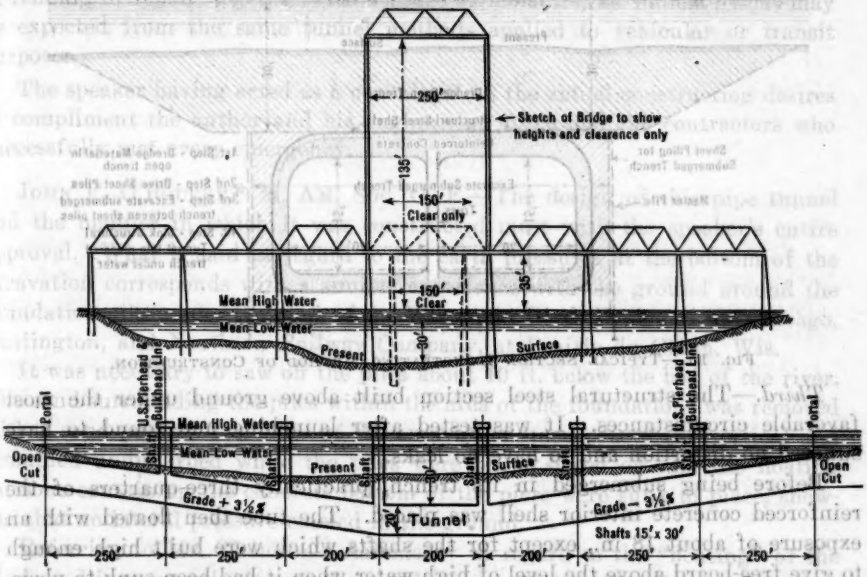

FIG. 14.—COMPARISON BETWEEN BRIDGE AND TUNNEL, HACKENSACK RIVER.

Some of the outstanding features developed at the Gowanus Tunnel are as follows:

First.—The adaptability and pliability of the steel interlocking sheet-piling in holding the sides of deep trenches, the piles being driven from 14 ft. to about 25 ft. below low water so that the top line of the submerged sheet-piling shall be in permanent position below the requirements of navigation.

At this tunnel not only the sheet-piles but the heavy 20-in. I-beams (the "master piles" which were placed 10 ft. apart for support against lateral earth pressure on the sides of the submerged trench) were driven well down by the aid of hammer, jet, and follower into a hard sand formation.

Second.—The minimum of open excavation under water required by this method. Obviously, the sheet-pile trench might be omitted, and the whole excavation made in open cut. This was considered but not adopted, not only on account of lack of control of the slopes in the river bottom itself, but on account of the danger of undermining the surrounding docks and structures.

The excavation was further complicated by the remains on each side of the stream of old cribwork of logs and rock. In most of the outlying districts where tunnels may be required, and notably at the Hackensack and Passaic Crossings, the problem of excavation (Fig. 15) is much simpler than at the

Gowanus Tunnel. At those sites there is a blanket of mud of moderate depth, underlaid with good sand. Therefore, the first under-water trench to create depth for navigation before the sheet-pile trench is started, could all be excavated most cheaply by pumping, using the material to be deposited on the adjacent marsh lands to make them usable and valuable.

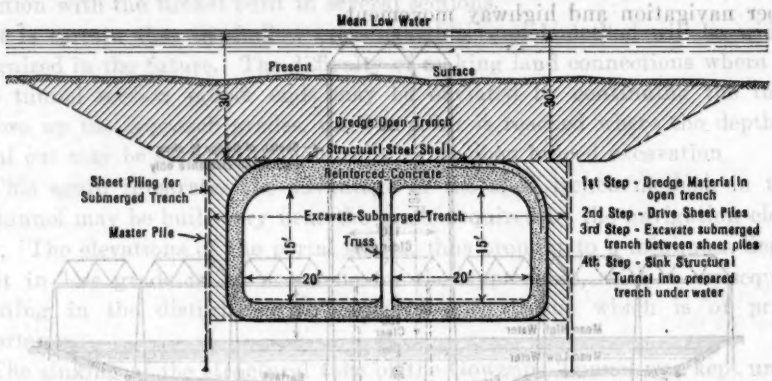

FIG. 15.—TYPICAL SECTION ILLUSTRATING METHOD OF CONSTRUCTION.

Third.—The structural steel section built above ground under the most favorable circumstances. It was tested after launching and found to have suffered no distortion and to have no leaks.

Before being submerged in its trench, practically three-quarters of the reinforced concrete interior shell was placed. The tube then floated with an exposure of about 18 in., except for the shafts which were built high enough to give free-board above the level of high water when it had been sunk to place. All this work was accomplished while the tube was in the preparatory and temporary sheet-pile trench. On the other hand the Passaic and Hackensack Tunnels, or other similar tunnels, could be constructed in the open, longitudinally along the river banks, on side launching ways.

The tentative section for these tunnels (Fig. 15) approximates the oval Gowanus Section. The section for the four traffic lines shown could be modified to approach more nearly a rectangular form, similar to large scows and railroad car-floats. The longitudinal trusses between roadways offer opportunity for study in the development of longer sections between shafts before launching, even of a combination of one intermediate and two end shafts, which would still leave the length short as compared with a moderate sized ship. Lengthening the sections before launching would measurably decrease the cost, on account of the lesser number of under-water connections to be made at the joints.

Fourth.—The side or lateral launching of this tube on a slope which caused comparatively rapid and free movement down the ways by gravity, the only control being tackle attached to anchors at intervals.

For future tunnels built on this principle it is easy to conceive of much longer units, by planning the side launching ways on a flatter slope, or on a slope whereby the tube would be pulled down the ways from the front, or pushed down by jacks in the rear.

Fifth.—The excellent general appearance that may be obtained with the simple steel shell, and reinforced concrete lining. With ordinary rubbing and painting after the removal of the forms, the interior presents the appearance of a well-lighted, white enameled room. The spiral stairs at each shaft, easy walkways for inspection, ample room for handling pipe, and for work, in case of renewal or repair, together with absence of moisture, all indicate what may be expected from the same tunnel methods applied to vehicular or transit purposes.

The speaker having acted as a consultant on the actual construction desires to compliment the author and his engineering force and the contractors who successfully met every emergency.

JOHN F. O'ROURKE,* M. AM. SOC. C. E.—The design of this pipe tunnel and the manner in which it was constructed meet with the speaker's entire approval. What is said in regard to the earth pressures at the bottom of the excavation corresponds with a similar experience with the ground around the foundation piles under the piers of the Wisconsin River Bridge of the Chicago, Burlington, and Northern Railway Company, at Prairie du Chien, Wis.

It was necessary to saw off the piles about 10 ft. below the bed of the river. The sand surrounding the piles within the area of the foundations was removed with a sand pump. The sides of the excavation stood practically vertical and remained undisturbed while the piles were being sawed off and the floating caissons containing the concrete bottom of the piers were sunk to place, showing that no lateral pressure existed in the ground.

Referring to Mr. Thomson's statement† in regard to the "floating" of the Hudson River tunnels, it was the speaker's experience that the ground there was also free from lateral pressures. This was shown by the fact that the clay of which the bed of the Hudson is composed, took almost a year to close in finally around the tunnels, with the air pressure in the tunnels during construction usually about equal to that due to the water above the river bottom, which balanced the water pressure and left the clay free to move if there was any lateral pressure.

The so-called floating of the tunnels is nothing more than the compressive action of the water of the river on the mass of clay resting on the rock about 200 ft. below the bottom of the tunnels. This was measured by means of a steel rod extending through the tunnel bottom and fastened in the rock, which showed that the tunnels moved up and down about ⅛ in. between high and low water. This movement was continuously recorded by means of a clock mechanism with a pointer which gave a continuous curve, considerably magnified, for each 24 hours. The bed of the river was shown to be so elastic and sensitive to the varying pressures of the water that it was possible to determine the actual rise and fall of the tide each day from the movement of the tunnel. This interesting fact shows how stable the undisturbed clay in the bed of the Hudson River really is and its reliability for carrying loads without settlement when they are properly limited to the ordinary practice for that kind of ground.

* Pres. and Chf. Engr., O'Rourke Eng. Constr. Co.; Cons. Engr., New York, N. Y.

† *Proceedings*, Am. Soc. C. E., May, 1926, Papers and Discussions, p. 1020.

MOMENTS IN RESTRAINED AND CONTINUOUS BEAMS BY THE METHOD OF CONJUGATE POINTS

Discussion*

By Jules D. Gedo, Esq.

Jules D. Gedo,† Esq. (by letter).‡—The authors' method is excellent so far as dead load is concerned. As to the live loads, the method does not differ much from that of fixed points. As soon as a span is not loaded the conjugate points fall into the beam and are called fixed points.

The writer has developed a method which allows the use of tabulations. These can be made for every possible case, no matter how many spans there are, whether they are equal or not, or whether the moments of inertia vary from span to span. This would seem to be impossible, because the cases to be covered are innumerable.

The writer's method is based on the fact that every multiple-span continuous beam can be conceived of as an aggregate of 2 and 3-span beams. According to the paper, to determine influence lines every span is loaded separately. Consider, for instance, Span *L-R* in Fig. 95.

If the two moments, M_L, and M_R, over the supports of the loaded span, are found, the moment diagram is determined inside and outside the span. Outside the loaded span the moments spread through the farther fixed points of the successive spans. To determine these two moments is a laborious task. Instead of determining them consider Figs. 96 and 97.

Fig. 96 shows how to develop Fig. 97 from Fig. 95. Fig. 97 represents a 3-span beam. This 3-span beam is equivalent to the mutiple-span beam for the case when Span *L-R* is loaded. In other words if Span *L-R* is loaded as a span of the multiple-span beam, and if it is loaded the same way as the middle span of the 3-span beam, in both cases the same moment diagram for Span *L-R* will result. This may be shown statically as follows: The problem of an n-span beam is to determine the $n + 1$ components of the load. This problem is reduced at once by recognizing that the resultant of R_G and R_E (reactions in Supports *G* and *E*) must pass through F_1''. As is shown in the paper the moments in the fixed points are zero, wherever the loads may be (certainly, the loads must be outside the span to which the fixed points belong and if it is a left fixed point, the load must be to the right).

* Discussion on the paper by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., continued from April, 1926, *Proceedings*.

† Structural Steel Detailer, Am. Bridge Co., Gary, Ind.

‡ Received by the Secretary, May 14, 1926.

How is it possible to get at F_l'' a moment that equals zero? Left from F_l'' are two reactions, R_G and R_E. If their moment relative to F_l'' is zero (wherever the load may be between L and R) it shows that their resultant passes through F_l''. Hence it is only necessary to find n-reactions instead of $n + 1$. For similar reasons the resultant of R_G, R_E, and R_D goes through F_l' and the resultant of R_C, R_B, and R_A is in F_r'. Therefore, only four reactions must be found instead of $n + 1$. Furthermore, the moment at L is the same as if it were considered as the moment of R_G, R_E, and R_D, or as the moment of the resultant of R_G, R_E, and R_D. Hence, the problem of the n-span beam is reduced to that of a 3-span beam.

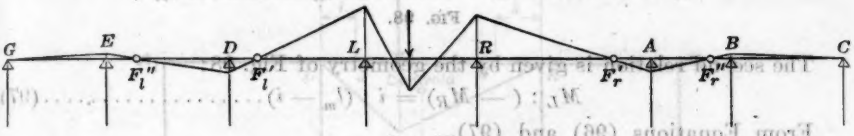

FIG. 95.

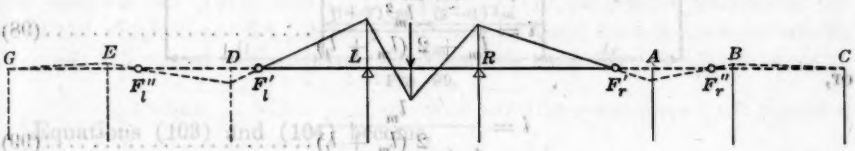

FIG. 96.

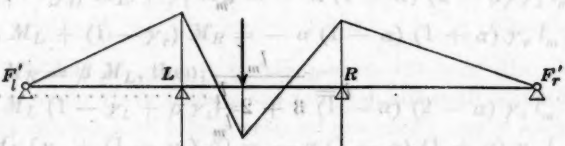

FIG. 97.

Considering the non-vertical dotted lines in Fig. 96 as funicular polygons for the reactions acting in the vertical dotted lines, it may be noted that the first and last rays are horizontal, which shows easily where the resultants of the reactions are. Thus, the fixed points are more than imaginary hinges. They play statically the same rôle as the cable of a suspension bridge, or the line of resistance of an arch.

The foregoing statements show that for every loaded span there may be substituted a 3-span beam. The middle span of this equivalent 3-span beam is identical with the loaded span of the multiple-span beam. The outside supports of the 3-span beam are fixed points of the n-span beam, namely, the farther fixed points of the neighboring spans. If there is only one neighboring span to the loaded span, that is, if a side span is loaded, there results a 2-span beam.

Therefore, to analyze an n-span beam, it is necessary to compute n-2 3-span beams and two 2-span beams. Since as far as calculation is concerned there are none but 3 and 2-span beams, furthermore, since in these 2 and 3-span beams only one span is loaded, it is both worth while and easy to prepare tables for them.

The 3-span beams may be studied with respect to their fixed points. To compute i, in Fig. 98, two relations are known. The first is given by the Theorem of Three Moments:

$$2\,M_L\,(l_l + l_m) + M_R\,l_m = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

Note that Span L-R is not loaded (a loaded span has no fixed points).

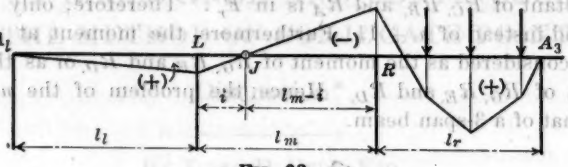

FIG. 98.

The second relation is given by the geometry of Fig. 98:

$$M_L : (-M_R) = i : (l_m - i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (97)$$

From Equations (96) and (97),

$$i = \frac{l_m^{\,2}}{l_m + 2\,(l_m + l_l)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (98)$$

or,

$$i = \frac{l_m}{1 + \dfrac{2\,(l_m + l_l)}{l_m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (99)$$

or,

$$i = \frac{l_m}{3 + 2\,\dfrac{l_l}{l_m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (100)$$

It is easy to remember Equation (100). The members, 3 and 2, are to be remembered from the fact that a multiple-span continuous beam, as already proved, is an aggregate of 3 and 2-span beams. Furthermore, if $l_l = 0$, $i = \frac{l_m}{3}$* (restrained beams).

It will be shown now how it is possible to prepare tables for the 3-span beams. In Fig. 99 load Span L-R (the middle span) with the load of unity. For this case according to the Theorem of Three Moments:

$$2\,M_L\,(l_l + l_m) + M_R\,l_m = -\,\alpha\,(1-\alpha)\,(2-\alpha)\,l_m^{\,2} \quad \ldots\ldots (101)$$

$$M_L\,l_m + 2\,M_R\,(l_m + l_r) = -\,\alpha\,(1-\alpha)\,(1+\alpha)\,l_m^{\,2} \quad \ldots\ldots (102)$$

Dividing Equation (101) by $l_m + 2\,(l_l + l_m)$, and Equation (102) by $l_m + 2\,(l_r + l_m)$,

$$\frac{2\,M_L\,(l_l + l_m)}{l_m + 2\,(l_l + l_m)} + \frac{M_R\,l_m}{l_m + 2\,(l_l + l_m)} = -\,\frac{\alpha\,(1-\alpha)\,(2-\alpha)\,l_m^{\,2}}{l_m + 2\,(l_l + l_m)} \quad \ldots (103)$$

$$\frac{M_L\,l_m}{l_m + 2\,(l_r + l_m)} + \frac{2\,M_R\,(l_m + l_r)}{l_m + 2\,(l_r + l_m)} = -\,\frac{\alpha\,(1-\alpha)\,(1+\alpha)\,l_m^{\,2}}{l_m + 2\,(l_r + l_m)} \quad \ldots (104)$$

* To determine the fixed points analytically for any given multiple-span beams, see Müller-Breslau or Strassner.

Introducing the two relations:

$$\gamma_l = \frac{i}{l_m} = \frac{l_m}{l_m + 2\,(l_m + l_l)} = \frac{1}{1 + \dfrac{2\,(l_m + l_l)}{l_m}}$$

and correspondingly,

$$\gamma_r = \frac{k}{l_m} = \frac{l_m}{l_m + 2\,(l_m + l_r)} = \frac{1}{1 + \dfrac{2\,(l_m + l_r)}{l_m}}$$

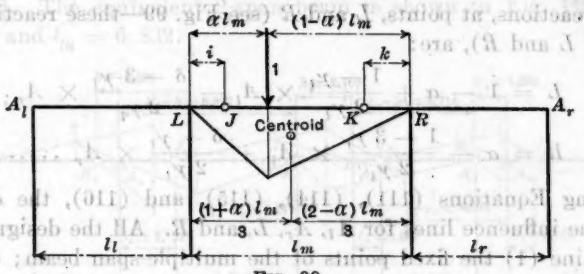

FIG. 99.

Equations (103) and (104) become,

$$(1 - \gamma_l)\, M_L + \gamma_l\, M_R = -\,\alpha\,(1 - \alpha)\,(2 - \alpha)\,\gamma_l\, l_m \ldots\ldots (105)$$

$$\gamma_r\, M_L + (1 - \gamma_r)\, M_R = -\,\alpha\,(1 - \alpha)\,(1 + \alpha)\,\gamma_r\, l_m \ldots\ldots (106)^*$$

Assuming $M_R = \beta\, M_L$, then,

$$M_L\,(1 - \gamma_l + \beta\,\gamma_l) = -\,\alpha\,(1 - \alpha)\,(2 - \alpha)\,\gamma_l\, l_m \ldots\ldots (107)$$

$$M_L\,[\gamma_r + (1 - \gamma_r)\,\beta] = -\,\alpha\,(1 - \alpha)\,(1 + \alpha)\,\gamma_r\, l_m \ldots\ldots (108)$$

Taking $M_L = A_l\, l_l$ (see Fig. 99), then,

$$A_l = -\,\frac{\alpha(1 - \alpha)\,(2 - \alpha)}{1 - \gamma_l + \beta\,\gamma_l} \times \frac{2\,\gamma_l^2}{1 - 3\,\gamma_l} \ldots\ldots (109)$$

Dividing Equation (107) by Equation (108) and taking,

$$\frac{\gamma_l}{\gamma_r} = \delta$$

then,

$$\beta = \frac{3\,\gamma_l - (1 + \alpha)}{3\,\gamma_l - \delta\,(2 - \alpha)} \ldots\ldots (110)$$

or,

$$A_l = \frac{-\,6\,\gamma_l^3\,\alpha\,(1 - \alpha) + 2\,\delta\,\gamma_l^2\,\alpha\,(1 - \alpha)\,(2 - \alpha)}{[\gamma_l - \delta\,(1 - \gamma_l)]\,(1 - 3\,\gamma_l)} \ldots\ldots (111)$$

From Fig. 99,

$$A_r = \frac{M_R}{l_r} = \frac{\beta\, M_L}{l_r} = \frac{\beta\, A_l\, l_l}{l_r} \ldots\ldots (112)$$

* The writer did not want to exploit the geometrical meaning of Equations (105) and (106).

but,

$$\frac{l_l}{l_r} = \frac{\frac{l_l}{l_m}}{\frac{l_r}{l_m}} = \frac{\frac{1-3\gamma_l}{2\gamma_l}}{\frac{1-3\gamma_r}{2\gamma_r}} = \frac{1-3\gamma_l}{\delta-3\gamma_l} \quad \ldots\ldots\ldots\ldots (113)$$

so that,

$$A_r = A_l \times \frac{3\gamma_l - (1+\alpha)}{3\gamma_l - \delta(2-\alpha)} \times \frac{1-3\gamma_l}{\delta-3\gamma_l} \quad \ldots\ldots\ldots (114)$$

As soon as A_l and A_r are known, the problem is statically determinate and the middle reactions, at points, L and R (see Fig. 99—these reactions are likewise termed L and R), are:

$$L = 1 - \alpha + \frac{1-\gamma_l}{2\gamma_l} \times A_l - \frac{\delta-3\gamma_l}{2\gamma_l} \times A_r \quad \ldots\ldots\ldots (115)$$

$$R = \alpha - \frac{1-3\gamma_l}{2\gamma_l} \times A_l + \frac{\delta-\gamma_l}{2\gamma_l} \times A_r \quad \ldots\ldots\ldots (116)$$

By tabulating Equations (111), (114), (115), and (116), the designer can determine the influence lines for A_l, A_r, L, and R. All the designer has to do is to determine (1) the fixed points of the multiple-span beam; (2) the fixed points of the 3-span beam with the formulas, $i = \frac{l_m}{3 + 2\frac{l_l}{l_m}}$ and $k = \frac{l_m}{3 + 2\frac{l_r}{l_m}}$; and (3) the relations, $\gamma_l = \frac{i}{l_m}$ and $\delta = \frac{\gamma_l}{\gamma_r} = \frac{i}{k}$. For these γ_l and δ-values the tabulations will give the corresponding A_l, A_r, L, and R. By knowing these four influence lines the designer can answer every possible question relative to the n-span beam. This will be shown later by a definite problem.

Generally, the fixed points of the 3-span beam are not identical with the fixed points of the n-span beam. This, of course, involves extra work, because tables could be prepared for the relations, $l_l : l_m : l_r = 1 : x : y$. However, the fixed points of the 3-span beam, which are secondary fixed points of the n-span beam, play an important and beautiful rôle as will be shown.

For the 2-span beams call the loaded span l_λ and the unloaded span l_u. The middle reaction is called A_m, that for loaded span, A_λ, and for the unloaded span, A_u. The secondary fixed point in the loaded span is determined by the formula:

$$i = \frac{l_\lambda}{3 + 2\frac{l_u}{l_\lambda}};$$

then,

$$\gamma = \frac{i}{l_\lambda} = \frac{l_\lambda}{3 + 2\frac{l_u}{l_\lambda}}$$

and so if by measuring α from A_m to A_λ,

$$A_u = -\alpha(1-\alpha)(2-\alpha)\frac{\gamma^2}{(1-\gamma)(1-3\gamma)} \quad \ldots\ldots\ldots (117)$$

$$A_m = \alpha + \alpha(1-\alpha)(2-\alpha)\frac{\gamma}{2(1-3\gamma)} \quad \ldots\ldots\ldots\

If l_2 is loaded, the equivalent 3-span beam has the spans, $l_l = 5$, $l_m = 9$, and $l_r = 4.714$ (see Fig. 100). l means left, m, middle, and r, right.

The secondary fixed points may then be found as follows: i falls into the fixed point of the multiple-span beam because l_2 is preceded in both cases by $l = 5$, hence,

$$i = \frac{9}{3 + 2\frac{4.714}{9}} = 2.189;$$

$$k = \frac{5}{3 + 2\frac{4.714}{9}} = 2.223$$

$$\gamma_l = \frac{2.189}{9} = 0.243$$

and,

$$\delta = \frac{i}{k}\frac{2.189}{2.223} = 0.985$$

For $\delta = 0.985$, the values are as given in Table 34.

TABLE 34.—VALUES FOR $\delta = 0.985$.

γ_l	α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.00
0.243	A_l (−)	0.000	0.089	0.145	0.172	0.176	0.162	0.135	0.100	0.063	0.028	0.000
	A_r (−)	0.000	0.031	0.069	0.110	0.148	0.177	0.192	0.187	0.157	0.097	0.000
	L (+)	1.000	1.023	0.990	0.911	0.797	0.660	0.509	0.358	0.215	0.092	0.000
	R (+)	0.000	0.097	0.225	0.372	0.528	0.679	0.818	0.930	1.005	1.033	1.000
	$L - A_l$ (+)	1.000	0.934	0.844	0.738	0.620	0.498	0.374	0.258	0.152	0.064	0.000
	$R - A_r$ (+)	0.000	0.066	0.156	0.262	0.380	0.502	0.626	0.742	0.848	0.936	1.000
	$\frac{A_l}{L - A_l}$	0.000	0.096	0.171	0.234	0.285	0.326	0.362	0.390	0.414	0.432	$\frac{0}{0}$
	$\frac{A_r}{R - A_r}$	$\frac{0}{0}$	0.465	0.444	0.420	0.389	0.353	0.307	0.252	0.185	0.103	0.000

If l_3 is loaded, the spans of the equivalent 3-span beams are: $l_l = 6.811$; $l_m = 6$; and $l_r = 5$.

The secondary fixed points are then,

$$i = \frac{6}{3 + 2\frac{6.811}{6}} = 1.138$$

k (identical with the fixed point of the 4-span) $= 1.286$

$$\gamma_l = \frac{i}{l_m} = \frac{1.138}{6} = 0.190$$

$$\delta = \frac{i}{k} = \frac{1.138}{1.286} = 0.885$$

For $\delta = 0.885$, the values are as given in Table 35.

If l_4 is loaded, the spans of the equivalent 2-span beam are: $l_u = 4.913$, and $l_\lambda = 5$.

The secondary fixed point in l_λ is,

$$i = \frac{5}{3 + 2\frac{4.913}{5}} = 1.007$$

$$\lambda = \frac{1.007}{5} = 0.201$$

For $\gamma = 0.201$, the values are as given in Table 36.

TABLE 35.—VALUES FOR $\delta = 0.885$.

γ_l.	α.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.190	A_l (−).....	0.000	0.032	0.052	0.062	0.065	0.060	0.051	0.039	0.025	0.011	0.000
	A_r (−).....	0.000	0.021	0.044	0.067	0.086	0.101	0.107	0.103	0.085	0.051	0.000
	L (+).....	1.000	0.951	0.875	0.778	0.667	0.544	0.420	0.298	0.182	0.082	0.000
	R (+).....	0.000	0.102	0.221	0.351	0.485	0.616	0.738	0.844	0.928	0.980	1.000
	$L - A_l$ (+)..	1.000	0.919	0.823	0.716	0.601	0.485	0.369	0.259	0.157	0.071	0.000
	$R - A_r$ (+)..	0.000	0.081	0.177	0.284	0.399	0.515	0.631	0.741	0.843	0.929	1.000
	$\frac{A_l}{L - A_l}$.....	0.000	0.035	0.063	0.087	0.107	0.124	0.138	0.150	0.160	0.168	$\frac{0}{0}$
	$\frac{R}{R - A_r}$.....	$\frac{0}{0}$	0.255	0.247	0.234	0.216	0.196	0.170	0.139	0.101	0.055	0.000

TABLE 36.—VALUES FOR $\lambda = 0.201$.

γ.	α.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.201	A_u (−).....	0.000	0.023	0.037	0.045	0.049	0.048	0.043	0.035	0.025	0.013	0.000
	A_m (+).....	0.000	0.145	0.273	0.390	0.497	0.595	0.685	0.769	0.849	0.926	1.000
	A_λ (+).....	1.000	0.878	0.764	0.655	0.552	0.453	0.358	0.266	0.176	0.087	0.000
	$A_m - A_u$ (+)	0.000	0.122	0.236	0.345	0.448	0.547	0.642	0.734	0.824	0.913	1.000
	$\frac{A_u}{A_m - A_u}$.....	$\frac{0}{0}$	0.182	0.156	0.133	0.110	0.088	0.070	0.048	0.030	0.014	0.000

For the sake of convenience the writer tabulated $L - A_l$ and $R - A_r$ for the 3-span beams and $A_m - A_u$ for the 2-span beams. These values are needed for the determination of the moments inside the loaded span. The values, $\frac{A_l}{L - A_l}$, $\frac{R}{R - A_r}$, and $\frac{A_u}{A_m - A_u}$, will facilitate the work greatly. Multiplying $\frac{A_l}{L - A_l}$ by l_l gives the point of application of $L - A_l$. The distance, $l_l \frac{A_l}{L - A_l}$, is to be measured from L to the right. Similarly, the distance, $l_r \frac{A_r}{R - A_r}$, is the point of application of the resultant of R and A_r, and is to be measured from R to the left. The distance, $l_\lambda \frac{A_u}{A_m - A_u}$, has similar meaning and is to be scaled from A_m to the loaded span. These distances may be scaled vertically as shown in Fig. 101.

For the inside spans of the multiple-span beam there are two curves. To get the lower one, for instance for l_2, multiply the values, $\frac{A_l}{L - A_l}$, by $l_l = 5$. Thus, this curve is the influence line for the location of the resultant of L and A_l. The influence line for the location of the resultant of R and A_r is the

upper curve which has been developed by multiplying $\frac{A_r}{L - A_r}$ by $l_r = 4.714$.

The vertical distances between the lower and upper curves show simple beams the left reaction of which is $L - A_l$ and the right reaction, $R - A_r$. For every position of the load there is a simple beam. The position of the load on these simple beams is shown by the straight line drawn at 45 degrees. These simple beams change their positions and their lengths.

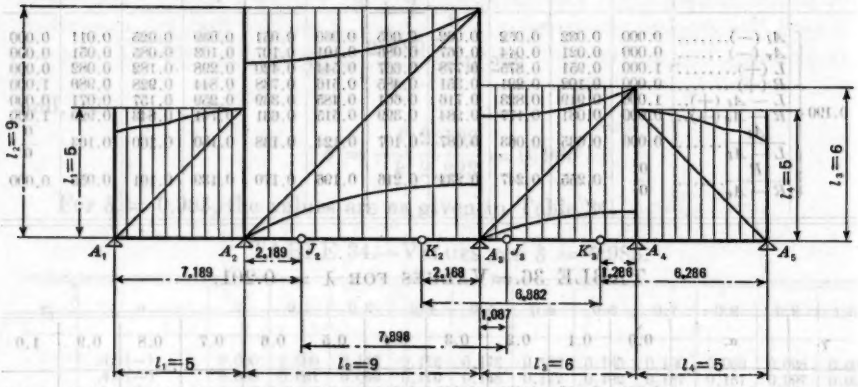

FIG. 101.

If $\alpha = 0$, the left end of this beam is at Point L. As the load moves to the right, the left end of the beam (point of application of the resultant of forces, L and A_l) is moving to the right too. Finally, the load arrives in Point R. In this case, $\alpha = 1$. Tables 34 and 35 show the value,

$$\frac{A_l}{L - A_l} = \frac{0}{0 - 0} = \frac{0}{0}$$

Without using higher mathematics, it can be shown statically that $\frac{0}{0}$ in this case equals i_2 (the index, 2, indicating that the second span of the n-span beam is loaded), or, in other words, when the load arrives in the right middle support of the equivalent 3-span beam the point of application of the resultant of the two left reactions, A_l and L, is in the left fixed point of the equivalent 3-span beam (secondary fixed point of the n-span beam). Similarly, if the load is in Point L, the right end of the simple beam (point of application of the right reactions, R and A_r) is in the right fixed point of the 3-span beam.

To prove this, imagine that the load arrived at Point L from the left (Fig. 102 (a) to (c)). From the theory of the fixed points the moments will pass through Point K (right fixed point of the middle span). Fig. 102 (b) and (c) shows the resulting 2-span beam. As the load is approaching Point L, K remains the right support of the 2-span beam; and, finally, if the load reaches Point L the left span of the 2-span beam may be omitted as being not stressed and thus the right end of the simple beam is at K or at the right fixed point of the middle span of the equivalent 3-span beam.

The influence lines of Fig. 101 will facilitate the work greatly inside the span. The tabulations give the reactions of the simple beam; Fig. 101 shows the distances of any section from the reactions and from the load if it is desired to determine influence lines for moments inside the span.

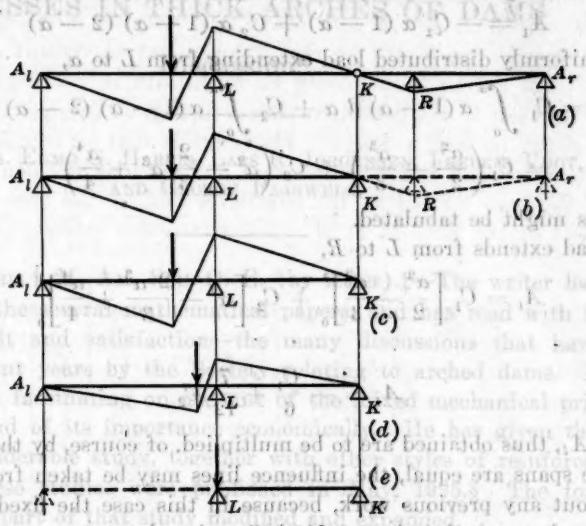

FIG. 102.

Outside the loaded span the influence line for A_l is the influence line for every moment, reaction, and shear left of the loaded span. Note, however, that every quantity has its own multiplier; and, correspondingly, on the right from the loaded span, A_r, similar relations hold.

To determine, for instance, the influence line for A_1 (Fig. 101)—as long as the load is between A_1 and A_2 the influence line for A_1 is identical with A_λ as given in Table 33. If the load is between A_2 and A_3 the influence line for A_1 is identical with A_l, as given in Table 34. If the load is between A_3 and A_4 the influence line for A_1 is that for A_l, as given in Table 35, multiplied by $-\frac{2.189}{7.189}$ (if l_3 is loaded, the outside left reaction is in I_2: this outside left reaction, A_l, is the resultant of A_1 and A_2). If the load is between A_4 and A_5, the influence line for A_1 is that for A_u, as given in Table 36, multiplied by $-\left(\frac{1.087}{7.898}\right) \times \left(-\frac{2.189}{7.189}\right)$.

As soon as the fixed points are found and the tabulations made, everything else may be computed by slide-rule.

Somewhat more complicated is the computation for variable moments of inertia. If the moment of inertia does not change inside the span, in order to use the tables, determine the fixed points of the n-span beam n times. If the moment of inertia of the loaded span be I_l, and that of any other span, I_p, multiply every span length by the corresponding $\frac{I_l}{I_p}$. Thus may be obtained

for every loaded span a different n-span beam. After the n-span is found the method is the same as before.

For uniformly distributed loads the functions may simply be integrated. For instance, by remodeling Equation (111):

$$A_1 = -C_1 \alpha (1-\alpha) + C_2 \alpha (1-\alpha)(2-\alpha)$$

and for a uniformly distributed load extending from L to α,

$$A_l = C_1 \int_0^\alpha \alpha (1-\alpha)\, d\alpha + C_2 \int_0^\alpha \alpha (1-\alpha)(2-\alpha)\, d_\alpha$$

$$= C_1 \left(\frac{\alpha^2}{2} - \frac{\alpha^3}{3}\right) + C_2 \left(\alpha^2 - \frac{2}{3}\alpha^3 + \frac{\alpha^4}{4}\right)$$

which values might be tabulated.

If the load extends from L to R,

$$A_l = C_1 \left[\frac{\alpha^2}{2} - \frac{\alpha^3}{3}\right]_0^1 + C_2 \left[\alpha^2 - \frac{2}{3}\alpha^3 + \frac{\alpha^4}{4}\right]_0^1$$

or,

$$A_l = -\frac{C_1}{6} + \frac{7}{12} C_2$$

The values, A_l, thus obtained are to be multiplied, of course, by the load.

If all the spans are equal, the influence lines may be taken from the tabulations without any previous work, because in this case the fixed points and, consequently, the equivalent beams are known in advance.

Conclusions.—

(1) There never will be a better method for dead loads than that of the authors, which is built up on the broadest mathematical vision.

(2) The writer claims originality in discovering the fact that every n-span beam is an aggregate of n-2 three-span and 2 two-span beams; and in introducing the secondary fixed points and the influence lines for the location of the equivalent simple beams.

(3) The writer's original problem was: Where are the points of application of certain groups of reactions of an n-span beam?

(4) The writer believes that this question is important for every kind of continuous structures, such as multi-story rigid frames, Vierendeel girders, etc.

STRESSES IN THICK ARCHES OF DAMS

Discussion*

By Messrs. Elmo G. Harris, Lars R. Jorgensen, Fredrik Vogt, and George Paaswell.

Elmo G. Harris,† M. Am. Soc. C. E. (by letter).‡—The writer has laboriously followed the several mathematical papers, and has read with interest—also some profit and satisfaction—the many discussions that have been published in recent years by the Society relating to arched dams. He has found the subject fascinating on account of the mixed mechanical principles involved in it, and of its importance economically. He has given this type of structure considerable study, together with other styles of reinforced concrete dams. These studies were published in May, 1925.§ The following is in substance a part of that study modified and expanded.

Several of the mathematical analyses alluded to are based on the assumption that one of the major mechanical principles involved is cantilever action in any small vertical prism of the dam. The writer does not concede that such cantilever action is of sufficient importance to justify the intricate analyses, but believes that conclusions thus reached may be quite misleading and that, instead of cantilever action, vertical-beam action‖ is a major mechanical principle active in carrying and distributing the loads on an arch dam. This conclusion results from the following considerations.

In Figs. 37, 38, and 39 are shown various types of arch dams. Fig. 37 gives three cross-sections; Fig. 38 represents a dam designed by the cylinder method, with no surplus material, whereas Fig. 39 shows a dam also designed as a cylinder, but with surplus material at the top. Referring to Fig. 37, let (*a*), (*b*), and (*c*) represent vertical cross-sections of the wall of a dam. The question as to whether or not the designer should consider cantilever action as a major factor will depend on whether or not the foundation, *C*, can be made so rigid that the vertical tangent to the face of the dam at *B* can be assumed as unmoved when the dam is loaded and all the stresses and strains are in action. It is the writer's opinion that it is not necessary for safety, and certainly not economical, to design the foundation with this intent. The designer will usually find no difficulty in obtaining a safe bearing stress under

* Discussion on the paper by B. F. Jakobsen, M. Am. Soc. C. E., continued from February, 1926, *Proceedings*.

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‡ Received by the Secretary, March 6, 1926.

§ "A Study of Reinforced Concrete Dams," Missouri School of Mines *Bulletin*, Vol. 8 No. 3.

‖ Analysis of vertical-beam dams constitutes part of the study alluded to previously.

the foundation. The more exacting condition will be that of providing an effective cut-off wall below *B D*. In this part of the study it should be noted that any unequal bearing on the foundation will cause a movement at *A* independent of the elastic deflection due to stresses above *B D*. Evidently, the more the cantilever action with consequent varying intensity of stress on the foundation, the more will be this independent movement at *A*. It is conceivable that a foundation could be made safe in bearing and safe against leakage and left in the condition shown in Fig. 37 (*c*), yet there would be but little rigidity in such a design. The point is that an infinitesimal movement of the tangent at *B* would render vain any attempt at calculating the proportion of the load carried by the cantilever and by the arch. Doubtless some movement of the tangent at *B* will occur under loading in the majority of dams.

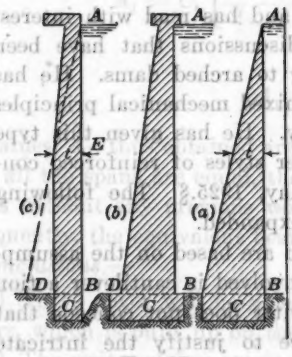

FIG. 37.

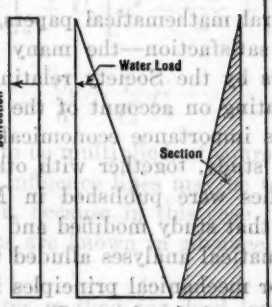

FIG. 38.

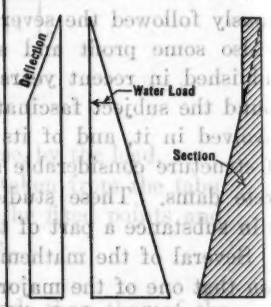

FIG. 39.

The cylinder formula would indicate a zero thickness or no stress at the water surface in an arched dam. If pure cantilever action occurs there must be a movement down stream at *A*. This down-stream movement would be resisted by the arch. Inevitably pure cantilever action cannot extend to the top of an arch dam.

At the top the vertical beam rests against a more or less elastic support. The more surplus materal there is at the top of the dam (as measured by the cylinder formula) the more rigid will be the support at the top end of the vertical beam and the farther down from the top will be the point of contraflexure of the beam considered as a cantilever.

Taking up now the question as to which is the more logical, the cantilever or the vertical beam, or which action is probably the more dominant in dams as built, or which should be developed by the design. Suppose Sections (*a*) and (*c*) (Fig. 37), be taken from straight walls. For the cylinder formula (in which *T*, the axial stress, is equal to the product of the radius, *r*, by the unit pressure, *p*), *r* would be infinite and, therefore, *T* would also be infinite. Evidently the cylinder formula fails to apply and in both cases simple beam action must carry all the load. In Section (*a*) the action would be purely that of a cantilever even if some movement of the tangent at *B* should occur. In Section (*c*), if the top girder be supported at intervals by piers or abutments

and the beams, *A B* (making up the wall), simply rest against the girder at the top and against the cut-off wall at *D*, the reactions will be, at *A*, one-third the total pressure between *A* and *B* and, at *D*, two-thirds. As such dams would be built the vertical beams (curtain wall, *A B*) would be monolithic with the girder at the top and with the cut-off wall or foundation, *C*, at the base. Whatever rigidity or fixedness of the beam may thus be obtained at the top and bottom will reduce the bending moment near the center, and therefore (theoretically) the beams could be made lighter the more nearly fixed they are at the ends.

Now assume that both dams, (*a*) and (*c*) (Fig. 37), are built on a circular arc in plan, thus becoming arched dams. It has been shown that Case (*a*) cannot then be under pure cantilever action in combination with arch action; but in Case (*c*) vertical beam action must continue although the load now carried by the beam will be reduced in amount and changed in relative intensity at all points due to the introduction of arch action.

That the beam must carry some of the load is evident by considering the fact that the arch stresses causing arch deflection are at right angles to the plane in which the arch is bent (plane of the paper in Fig. 40 and Fig. 42). The beam could not be held in this bent condition except by loads in the plane of the paper. Of course, if vertical beam action is to be utilized, the wall must be reinforced accordingly.

A possible fallacy exists as to the limit in the length of radius for an arched dam. Without exception all studies on the subject that the writer has followed fix, by the cylinder formula alone, the maximum radius that may be economically used. With a conservative permissible average unit stress this maximum radius is usually put at 300 or 400 ft. The fallacy is in failing to take account of vertical beam action which will be present in arched dams as usually proportioned, and also in failing to utilize this additional recourse available to the engineer if he will modify his design with the purpose of bringing into action and economically utilizing vertical beam action.

To illustrate: In the writer's studies* it was shown that for straight vertical-beam dams the appropriate thickness at mid-height is $0.018\ h_2^2$ (h being the total height) and that that thickness may be made constant from mid-height down to the base. For a beam 50 ft. high this thickness would be 6.3 ft. By the cylinder formula, assuming about 50 000 lb. per sq. ft. as the average horizontal thrust in the arch, the radius that would require such a thickness at 25-ft. depth would be 200 ft. and for the same thickness at 50-ft. depth the radius would be 100 ft. Yet the section designed as a vertical beam would be safe with an infinite radius (straight wall). Evidently any degree of curvature will add strength instead of reducing it. A rational design of arched dams will not be accomplished until vertical beam action is recognized and utilized.

From an economic point of view the combination of vertical beam and of arch actions is not as promising as might seem from the foregoing, for, to carry the top reaction of the vertical beams a horizontal arch rib must be

* Missouri School of Mines *Bulletin*, Vol. 8, No. 3, p. 1.

placed along the top of the dam (replacing the top girder of the straight vertical-beam dam). For long radii this arch rib will assume large proportions.

The top reaction of the vertical beams in a combination vertical-beam and arch dam depends on several factors, among which are:

(*a*) The rigidity of the base or foundation (Fig. 37 (*c*)). The more rigid the base the less will be the top reaction.

(*b*) The radius of curvature—everything else holding constant, the flatter the arc the greater will be the top reaction.

(*c*) The lateral location of the beam between the crown and the abutment. It will probably be greatest for a vertical beam at the crown and will gradually reduce to zero at the abutment.

(*d*) The rigidity of the arch rib against deflection radially inward. This depends on the radius and the unit stress or area; the more rigid the rib, as compared to the wall below, the greater will be the top reaction.

From these considerations it is evident that a satisfactory determination of the reactions will be difficult and may be impossible.

Since the load on the top arch rib is not constant the cylinder formula cannot be applied to that rib. All the thrust going through the top arch rib is so much relief to the arch wall below.

One important matter that seems to be generally ignored (intentionally or otherwise) is the large portion of the water load on an arched dam that goes into the base or foundation without affecting the arch. This is due to fixedness of the base against horizontal movement inward or down stream. Consider a horizontal slice, say, 1 ft. vertically, adjacent to and above the base. If the cylinder formula applies to this slice it would be under compression in the direction of the axis of the arch; the arch would be shortened and would slide (deflect) inward a determinate distance. This motion is prevented because the slice is rigidly fixed at the base. It may then be fairly assumed that all the water pressure on the bottom foot of the arch goes into the base as pure punching shear. Likewise, the next horizontal slice above will be fixed to the slice below but some deflection will develop; and this same relation will increase as points are taken progressively up the face. This deflection will be the sum of the effects due to pure shear, vertical-beam action, and arch action. Evidently, the water pressure at any point (as at *P*, Fig. 40 and Fig. 42) will likewise be divided, Part *s* going down into the base without affecting the beam or arch, Part *b* going into the vertical beam, and the remainder, Part *a*, going into the arch.

If any one has difficulty in understanding the difference between the shear, *s*, and that in a simple vertical beam it can be made clear by imagining two adjacent horizontal slices of the arch (say, 1 ft. thick) as at *J*, Fig. 40, without cohesion or friction in the dividing plane, but having dowel pins set between the two slices. Evidently the lower slice is more rigidly held against arch deflection than the upper one. Thus will be developed a pure shear in the dowel pins. It is this shear that is represented by *s* in Figs. 40 and 42.

In Figs. 39 to 42, inclusive, the writer presents some suggestive diagrams illustrating what takes place in an arched dam, such as distribution of the

water load, deflections, and shears. Of course the proportions in the diagrams may not be correct even for any one dam, and they would change with every change in the proportions of a dam, in the rigidity of the base, and with the place in the dam at which the section is taken.

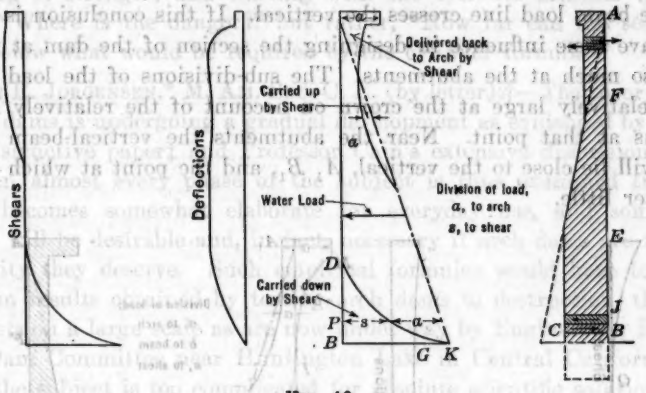

FIG. 40.

For Fig. 40 the cylinder formula was used taking account of shear and of resistance to deflection by surplus material placed at the top of the dam; vertical beam action is ignored. In Fig. 41 on the other hand, vertical beam action alone is considered, with due recognition of the arch deflection in the rib at the top. In Fig. 42 the two previous effects are combined and the deflections in general are less, due to the divided load. The distance, *G K*, allows for the arch action from the horizontal compression of the trench wall below *C*. The reaction, *M-N*, will include all the areas, *D G B*, and the greater part of the area, *I G D*.

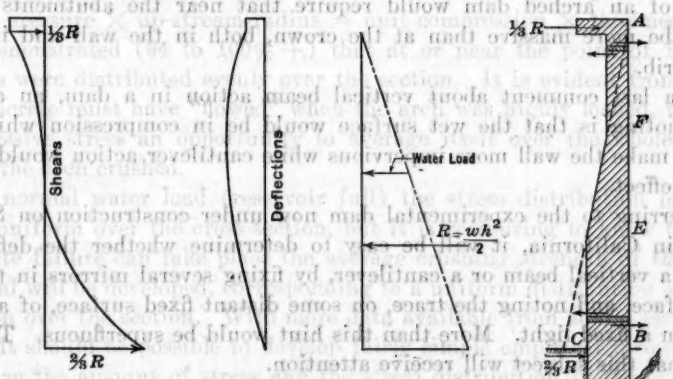

FIG. 41.

The point, *D* (Fig. 42), will not be at the same height in all the vertical sections. It will descend toward the base as the section approaches the abutments. The beam load, *b*, will become less as the section approaches the abutment. The reason for these variations is that the inward deflection of the

arch becomes less as the abutment is approached and therefore the deflections causing beam and shear loads will be less.

It is with some temerity that the writer shows that the cylinder formula applies correctly at only one point in a vertical section, the point, I (Fig. 42), where the beam load line crosses the vertical. If this conclusion is correct, it should have some influence in designing the section of the dam at the crown, but not so much at the abutments. The sub-divisions of the load (s and b) will be relatively large at the crown on account of the relatively large arch deflections at that point. Near the abutments the vertical-beam load line, $G\ I\ M$, will lie close to the vertical, $A_1\ B_1$, and the point at which they cross will matter little.

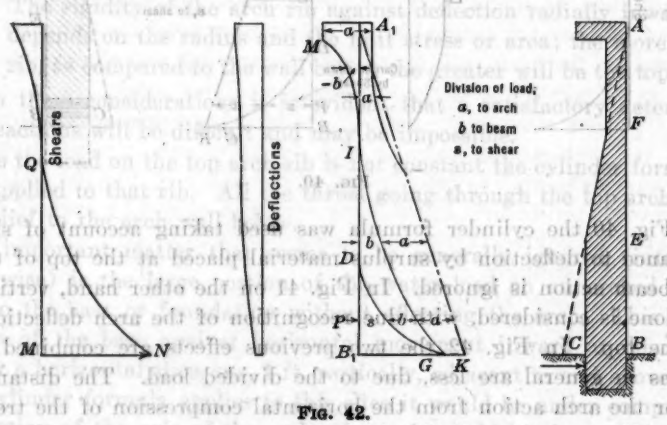

FIG. 42.

If the foregoing conclusions are correct it would indicate that the economic design of an arched dam would require that near the abutments the dam should be more massive than at the crown, both in the wall and in the top arched rib.

As a last comment about vertical beam action in a dam, an advantage worth noting is that the wet surface would be in compression which would tend to make the wall more impervious while cantilever action would have the reverse effect.

Referring to the experimental dam now under construction on Stevenson Creek, in California, it will be easy to determine whether the deflection is that of a vertical beam or a cantilever, by fixing several mirrors in the down-stream face, and noting the trace, on some distant fixed surface, of a reflected ray from a fixed light. More than this hint would be superfluous. The writer hopes that this subject will receive attention.

Thus far, the writer has not alluded to the effect of temperature changes, nor will he go further than to express the belief that temperature stresses can be practically eliminated by one or more flexible joints. These would usually need to go only part way down from the top. If it is practicable to put "hinged" joints in a concrete arched bridge surely it could be done with safety in an arched dam where there are no shocks or vibrations.

Although the numerical determination of the stresses in an arched dam seems hopeless, the writer wishes, in conclusion, to call attention to the comforting and reassuring fact that, excepting temperature stresses, each and every one of the mechanical agencies coming into play in an arched dam co-operate in giving it strength, and starting with the cylinder formula the question is not, "Where is the danger?" but rather, "How far can the section be reduced below what would be required by the cylinder formula?"

LARS R. JORGENSEN,* M. AM. SOC. C. E. (by letter).†—The theory of stresses in arch dams is undergoing a gradual development as evidenced by Mr. Jakobsen's instructive paper‡ and Professor Cain's extensive discussion.§

When almost every phase of the subject is thus examined the resulting theory becomes somewhat elaborate for everyday use, and some short-cut method will be desirable and, in fact, necessary if arch dams are to enjoy the popularity they deserve. Such empirical formulas would have to be worked out from results obtained by testing arch dams to destruction, that is, from such tests on a large scale as are now under way by Engineering Foundation's Arch Dam Committee near Huntington Lake in Central California.

As the subject is too complicated for absolute scientific solution, empirical formulas based on actual test data will undoubtedly prove expedient for designing purposes. Several tests to destruction were made on small arch dams in connection with the construction of Lake Cushman Dam, a constant angle arch, 275 ft. high, built by the City of Tacoma, Wash. While these test dams (1 in. thick, 36 in. span, and 36 in. radius, first series, and 15 in. thick, with a 7-ft. up-stream radius) were too small to use as a foundation for empirical formulas based on results from crushing tests, it was rather interesting to find that the unit crushing strength of these arches was from 94 to 100% + of the unit crushing strength of cylinders made from the same material.‖ The crushing strength of the arches was obtained by using the simple arch formula (water pressure $\times$ up-stream radius = unit compression $\times$ thickness), and it was demonstrated (94 to 100% +) that at or near the point of failure the stresses were distributed evenly over the section. It is evident from this that the concrete must have "flowed" when the arch was highly loaded, to give the compressive stress an opportunity to average itself over the whole area just before the arch crushed.

At normal water load (reservoir full) the stress distribution is far from being uniform over the cross-section, but it is reassuring to know that before complete failure can take place the average crushing strength of the building material will be developed, corresponding to a uniform distribution of the compression over the section. With more data available from actual tests on large dams, it should be possible to develop fairly simple empirical formulas, which will give the amount of stress and the stress distribution in the building material for normal loads on the arch.

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* Cons. Engr., Constant Angle Arch Dam Co., San Francisco, Calif.

† Received by the Secretary, April 8, 1926.

‡ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 206.

§ *Loc. cit.*, p. 252.

‖ These data will soon be published by B. E. Torpen, Assoc. M. Am. Soc. C. E., who was largely responsible for making the tests on these dams.

It is not safe to assume the cantilever as taking any load, since whether it does or does not depends on the degree of water-soaking and its distribution between the up-stream and the down-stream face of the dam body; and this cannot be calculated. It seems fair to assume, however, that the effect of the water-soaking (swelling) is distributed similarly to the effect of uplift, or that it is a maximum at the up-stream face decreasing to zero at the down-stream face.

For the sort of concrete used for massive dams (1 : 3 : 6 approximately) the coefficient of swelling can be taken at 0.0003.* Then if a dam were 300 ft. high before the water came on its up-stream face, it would become $300 \times 0.0003 \times 12 = 1.08$ in. higher along the up-stream face with the reservoir full. As the down-stream face remains at its original length the dam would deflect in a down-stream direction without developing any secondary stresses to resist this movement. The amount of this deflection depends on the shape of the cross-section of the dam and may easily be as much as the deflection of the arch carrying the full water load, in which case the cantilever cannot relieve the arch of any load. This also explains why the cantilever never cracks along the up-stream face at or near the bottom, or at any other place, and why apparent tension in the arch along the up-stream side is not serious; tension simply does not exist in either place, due to the swelling of the wet concrete. Since it is doubtful whether the cantilever relieves the arch of any load, the arch should be calculated to support the entire load.

DR. FREDRIK VOGT† (by letter).‡—The writer has studied this valuable paper with the greatest interest. Its reference to his paper, "Ueber die Berechnung der Fundamentdeformation",§ gives him the opportunity of offering some supplementary remarks on the influence that the yielding of the foundation has upon these thick arches. For so complicated a body statically as an arch dam, it would be fiction to claim any exact stress calculations. The writer's calculations, therefore, do not pretend to be exact, but only to show approximately how the stresses are changed due to the yielding of the foundation. At first glance it might seem obvious to consider this yielding, as negligible. Closer investigations, however, show that if a more or less correct calculation is wanted, it should by no means be neglected in the case of thick arches.

The writer agrees with Mr. Jakobsen's remarks relative to division of loads between the cantilevers and the arches.‖ The following, therefore, refers to the case in which the arches have a known, uniformly distributed pressure, p_a (the author's Fig. 2¶). In the near future the writer expects to publish a paper on the influence of the yielding of foundation on the division of load between the cantilevers and the arches. Consequently, that part of the question is not discussed.

* See Campbell and White's experiments, *Engineering News*, July 6, 1911.
† Civ. Engr., Trondhjem, Norway.
‡ Received by the Secretary, April 23, 1926.
§ *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.
‖ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 251.
¶ *Loc. cit.*, p. 211

(*A*).—In Fig. 43 a cantilever is acted on by a force, P, per unit length (that is, load = $P\,b$). The influence of the yielding of the foundation on the deflection of this cantilever will be approximately equivalent to the replacement of the elastic foundation by a continuation of the cantilever with the length, h_1, down to an imaginary unyielding foundation. The calculations assume that the foundation is bounded by an infinite plane surface, and that the material of the foundation follows Hooke's law. To simplify matters, it is assumed that the modulus of elasticity, E, is the same for the foundation as for the cantilever, a condition which will be nearly fulfilled in the case of a dam. If the difference is appreciable, the displacements of the foundation here given may be corrected by multiplying them by the ratio of E for the arch to E for the foundation. If the rock in the foundation is shattered, the yielding will be greater of course than is here given, since the tensile strength in this case has disappeared. A calculation on this assumption is obviously impossible. Further, the following calculations assume an arch without cracks or open contraction joints.

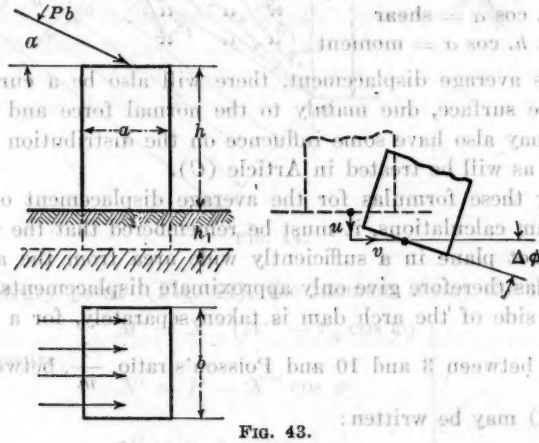

FIG. 43.

When the moment is of main importance for calculating the deflection, and when Poisson's ratio, $\frac{1}{m_1}$ is small:

$$h_1 = 0.45\ t \text{ (approximately)} \qquad (110)$$

When the normal force or the shear is of importance, h_1 must be given a larger value.

If the deformation due to the normal compression is considerable as compared with the deflection due to the moment, the calculation of the deflection based on Equation (110) will be as given in Article (*D*).

Approximate formulas for the average displacement and rotation of the loaded part of the surface, Fig. 43, are* for values of b varying from a to 30 a:†

* See *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2, Equations 8, 17, 20, 23, and 27. The pressure, p, per square inch in these formulas is here expressed by N, S, and M, in the usual manner. The formulas are developed by means of the known equations from Boussinesq and Cerrutti.

† For values of b varying from a to $\frac{a}{30}$, the expression, $\sqrt[3]{a^2\,b}$, may be replaced by $\sqrt[3]{a\,b^2}$.

$$\left.\begin{aligned} u &= \frac{m^2-1}{m^2} \times \frac{N}{E\,a} \times \sqrt[3]{a^2\,b} \\ v &= \frac{m^2-1}{m^2} \times \frac{S}{E\,a} \times \sqrt[3]{a^2\,b} + \frac{(m-2)\,(m+1)}{m^2} \times \frac{M}{E\,a} \times \frac{1}{1+1.1\frac{a}{b}} \\ \Delta\,\phi &= \frac{18}{\pi} \times \frac{m^2-1}{m^2} \times \frac{M}{E\,a^2} \times \frac{1}{1+0.25\frac{a}{b}} \\ &\quad + \frac{(m-2)\,(m+1)}{m^2} \times \frac{S}{E\,a} \times \frac{1}{1+1.1\frac{a}{b}} \end{aligned}\right\} \quad \ldots\ldots(111)$$

in which,

$N = P.\ \sin\alpha$ = normal force per unit length,
$S = P.\ \cos\alpha$ = shear " " "
$M = P.\ h.\ \cos\alpha$ = moment " " "

Besides this average displacement, there will also be a curvature on the originally plane surface, due mainly to the normal force and the moment.* This curving may also have some influence on the distribution of stress near the abutments, as will be treated in Article (*C*).

In applying these formulas for the average displacement of the foundation in arch dam calculations, it must be remembered that the surface of the foundation is not plane in a sufficiently wide area from the abutment, and that the formulas therefore give only approximate displacements.

When each side of the arch dam is taken separately, for a dam with the ratio of b to a between 3 and 10 and Poisson's ratio, $\frac{1}{m}$, between $\frac{1}{6}$ and $\frac{1}{10}$, Equations (111) may be written:

$$\left.\begin{aligned} u &= \xi \times \frac{N}{E} \\ v &= \xi \times \frac{S}{E} + \eta \times \frac{M}{E\,a} \\ \Delta\,\phi &= \mu \times \frac{M}{E\,a^2} + \eta \times \frac{S}{E\,a} \end{aligned}\right\} \quad \ldots\ldots(112)$$

in which,

$\xi = 1.40$ to 2.14
$\eta = 0.57$ to 0.79
$\mu = 5.14$ to 5.53

(*B*).—The yielding of the foundation due to the direct water pressure on the sides of the valley will be considered subsequently. In the meantime, the yielding of the foundation due to the reaction of the arch ring on the abut-

* See Figs. 4, 5, and 6, "Ueber die Berechnung der Fundamentdeformation," in *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

ments only will be considered (Fig. 44). As in the paper, the force, X', per unit length is denoted by X'', and its distance from the center, C, of the arch by A''. The abutment is assumed to be vertical, making an angle, ψ_1, with the arch radius. For convenience, the water pressure is assumed to be uniformly distributed over the whole length, E_0-E_1, and consequently also over E_1-E_2. As the angle, ψ_1, usually will be small, the resulting error will be negligible.

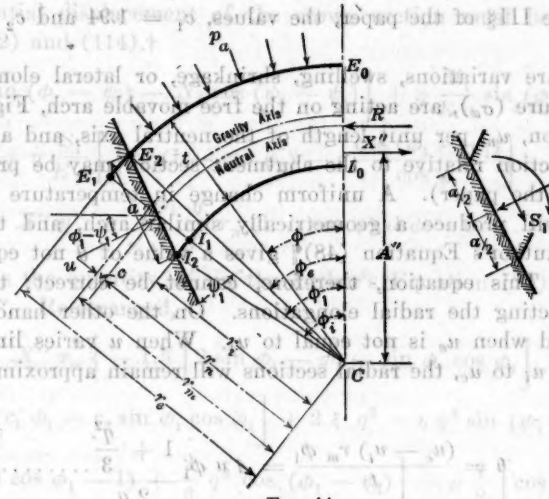

FIG. 44.

At an arbitrary point (Fig. 44), the moment about the neutral axis is:

$$M' = X''\,(A'' - r_n \cos \phi)$$

the normal pressure,

$$N' = R - X'' \cos \phi$$

in which,

$$R = p_a\, r_e \qquad \ldots\ldots\ldots\ldots\ldots\ldots (113)$$

and the shear,

$$S' = X'' \sin \phi$$

The moment, etc., at the abutment, referred to the center of gravity* and plane of the abutment, is:

$$M_1 = X''\left[A'' - r_n \cos \phi_1 - c\,\frac{\cos(\phi_1 - \psi_1)}{\cos \psi_1}\right] + R\,c \text{ (approx.)}$$
$$N_1 = R \cos \psi_1 - X'' \cos(\phi_1 - \psi_1) \qquad \ldots\ldots (114)$$
$$S_1 = R \sin \psi_1 + X'' \sin(\phi_1 - \psi_1)$$

The angle, ψ_1, is usually relatively small, and, therefore,

$$a = \frac{t}{\cos \psi_1} \text{ (approx.)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (115)$$

* M_1 should be taken about the center of gravity and not about the neutral axis of the curved arch, so that it may be used in Equations (112). As the distribution of stress in curved beams is not linear, Equations (112) are only approximate.

With the notation, $q = \frac{t}{2\,r_n}$, see the author's Equation (23)* :

$$\frac{c}{r_n} = \frac{q^2}{3}, \text{ and } \frac{c}{t} = \frac{q}{6} \text{ (approx.)}\dagger \quad \ldots\ldots\ldots\ldots\ldots\ldots (116)$$

Further notations are as follows:

$$c_1 = \frac{1}{2}\left(\frac{\kappa\,E}{G} + 1\right) \text{ and } c_2 = \frac{1}{2}\left(\frac{\kappa\,E}{G} - 1\right)\ddagger$$

From Article III§ of the paper, the values, $c_1 = 1.94$ and $c_2 = 0.94$, are obtained.

If temperature variations, swelling, shrinkage, or lateral elongations due to vertical pressure (σ_x), are acting on the free movable arch, Fig. 44, a tangential elongation, u_n, per unit length of the neutral axis, and a rotation, θ, of the crown section relative to the abutment section may be produced (see Article XI‖ of the paper). A uniform change in temperature on the free movable arch will produce a geometrically similar arch, and therefore no rotation. The author's Equation (48)¶ gives a value of θ not equal to zero, when $u_e = u_i$. This equation, therefore, cannot be correct; the error is caused by neglecting the radial elongations. On the other hand a rotation may be produced when u_e is not equal to u_i. When u varies linearly along the radius from u_i to u_e, the radial sections will remain approximately plane, and

$$\theta = \frac{(u_e - u_i)\,r_m\,\phi_1}{t} = \Delta\,u\,\phi_1\,\frac{1 + \frac{q^2}{3}}{2\,q} \quad \ldots\ldots\ldots\ldots\ldots (117)$$

in which, $\Delta\,u = u_e - u_i$, and

$$u_n = u_e\left(\frac{1}{2} - \frac{c}{t}\right) + u_i\left(\frac{1}{2} + \frac{c}{t}\right) \quad \ldots\ldots\ldots\ldots\ldots (118)$$

From symmetry, the crown section has no rotation, so that (see Equations (112)),

$$\mu \cdot \frac{M_1}{E\,a^2} + \eta \times \frac{S_1}{E\,a} + \int_0^{\phi_1} \frac{M'\,r_n\,d\,\phi}{E\,I'} + \theta = 0 \quad \ldots\ldots\ldots (119a)$$

or, re-arranged (see Equations (114)),

$$X''\,r_n\left\{\sin\phi_1 + \mu\,\frac{q}{6}\cos\psi_1\left[\cos\phi_1\cos\psi_1 + \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right] - \eta\,\frac{q^2}{3}\cos\psi_1\sin(\phi_1 - \psi_1)\right\} - X''\,A''\left\{\phi_1 + \mu\,\frac{q}{6}\cos^2\psi_1\right\}$$

$$= R\,r_n\,\frac{q^2}{3}\cos\psi_1\left\{\mu\,\frac{q}{6}\cos\psi_1 + \eta\sin\psi_1\right\} + \Delta\,u\,E\,{r_n}^2\,\frac{q^2}{3}\left(1 + \frac{q^2}{3}\right)\phi_1. \quad (119b)$$

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* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 216.

† More accurately,

$$\frac{c}{r_n} = \frac{q^2}{3}\left[1 - \frac{q^2}{15} + \frac{2}{315}\frac{q^4}{} - + \cdots\right]$$

‡ For uniform distribution of shear, $\kappa = 1$; for parabolic distribution in a rectangular section, $\kappa = 1.2$. The author assumes $\frac{E}{G} = 2.4$.

§ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 212.

‖ *Loc. cit.*, p. 222.

¶ *Loc. cit.*, p. 223.

From the author's Equations (38) and (39),* etc., the radial displacement in a cylinder may be given by,

$$\left.\begin{aligned} e' &= -r_n\,\varepsilon' = \sigma\,\frac{R\,r_n}{E\,t} - r_n\,u_n \\ \sigma &= \left[\frac{m-1}{2_m} + \frac{m+1}{2_m}\left(\frac{r_i}{r_n}\right)^2\right]\frac{r_e}{r_m} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (120)$$

in which,

The tangential displacement of the crown section must be zero, or, see Equations (112) and (114),†

$$\begin{aligned} &\frac{\xi}{E}\left[S_1 \sin(\phi_1 - \psi_1) - N_1 \cos(\phi_1 - \psi_1)\right] + \eta\,\frac{M_1}{E\,a}\sin(\phi_1 - \psi_1) \\ &+ \left[\mu\,\frac{M_1}{E\,a^2} + \eta\,\frac{S_1}{E\,a}\right]\left[r_n(1 - \cos\phi_1) - c\,\frac{\cos(\phi_1 - \psi_1)}{\cos\psi_1}\right] - e' \sin\phi_1 \\ &+ X''\,y'' + \int_0^{\phi_1} \frac{\theta}{\phi_1}\,r_n(1 - \cos\phi)\,d\phi = 0 \ldots\ldots\ldots\ldots (121a) \end{aligned}$$

in which, y'' is the quantity, y', in the author's Equation (29),‡ when A' is replaced by A''. Re-arranged,

$$\begin{aligned} &X''\,r_n \left\{ -1.5\left[2\sin\phi_1 - \phi_1 - \sin\phi_1\cos\phi_1\right]\right. \\ &+ q^2\left[c_1\,\phi_1 - c_2 \sin\phi_1\cos\phi_1\right] + 2\,\xi\,q^3 - \eta\,q^2 \sin(\phi_1 - \psi_1) \\ &\left[\cos\psi_1\,(2\cos\phi_1 - 1) + \frac{2}{3}\,q^2\cos(\phi_1 - \psi_1)\right] - \mu\,\frac{q}{2}\left[\cos\phi_1\cos\psi_1\right. \\ &\left.\left.+ \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\left[\cos\psi_1(1 - \cos\phi_1) - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\right\} \\ &+ X''\,A''\left\{3\left[\phi_1 - \sin\phi_1\right] + \eta\,q^2\cos\psi_1\sin(\phi_1 - \psi_1)\right. \\ &\left.+ \mu\,\frac{q}{2}\cos\psi_1\left[\cos\psi_1(1 - \cos\phi_1) - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\right\} \\ &= R\,r_n\,q^2\left\{\sigma\sin\phi_1 + 2\,\xi\,q\cos\phi_1 - \eta\left[\sin\psi_1\left[\cos\psi_1(1 - \cos\phi_1)\right.\right.\right. \\ &\left.\left.\left. - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right] + \frac{q^2}{3}\cos\psi_1\sin(\phi_1 - \psi_1)\right]\right. \\ &\left. - \mu\,\frac{q}{6}\cos\psi_1\left[\cos\psi_1(1 - \cos\phi_1) - \frac{q^2}{3}\cos(\phi_1 - \psi_1)\right]\right\} \\ &- u_n\,E\,r_n^2\,2\,q^3\sin\phi_1 - \Delta\,u\,E\,r_n^2\,q^2\left(1 + \frac{q^2}{3}\right)(\phi_1 - \sin\phi_1) \ldots (121b) \end{aligned}$$

From Equations (119) and (121), X'' and A'' may be found. The arch stresses due to X'' and $R = p_a\,r_e$ may be found from the author's Equations (34) and (36).§

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 219.

† The quantity, u_i in Equation (112) is the displacement normal to the abutment surface, $E_2\,I_2$, in Fig. 44, and $-\mu\cos(\phi_1 - \psi_1)$ is the displacement in the direction of X'' resulting from the displacement, u.

‡ *Proceedings*, Am. Soc. C. E., February, 1926, papers and Discussions, p. 217.

§ *Loc. cit.*, p. 218.

For instance, when $b = 4.5a$ and $m = 8$, see Equations (111) and (112),

$$\xi = 1.6,\ \eta = 0.7,\ \text{and}\ \mu = 5.4$$

With these values, using the example of Article XII* ($r_m = 100$ ft., $t = 50$ ft., $\phi_1 = 45°$, $p_a = 69.5$ lb. per sq. in., and $u_e = u_i = 0$ (that is, $\sigma_z = 0$), which gives $c = 2.118$ ft., $r_n = 97.882$ ft., and $q = 0.25541$), and with $\psi_1 = 0$,

$$R = 8687.5 \text{ lb. per arch slice } \tfrac{1}{12} \text{ in. thick;}$$
$$X'' = 0.47896\ R = 4\,161 \text{ lb.; and}$$
$$A'' = 0.83918\ r_n = 82.140 \text{ ft.}$$

Consequently, the yielding of the foundation causes a reduction in the force, X, and at the same time the force is acting considerably nearer the center, C, of the arch (Fig. 44). The moment due to X is therefore increased at the crown and reduced at the abutments. It follows that the tensile stress in the intrados is increased and that in the extrados is reduced.

Take now $\psi_1 = 20°$ instead of 0°; then,

$$X'' = 0.48856\ R = 4\,244 \text{ lb.; and}$$
$$A'' = 0.83956\ r_n = 82.178 \text{ ft.}$$

TABLE 11.—COMPARISON OF STRESSES ACCORDING TO THREE METHODS OF COMPUTATION.

	X, in pounds.	A, in feet.	Tensions, in Pounds per Square Inch.			
			Crown Section.		Abutment Section.	
			Extrados.	Intrados.	Extrados	Intrados.
With $\psi_1 = 0°$						
Cain	4 566	90.032	−191.7	+26.8	+102.5	−320.9
Jakobsen	4 420	88.125	−166.2	+21.3	+ 71.3	−375.2
Vogt	4 161	82.140	−216.0	+79.0	+ 8.1	−294.7
With $\psi_1 = 20°$						
Cain	4 566	90.032	−191.7	+26.8	+ 49.4	−421.5
Jakobsen	4 420	88.125	−166.2	+21.3	+ 28.8	−490.3
Vogt	4 244	82.178	−217.0	+84.5	− 29.8	−407.0

The numerical values of the stresses are given in Table 11, calculated from Mr. Jakobsen's Equations (34) and (36), for the abutment section with the angles, ϕ_e and ϕ_i (see Fig. 44). When ψ_1 is not equal to zero these formulas are certainly not exact even when the abutment section is a plane, since the abutment section and the radial sections cannot at the same time remain plane. The writer supposes, however, that the error is negligible, when ψ_1 is small. The corrections that follow from the curving of the abutments are treated in Article (*C*).

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 224.

The coefficients, η and μ, are only slightly changed by a change in the ratio of b to a, and the values, $\eta = 0.7$ and $\mu = 5.4$, may be used as approximate values in most cases. In any case the calculations are only approximate, and therefore the coefficients are taken only to the first decimal. The coefficient, ξ, on the other hand, depends more on the ratio of b to a, that is, on the length of the loaded part of the surface. For instance, with $b = 6a$ instead of $4.5a$, η and μ are unchanged (to the nearest tenth), but ξ is increased from 1.6 to 1.8. The resulting additional stresses at the crown and abutment sections are $-1.1; +5.2; +3.0; -1.6$ lb. per sq. in. The stresses are thus increased, but not to such an extent that it is of importance.

For comparison the calculations are recapitulated in Table 12 for the same value of $\frac{t}{r_m}$, but with $\phi_1 = 15°$ and using the values, $\xi = 1.6$, $\eta = 0.7$, and $\mu = 5.4$.

TABLE 12.—COMPARISON OF STRESSES ACCORDING TO THREE METHODS OF COMPUTATION.

	X, in pounds.	A, in feet.	Tensions, in Pounds per Square Inch.			
			Crown Section.		Abutment Section.	
			Extrados.	Intrados.	Extrados.	Intrados.
With $\psi_1 = 0°$						
Cain	8 200.5	98.862	−32.1	+12.7	+29.3	− 60.0
Jakobsen	7 871.4	96.768	−42.1	+13.2	+ 7.8	− 69.1
Vogt	6 233.5	94.520	−92.7	+ 5.4	−53.6	− 59.7
With $\psi_1 = 15°$						
Cain	8 200.5	98.862	−32.1	+12.7	+ 5.8	−112.7
Jakobsen	7 871.4	96.768	−42.1	+13.2	−12.1	−128.7
Vogt	8 260.8	94.929	−67.9	+68.1	−36.3	+ 80.8

With $t = 0.5\ r_m$ and $\phi_1 = 15°$, the span of the arch is about equal to its thickness, so that the dam is acting more like a plug or wedge than like an arch. As may be seen from Table 12, the tension in this case materially depends on the angle, ψ_1. By such a wedge the displacements mentioned in Article (*F*), also may be of importance, and the calculations may be somewhat uncertain.

(*C*).—As already mentioned, the abutment section, besides having the mean displacements given by Equations (112), will also suffer a curvature, even if the distribution of the load is linear.* This curving is caused more by the normal force and the moment than by the shear. The deformation and the mean displacements of the loaded part of the surface are shown diagrammatically (dotted) in Fig. 45. As an average over the whole length

* See Figs. 4, 5, and 6, "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

for the edges, E and I, the deformation due to the normal force, N_1, is about 8% less for $\xi = 1.6$ than the mean displacement, and for the gravity axis, A, it is about 4% greater.*

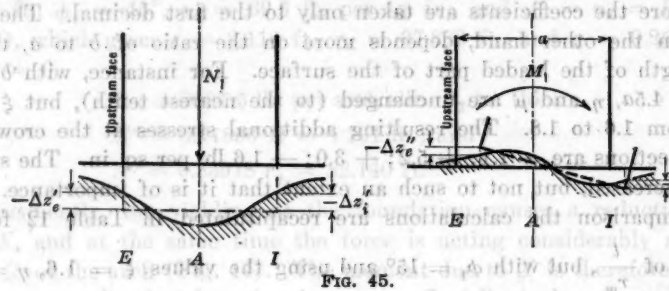

FIG. 45.

Consequently,

$$\Delta z_e' = \Delta z_i' = -0.08 \times 1.6 \times \frac{N_1}{E} = -0.128 \frac{N_1}{E} \text{ (approx.)}$$

When the moment is acting alone, the deformation for the edges† will be:

$$\Delta z_e'' = -\Delta z_i'' = \frac{1}{3} \Delta \phi \frac{a}{2} = \frac{1}{3} \times 5.4 \frac{M_1}{E a^2} \times \frac{a}{2} = 0.9 \frac{M_1}{E a} \text{ (approx.)}$$

Hence, for the numerical example, with $r_m = 100$ ft., $t = 50$ ft., $\phi_1 = 45°$, $\psi_1 = 0°$, and $p_a = 69.5$ lb. per sq. in.,

$$\Delta z_e = \Delta z_e' + \Delta z_e'' = -0.128 \frac{N_1}{E} + 0.9 \frac{M_1}{E a} = \frac{p_a r_e}{E} \times 0.009$$

$$\Delta z_i = \Delta z_i' + \Delta z_i'' = -0.128 \frac{N_1}{E} - 0.9 \frac{M_1}{E a} = -\frac{p_a r_e}{E} \times 0.265$$

The distribution of stress is then supposed to be approximately linear. The deformations, Δz_e and Δz_i, must be produced by additional stresses, which have their resultant and their moment equal to zero, since they have to change only the distribution of stress across the section. It follows from St. Venant's principle that the space through which these stresses act, in the direction of the arch axis, is small. Let Δz be produced by the stress, σ, on a length equal to t (for instance, 0.5 t in the arch and 0.5 t in the foundation); the corresponding additional stresses may be found from:

$$\frac{\sigma_e}{E} t = \Delta z_e = \frac{p_a r_e}{E} \times 0.009$$

$$\frac{\sigma_i}{E} t = \Delta z_i = -\frac{p_a r_e}{E} \times 0.265$$

or,

$$\sigma_e = p_a \frac{r_e}{t} \times 0.009 = +1.5 \text{ lb. per sq. in.}$$

$$\sigma_i = -p_a \frac{r_e}{t} \times 0.265 = -46.0 \text{ lb. per sq. in.}$$

* See Equation (5), "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

† *Loc. cit.*, Equation (10).

The length on which the additional stresses act is chosen arbitrarily and is rather uncertain. The numerical values of these stresses then also become uncertain. The calculation is given here only to show that the local distribution of stress in the abutment section, especially at the intrados, will be materially influenced by such conditions, which are difficult to estimate.

At a distance from the abutments—at the crown, say—this correction, according to St. Venant's principle, will not appreciably influence the distribution of stress. The curving of the radial sections due to shear, mentioned in the author's Article XVI,* will also cause a change in the distribution of stress. The corresponding additional stresses are, however, usually small.

(*D*).—If the temperature of the arch is changed and the temperature variation is linear along the radius, from Equations (117) and (118), for the free movable arch the values, u_n and θ, may be expressed by the temperature variation at the extrados, T_e, and at the intrados, T_i, and the coefficient of expansion, β. The corresponding values for X'' and A'' may be found from Equations (120) and (121).

A numerical example, for $r_m = 100$ ft., $t = 50$ ft., $\phi_1 = 45°$, $\psi_1 = 0°$, and $\beta E = 12$ lb. per sq. in., is given in Table 13. The values are calculated for $T_e = 0°$ Fahr. and 1° Fahr., and for $T_i = 0°$ Fahr. and 1° Fahr. The actual stresses can be found by multiplying these values by the actual temperature variations.

TABLE 13.—NUMERICAL EXAMPLE.—STRESSES FOR TEMPERATURE VARIATIONS.

	X, in pounds.	A, in feet.	Tensions, in Pounds per Square Inch.			
			Crown Section.		Abutment Section.	
			Extrados.	Intrados.	Extrados.	Intrados.
Yielding of Foundation Taken into Account.						
$T_e = 1°$, $T_i = 0°$..	−57.565	117.469	−3.201	+1.732	−6.301	+6.902
$T_e = 0°$, $T_i = 1°$..	−109.460	65.096	+5.599	−13.142	−0.295	−3.310
$T_e = T_i = 1°$..	−167.025	83.146	+2.398	−11.410	−6.596	+3.592
Yielding of Foundation Neglected.						
$T_e = 1°$, $T_i = 0°$..	−145.630	105.663	−4.594	−0.548	−12.435	+12.533
$T_e = 0°$, $T_i = 1°$..	−172.588	73.326	+5.932	−16.649	−3.361	−1.147
$T_e = T_i = 1°$..	−318.218	88.125	+1.338	+17.197	−15.796	+11.386

For these stresses, too, it is seen from Table 13, that the yielding of the foundation is of importance, reducing the stresses to a marked degree at most points.

Corresponding values may be obtained of course for the equivalent shrinkage and swelling, and for the lateral elongations due to a vertical pressure, σ_x. For instance, a swelling of the up-stream face due to water-soaking produces a compression at the extrados and a tension at the intrados, but the maximum

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 236.

stresses at the abutment are for the given example only about one-half the stress found by neglecting the yielding of the foundation (see Table 13).

(*E*).—The radial deflection at the arch crown relative to the abutment due to water pressure alone will become:*

$$\Delta r_{0.1} = \int_0^{\phi_1} \frac{M' r_n^2 (\sin\phi - \sin\phi_1)\, d\phi}{E I'} - \int_0^{\phi_1} \frac{X'' r_n \sin\phi \cos\phi\, d\phi}{E t} + \frac{\kappa E}{G} \int_0^{\phi_1} \frac{X'' r_n \sin\phi \cos\phi\, d\phi}{E t} + \sigma \frac{R r_n}{E t} (1 - \cos\phi_1)$$

$$= \frac{R r_n}{E t} \left\{ \sigma (1 - \cos\phi_1) + \frac{X''}{R} e_2 \sin^2\phi_1 + \frac{X''}{R} \frac{3}{2 q^2} \left[\sin^2\phi_1 - 2 \frac{A''}{r_n} (\phi_1 \sin\phi_1 + \cos\phi_1 - 1) \right] \right\} \quad \ldots (122)$$

In addition to this there is the displacement of the abutment in the downstream direction

$$\Delta r_{0.2} = \frac{\xi}{E} [N_1 \sin(\phi_1 - \psi_1) + S_1 \cos(\phi_1 - \psi_1)] + \eta \frac{M_1}{E a} \cos(\phi_1 - \psi_1)$$

$$- \frac{R r_n}{E t} \left\{ 2 q \xi \sin\phi_1 + \eta \cos(\phi_1 - \psi_1) \left[\frac{q^2}{3} \cos\psi_1 + \frac{X''}{R} \left[\left(\frac{A''}{r_n} - \cos\phi_1 \right) \cos\psi_1 - \frac{q^2}{3} \cos(\phi_1 - \psi_1) \right] \right] \right\} \quad \ldots (123)$$

The total radial deflection for the arch crown is:

$$\Delta r_0 = \Delta r_{0.1} + \Delta r_{0.2} \quad \ldots (124)$$

With $t = 0.5\ r_m$, and $\phi_1 = 45°$;

For $\psi_1 = 0°$:

$$\Delta r_0 = \frac{R r_n}{E t} [1.1613 + 0.6164] = \frac{R r_n}{E t} \times 1.7777$$

For $\psi_1 = 20°$:

$$\Delta r_0 = \frac{R r_n}{E t} [1.1767 + 0.6234] = \frac{R r_n}{E t} \times 1.8001$$

It is apparent that this deflection does not, to any great extent, depend on the angle, ψ_1.

For unyielding foundations the expression may be put in the form:†

$$\Delta r_0 = \lambda_0 \sigma \frac{R r_n}{E t} \quad \ldots (125)$$

in which,

$$\lambda_0 = \frac{(1 - \cos\phi_1) [1.5 (\phi_1 - \sin\phi_1) + q^2 (c_1 \phi_1 + c_2 \sin\phi_1)]}{1.5 \left(\phi_1 + \sin\phi_1 \cos\phi_1 - 2 \frac{\sin^2\phi_1}{\phi_1} \right) + q^2 (c_1 \phi_1 - c_2 \sin\phi_1 \cos\phi_1)}$$

In Fig. 46, λ_0 is shown as a function of q and ϕ_1, calculated from $c_1 = 1.9$ and $c_2 = 0.9$. These coefficients correspond to $\frac{\kappa E}{G} = 2.8$, for which Mr.

* The calculation is carried out for the displacement of the foundation relative to the arch crown to get an expression, which holds also when the abutment is turned through the angle $\Delta\phi$.

† See, also, "The Circular Arch under Normal Loads," by William Cain, M. Am. Soc. C. E., *Transactions*. Am. Soc. C. E., Vol. LXXXV (1922), p. 233.

Jakobsen uses 2.88 (see Article III*); the difference in the final result is small.†

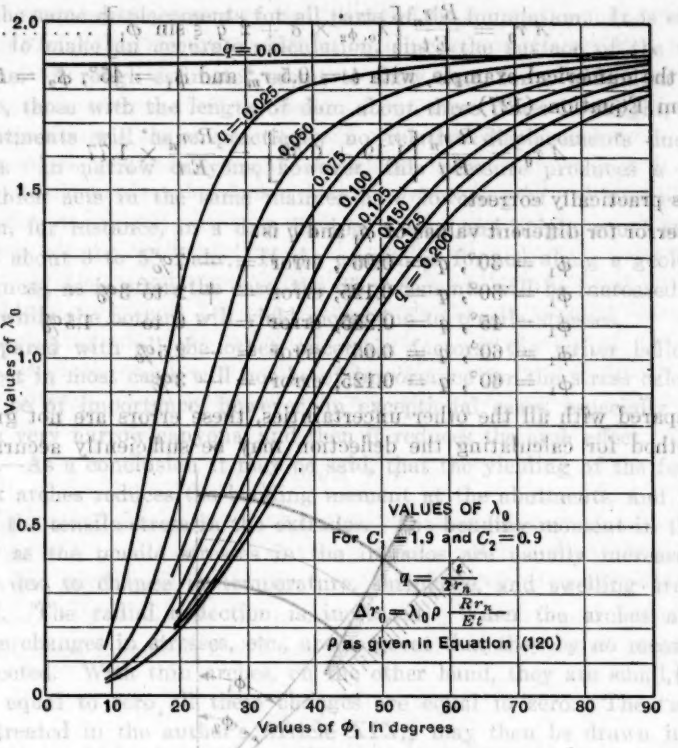

FIG. 46.

For unyielding foundations in the numerical example previously given:

$$\Delta r_0 = \frac{R\, r_m}{E\, t} \times 0.876 \text{ when } \frac{\kappa E}{G} = 2.88$$

or,

$$\Delta r_0 = \frac{R\, r_m}{E\, t} \times 0.871 \text{ when } \frac{\kappa E}{G} = 2.80$$

which is less than one-half the correct value.

Suppose the arch to be continued to an imaginary unyielding foundation at a depth, $h_1 = \frac{\mu}{12} t = 0.45\ t$, below the actual abutment (see Fig. 43 and Equation (110)), one-half the corresponding central angle, ϕ_2 (see Fig. 47), is:

$$\phi_2 = \phi_1 + \frac{\mu}{12} \frac{t}{r_m} = \phi_1 + 0.45 \frac{t}{r_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (126)$$

In such an arch the displacements due to the moment are approximately equal to the displacement in the actual arch; but the displacement due to the normal force and the shear in the foundation gives an additional dis-

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 212.

† The writer had these curves drawn before Mr. Jakobsen's paper had appeared, hence the reason for using the coefficients, 1.9 and 0.9.

placement of the crown section. The total deflection, Δr_o, of the crown may be found approximately from the expression:

$$\Delta r_0 = \frac{R r_m}{E t}\left[\lambda_{\phi_2} \times \sigma + 2 q \xi \sin \phi_1\right] \ldots\ldots\ldots\ldots (127)$$

For the numerical example, with $t = 0.5 r_m$ and $\phi_1 = 45°$, $\phi_2 = 57° 53.5'$, and from Equation (127):

$$\Delta r_0 = \frac{R r_m}{E t}\left[1.19 + 0.58\right] = \frac{R r_m}{E t} \times 1.77$$

which is practically correct.

The error for different values of ϕ_1 and q is:

$\phi_1 = 30°$, $q = 0.05$, error = — 7%
$\phi_1 = 30°$, $q = 0.125$, error = + 2 to 3%
$\phi_1 = 45°$, $q = 0.255$, error = 0 to — 1.5%
$\phi_1 = 60°$, $q = 0.05$, error = — 2.5%
$\phi_1 = 60°$, $q = 0.125$, error = — 3%

Compared with all the other uncertainties, these errors are not great, and this method for calculating the deflection may be sufficiently accurate.

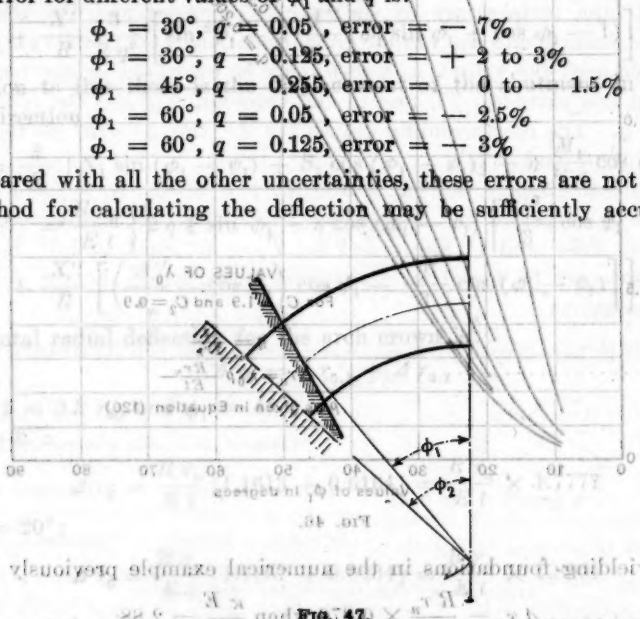

FIG. 47.

The calculation of Δr_0 is of vital importance for finding the division of load between the cantilevers and arches, and it is therefore desirable to get a simple approximate method like the one here given. For calculating the stresses, the writer would prefer the more accurate methods given in Article (*C*).

(*F*).—Since it has been made clear that even for a rock foundation the yielding must not be neglected in calculating the stresses or the deflections in dams, a study should be made as to what displacements the water pressure on the bottom and the sides of the valley produce at the abutments. Evidently it is not the absolute displacements of the foundation that are of interest for the stress calculations, but only the displacements of the different parts of the arch foundation relative to one another.*

*The following may serve as an illustration of the absolute displacements in large reservoirs: In a reservoir having an area of 1 by 2 miles and a depth of 150 ft. (that is, about 8 400 000 000 cu. ft.), the filling of the reservoir will give a mean lowering of the ground of about 3 in., due to the elastic deformations alone. This calculation assumes $E = 2.10^6$ lb. per sq. in., and follows from Equation (111). The measurements of such displacements may be used to determine the average value of E for solid rock.

From this it follows that only the pressure in the neighborhood of the dam counts, since a force acting at a great distance from the dam will give nearly the same displacements for all parts of the foundation. It is extremely difficult to make an accurate calculation, since the surface of the valley is not plane. A rough estimate* seems to show that, for a wide dam site, for instance, those with the length of dam about three times its height, or more, the abutments will have practically no relative displacements due to the pressure. In narrow canyons, however, this pressure produces a displacement which acts in the same manner as a lowering of the temperature of the dam, for instance, in a dam having a span and height equal to 150 ft., perhaps about 3 to 5° Fahr. If the canyon is formed along a geologic line of weakness, as is often the case, the displacements will be increased to some extent, while the bottom will yield more, due to tensile stresses.

Compared with all the other uncertain factors, the writer believes that this effect in most cases will not be of importance for the stress calculations. It may be of importance, however, in exceptional cases, especially for high dams in very narrow canyons, and then it reduces the arch effect.

(*G*).—As a conclusion it may be said, that the yielding of the foundation in thick arches reduces the bending moment at the abutments, and therefore reduces the tensile stress in the extrados. The bending moment in the crown as well as the tensile stresses in the intrados are usually increased. The stresses due to change in temperature, shrinkage, and swelling are usually reduced. The radial deflection is increased. When the arches are thick, all these changes in stresses, etc., are so great that they by no means should be neglected. With thin arches, on the other hand, they are small,† because with q equal to zero all these changes are equal to zero. The "secondary arch", treated in the author's Article XIX,‡ may then be drawn in accordance with the alterations in stresses.

The yielding of the foundation obviously makes the calculations more complicated, and it has been shown that small variations in the conditions, for instance in the angle, ψ_1, in short thick arches alter the stresses. Further, a number of corrections have been mentioned that are difficult to estimate accurately. In addition, there is the influence on the division of load between the cantilevers and arches, depending on the deflection. It should be realized therefore that there will always be considerable uncertainty in the calculations, even if there is complete and accurate knowledge, not only of the water pressure and the temperature, but also of the elastic properties of the rock and concrete, and the shrinkage and swelling of concrete. No calculation of this kind may claim to be more than a rough approximation.

GEORGE PAASWELL,§ M. AM. SOC. C. E. (by letter).‖—The writer is keenly interested in this analysis of the arch and fully appreciates the vast amount

* Based on Equations (4*a*), (4*b*), and (4*c*) in "Ueber die Berechnung der Fundamentdeformation," *Det Norske Videnskapsakademi*, Oslo, 1925, No. 2.

† For instance, for the test dam in Stevenson Creek, California, the yielding of the foundation will hardly be of great importance.

‡ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 241.

§ Asst. Engr., Board of Transportation, New York, N. Y.

‖ Received by the Secretary, May 8, 1926.

of work entailed in the presentation of a paper of this kind. He presents herewith some minor notes of discussion introducing a novel analysis of the arch, with the hope that his method will lead to a more thorough analysis of the actual problem and the correlation of the results with the data gathered by experiments.

The author has stated* that $\sigma_y = 0$, or that the Poisson ratio is zero. These are not synonymous statements. The Poisson ratio is not a stress equivalent. The fact that $\sigma_y = 0$, is either an assumption or a corollary of an assumed stress distribution. The Poisson effect is a molecular phenomenon and its only function in a deformation equation is to note the shift due to a distortion in a direction normal to its effect. It is well to clear up the confusion attendant on the introduction of this ratio in deformation equations.

As regards temperature changes in large masses,* the writer would like to call attention to some equations he has derived to show the variation in the range of temperature as the distance from the exposed face increases.† The periodic cycle of temperature changes may be given by an equation of the form:

$$u = A + B \cos \frac{2\pi}{T} t$$

in which, u is the temperature, A and B are constants determined from the range, T is the period of the season, and t is the time when u is measured. If x is the distance, in feet, of any point within the mass from the surface, the ratio of the range at this point to that at the surface is $e^{-kx} \cos kx$, in which, e is the base of natural logarithms and k is a coefficient such that,

$$k = \frac{1}{a}\sqrt{\frac{\pi}{T}}$$

The quantity, a^2, is the coefficient of thermal diffusivity, which, in the centimeter-gramme-second system for concrete is 0.0058. For a period of one year, with T measured in seconds, $k = 0.00413$; and for a period of one day, $k = 0.079$. From the experiments of Messrs. Paul and Mayhew quoted by the author, Table 11 has been compiled to show a comparison between the actual and the theoretical ranges.

TABLE 11.

Yearly Range.				Daily Range.			
x.	$e^{-kx} \cos kx$.	Theoretical.	Actual.	x.	$e^{-kx} \cos kx$.	Theoretical.	Actual.
0	1.00	75	75	0	1.00	50	50
1.0	0.87	65	..	0.25	0.45	22	..
2.0	0.76	57	..	0.50	0.11	5	..
3.5	0.57	43	32	1.00	0.07	3	2
5.0	0.42	31	..	1.5	0.02	1	..
10.0	0.09	7	12	2.0	0.01	1	1
20.0	0.04	3	0	2.5	0.00	0	..
....	....	..	..	3.0	....	..	..
....	....	..	..	3.5	....	..	0

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 207.

† "Retaining Walls: Their Design and Construction," p. 151.

It would appear that the formula, $e^{-kx} \cos kx$, is simple enough, and sufficiently accurate to warrant its use in the determination of temperature ranges.

To design an arch barrel by the usual formula is, of course, as approximate as to design a plate by the beam formula. The writer has attempted, therefore, to analyze the arch as a surface rather than as a ring and, in spite of the intricate mathematical analysis involved, hopes that at least qualitative results may be obtained. In a general study of flexure it is assumed as the basis of the common theory that the middle surface is unextended, that is, the neutral and the gravity surfaces coincide. The author has shown that this assumption is invalid for an arch of finite ratio of thickness to radius. However, the writer has predicated his analysis on the common theory. Take a typical arch barrel of average central radius, a, and thickness, $2h$, subjected to water pressure. For simplicity in computation the radius, a, is taken as unity and the height of the arch, L, as 2. The arch is assumed to be fixed at the bottom, and freely supported at the sides. Measure the co-ordinate, x, vertically downward from the crest of the dam and the angle, ϕ, in the horizontal plane from a central plane bisecting the arch sector. Let the total sector angle be $2\,\phi_0$. The flexure of the arch surface introduces a change in curvature vertically, and circumferentially, and also a twist of the normal to the arch about itself. Let k be the change in curvature in the x direction; K, in the ϕ direction; and t, the twist; and let w measure the radial deflection at any point. The changes in curvature may be represented (see any text, such as Love, for a demonstration) by:

$$k = \frac{d^2 w}{d x^2}$$

$$K = \frac{1}{a^2}\left(\frac{d^2 w}{d \phi^2} + w\right)$$

$$t = \frac{1}{a}\frac{d^2 w}{d x\, d \phi}$$

The energy, $d W$, expended by this change in curvature may be expressed in terms of these coefficients by:

$$d W = \frac{C}{2}\left[(k + K)^2 - 2 (1 - \sigma) (k K - t^2)\right]$$

in which, C is the flexural rigidity, and σ the Poisson ratio; so that

$$C = \frac{E I}{1 - \sigma^2}$$

If p is the intensity of water pressure at any point with the co-ordinates, x and ϕ, the theorem of virtual work states,

$$p w - d W = 0$$

The symbol, d, is here a coefficient of variation and may be applied to the usual differential coefficients previously given. Substituting the values of the coefficients, k, K, and t, and simplifying by applying some theorems in the calculus of variation, there results a general differential equation:

$$p = \frac{E I}{1 - \sigma^2}\left(\frac{d^4 w}{d x^4} + \frac{2}{a^2}\frac{d^4 w}{d x^2 d \phi^2} + \frac{1}{a^4}\frac{d^4 w}{d \phi^4} + \frac{2}{a^2}\frac{d^2 w}{d x^2} + \frac{2}{a^4}\frac{d^2 w}{d \phi^2} + \frac{w}{a^4}\right)$$

This may be termed a general flexural equation. If the radius is made infinite, the general equation of the plate becomes:

$$p = \frac{E\,I}{1-\sigma^2}\left(\frac{d^4 w}{d x^4} + 2\,\frac{d^4 w}{d x^2\,d y^2} + \frac{d^4 w}{d y^4}\right)$$

Again, if the plate is narrowed down to the beam,

$$p = E\,I\,\frac{d^4 w}{d x^4}$$

If the cylinder is taken of uniform thickness and of infinite length (the pipe problem, or tunnel lining), the equation is directly solvable and gives the stress system with due consideration for the continuity of the structure. If the length dimension is suppressed there results the usual equations for the arch ring. Unfortunately, there is no direct solution for the general equation given and a particular form of solution is assumed, satisfying the equation

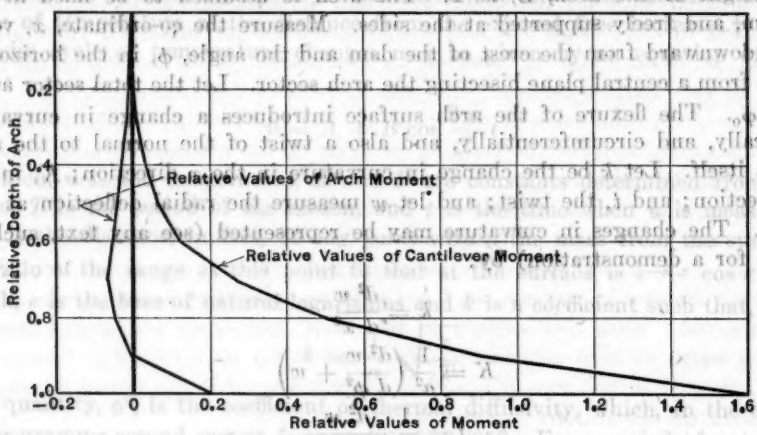

FIG. 48.

and the conditions at the edges and containing a constant which may be computed for specific values of the thickness and conditions of loading. Assume a solution in the form,

$$w = f\,(L - x)\,\sin\theta\,[\sinh\,(L - x) - j\,\cosh\,(L - x)]$$

in which, $\theta = \sqrt{2}\,(\phi_o - \phi)$ and j is a constant which has been evaluated for the condition that the cantilever moment must vanish at the crest. Let g represent the moment in the vertical direction, that is, the so-called cantilever moment, and let G represent the ordinary radial or arch moment:

$$g = C\,(k + \sigma\,K),$$
$$G = C\,(K + \sigma\,k)$$

The values of k and K are obtained from the expression for w and the moments read:

$$g = C f \sin\theta \left\{[2 + (1 - \sigma)\,j\,y]\,\cosh\,y + [y\,(1 - \sigma) - 2\,j]\,\sinh\,y\right\}$$

$$G = C f \sin\theta \left\{[y\,(1 + \sigma) - 2\,j]\,\sinh\,y + [2 - j\,y\,(1 + \sigma)]\,\cosh\,y\right\}$$

$$y = L - x.$$

These enable a qualitative study to be made of the moment variations. In Fig. 48 is shown the variation of the cantilever and of the arch moments taken vertically along the crown line of the arch. At any other vertical plane the moments are the crown moments multiplied by the sin θ. This study shows that the cantilever moment acquires a maximum value at the foot far exceeding the maximum arch moment. It is noticed that the arch moment changes sign toward the bottom.

It may be that these results are based on assumptions too approximate for the actual conditions. The writer therefore has contented himself with merely presenting the outline of moment values with the hope that data may be found to give sufficient color to the study to justify its further extension.

TREND OF CONSTRUCTION COST OF CERTAIN PUBLIC UTILITIES

Discussion*

By Messrs. Frank C. Wight and J. W. Hummer

Frank C. Wight,† M. Am. Soc. C. E. (by letter).‡—In the last paragraph of his interesting paper, Mr. Breuer says:

"That all these utilities, power plants, and distribution systems would trend so much alike, was not expected, but if similar research of others were readily available, the writer is very confident that their experience would be but a substantiation of these findings."

This last stated opinion the writer shares. Construction costs are made up of a number of items, each of which varies individually and in the proportion it bears to the total. Nevertheless, except for unusual types the total cost of construction seems to follow about the same curve over a period of years.

The writer is confirmed in this view by a study of the *Engineering News-Record* Construction Cost Index and its relation to the data Mr. Breuer presents. That index is purely an index; that is, it is not the cost of any specific project or even of typical projects or structures. It is a simple mathematical derivation of the composite costs of four of the items which go to make up construction cost, each weighted arbitrarily according to the part it is judged to take in the total cost of an engineering work. The value of the index, therefore, being based on judgment, must be proved by the accuracy with which it gauges actual cost and not by any individual opinion of the propriety of the relative weights of each item on the selection of the several items. The index has been worked out for the years 1914 to 1926 and is based on an assumption of normal costs in 1913. The index for any month of any year, therefore, indicates the cost of a construction project built at that time in its relation to the cost of the same project as built in 1913.

The index is based on the prices of steel, cement, lumber, and common labor in 1913, each item being weighted as follows: Steel, 37.50%; cement, 7.14%; lumber, 17.10%; and common labor, 38 per cent. The weightings were determined by studying the annual production of the three prime materials and the number of common laborers (excluding farm labor) in the United States. The

* This discussion (of the paper by William Breuer, Jun. Am. Soc. C. E., published in March, 1926, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Editor, *Engineering News-Record*, New York, N. Y.

‡ Received by the Secretary, March 30, 1926.

markets considered are structural steel in Pittsburgh, Pa.; cement without bags f. o. b. Chicago, Ill.; 3 by 12-in. to 12 by 12-in. Southern pine to contractors in New York, N. Y.; and the average price of common labor in twenty cities listed in *Engineering News-Record* in the first issue of each month. The prices are those obtaining in the last week of each month, published in the first issue of each month. The quantity of these materials and the number of man-hours used in computing the index were selected so that the total cost (quantities multiplied by prices and added together) for 1913 would be $100 for a "unit of construction".

On that basis, the Construction Cost Index for the years for which it has been computed is, as follows:

1913	100.00
1914	88.56
1915	92.58
1916	147.35
1917	181.24
1918	189.20
1919	198.42
1920	251.28
1921	201.78
1922	174.45
1923	214.07
1924	215.36
1925	206.68

A study of these figures will reveal that they form a curve remarkably similar to the curves of construction costs of actual properties presented in the paper. This will be evident from an inspection of the curves of Fig. 6, a series

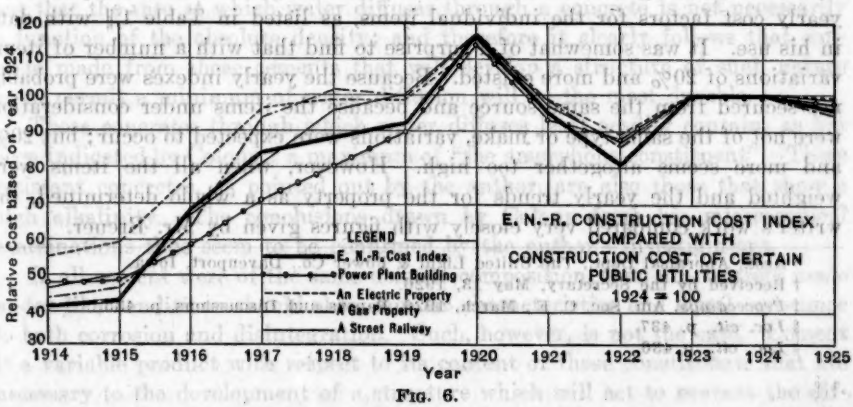

FIG. 6.

of graphs showing the trends of cost of (1) a power plant building; (2) an electric property; (3) a gas property; and (4) a street railway property, for the years 1914 to 1925, all plotted from Mr. Breuer's figures. Superposed on these graphs is the graph of *Engineering News-Record's* Construction Cost Index through the same years, with its base referred to the year 1924 instead

of to 1913, because 1924 has been used as the base of the relative figures presented by Mr. Breuer.

It is submitted that this similarity of trend amounts almost to identity, indicating not only the truth of Mr. Breuer's observation that there is a similar trend in all construction costs, but also that the fundamental characteristics of cost trends are fairly portrayed in the basic elements which go to make up the *Engineering News-Record* Construction Cost Index.

J. W. HUMMER,* JUN. AM. SOC. C. E. (by letter).†—A number of interesting comparisons are brought out in Mr. Breuer's paper. It seems rather a coincidence that the trends for an electric property, a gas property, and a street railway property should parallel from 1919 to 1925. One would expect them to be somewhat similar, but the maximum variation during the 7-year period, as shown in Fig. 1,‡ is 4 per cent. The writer made a similar study of the data at his command, and the results were approximately the same. The trends for the three types of utilities from 1919 to date, closely paralleled, the maximum variation being 10 per cent. Also, the data for the period from 1914 to 1919 showed a much closer correlation between the three types than is shown in Fig. 1. The gas and street railway trends were almost coincident for that 6-year interval and the electric property trend, with the exception of two years did not differ from the other two by more than 7 per cent.

The fact, as shown in Fig. 2,§ that the trends for a super-power station and a small power station extend so evenly over the 12-year period seems unusual. With the very decided change to large central stations, and the abandonment of small stations, one would expect that the economies, due to improved design, would cause the trend of the former to deviate more and more from that of the latter.

In determining the trend for the electric utility the writer compared the yearly cost factors for the individual items, as listed in Table 1,‖ with data in his use. It was somewhat of a surprise to find that with a number of items variations of 20% and more existed. Because the yearly indexes were probably not secured from the same source and because the items under consideration were not of the same type or make, variations were expected to occur; but 20% and more seems altogether too high. However, when all the items were weighted and the yearly trends for the property as a whole determined, the writer's work compared very closely with figures given by Mr. Breuer.

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* Asst., Appraisal Dept., The United Light & Power Co., Davenport, Iowa.

† Received by the Secretary, May 13, 1926.

‡ *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 435.

§ *Loc. cit.*, p. 437.

‖ *Loc. cit.*, p 436.

CORROSION OF CONCRETE

Discussion*

By Messrs. Thaddeus Merriman, John G. Ahlers, M. N. Clair, and R. H. Bogue

Thaddeus Merriman,† M. Am. Soc. C. E.—The author has done a great and real service to the science of concrete. No contribution of which the speaker is aware has so thoroughly developed the many phases involved.

Like all materials, concrete has its limitations. The author pointedly speaks of "corrosion". Limestone and steel corrode from the surface inward. When, however, "corrosion" is active not only on the surface but also within the mass, then it is that "disintegration" begins.

Perhaps the most potent influence which has operated to build up in the minds of people everywhere the idea that concrete is an imperishable material is the advertising slogan "Concrete for permanence". The author, however, has clearly shown that, in the absolute sense, no concrete is permanent and that the word, "concrete", is only a term which embraces everything from the best down to the poorest.

All other things being equal, the densest concrete will probably be the strongest and the most durable. The author, however, has forcibly pointed out that the rate at which water diffuses through a concrete is not necessarily a function of the absolute density; and therefore it clearly follows that concrete made from those cements that will develop a structure of such nature as to permit a minimum of water diffusion will be the most durable.

Those concretes through which water diffuses least readily contain, as has been indicated by Colony,‡ a maximum of "the amorphous constituent". These resistant concretes, as pointed out by the author, are also those that show a high alkalinity. The conclusions drawn by Colony from his microscopical examinations thus seem to be confirmed by the author's investigations.

If all cement were of the same definite composition, then all concrete made under like conditions should show similar characteristics and equal resistance to both corrosion and disintegration. Such, however, is not the case. Cement is a variable product with respect to its content of those constituents that are necessary to the development of a structure which will act to prevent the diffusion of water. One cement, which, by way of example, contains only 10%

* This discussion (of the paper by John R. Baylis, Assoc. M. Am. Soc. C. E., published in April, 1926, *Proceedings*, and presented at the meeting of May 5, 1926), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Chf. Engr., Board of Water Supply, City of New York, New York, N. Y.

‡ "Judging the Quality of Portland Cement," *Transactions*, Am. Inst. Min. and Metallurgical Engrs., 1921.

of the compounds that go to develop the amorphous constituent, cannot possibly make a concrete which will be as resistant as that made from a cement containing 25% of these compounds.

The speaker in a recent paper* has indicated some of the great differences among present-day cements. These differences are not disclosed by the standard specification under which all cements meeting the minimum requirements are labeled as equals. The science of cement has lagged far behind in the general march of progress and the almost universal demand for better concrete will not come to be satisfied until better cement is available for its making.

The consumer must learn to specify his cement just as he now specifies his steel, his coal, his wheat, and his oranges. A single standard for any commodity can do no less than promote mediocrity of quality.

JOHN G. AHLERS,† Esq.—From the contractor's viewpoint, the speaker disagrees with the author's opinion, which seems to indicate that much of the concrete being produced is bad. It is poor policy for the Engineering and Contracting Professions to illustrate disintegrating concrete and not to show a proportionate amount of good work.

After sixteen years of practical building experience, the speaker offers to go back over the structures constructed either directly by him or under his supervision and challenges critics to find anything like the conditions described. The reason for this is neither because a better quality of cement was used nor a better kind of aggregate, but merely because the concrete was made with reasonable care under good standard conditions; because standard brands of Portland cement were used and, what now becomes increasingly clear and more important every day, because not too much water was used in mixing the concrete. The cause of every case of bad concrete mentioned in the discussions, probably may be traced back, not to inferior cement, but to improper mixing, placing, or exposure to freezing during the setting period, with a poor grade of aggregate. It is entirely too much to expect the cement to overcome such obstacles and, in spite of them, to make durable concrete.

When the water-cement-ratio is kept constant for any definitely desired high strength, uniform, durable concrete will result. To illustrate the simplicity of principles involved, Fig. 14 shows how the concrete mixed by the speaker's Company is at present controlled.

Every job has its own "job constant", fixed by the water-cement-ratio of 1, from which increases or decreases in strength can be obtained by automatically moving the water-measuring tank; the strengths will follow the curve, known as a "job curve". Only the job constant is affected by the quality of the aggregate and the cement used.

Engineers, therefore, do not need different standards of testing cement or different gradings of cement. If any one believes that one brand is better than another, he can buy the grade that he believes is better and, by combining it with the aggregate available, establish his own job constant. Innumerable tests made by the Lewis Institute and many others show that the quantity and grading of the aggregate have no effect on the strength as long as the

* *Engineering News-Record*, April 22, 1926.
† Barney-Ahlers Constr. Co., New York, N. Y.

concrete is workable and plastic; also, the strength, and therefore the lasting quality of the concrete, is merely controlled by the water-cement-ratio used in producing the concrete.

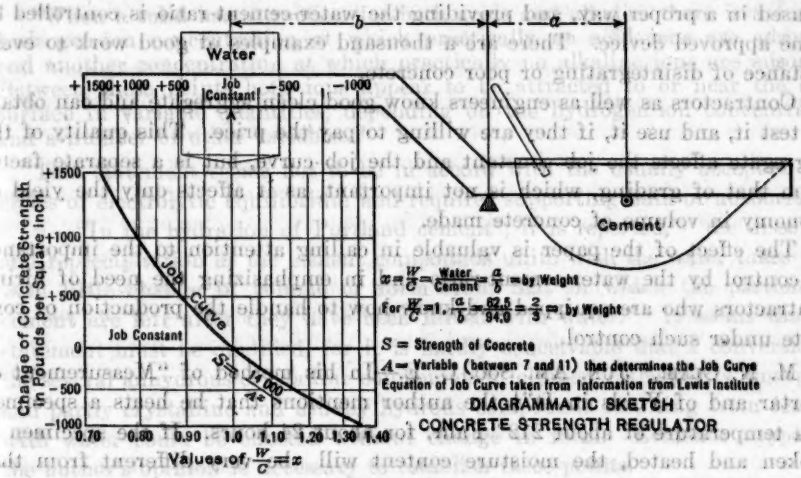
b
a
Water
+1500 +1000 +500 Job Constant −500 −1000
Cement
+1500
+1000
+500
0
−500
−1000
Change of Concrete Strength in Pounds per Square Inch
Job Curve
Job Constant
$S = \frac{14\,000}{Ax}$
0.70 0.80 0.90 1.0 1.10 1.20 1.30 1.40
Values of $\frac{W}{C} = x$
$x = \frac{W}{C} = \frac{\text{Water}}{\text{Cement}} = \frac{a}{b}$ = by Weight
For $\frac{W}{C} = 1$, $\frac{a}{b} = \frac{82.5}{94.0} = \frac{2}{3}$ = by Weight
S = Strength of Concrete
A = Variable (between 7 and 11) that determines the Job Curve
Equation of Job Curve taken from Information from Lewis Institute
DIAGRAMMATIC SKETCH
CONCRETE STRENGTH REGULATOR

FIG. 14.

Judging the paper from the angle of field experience since 1924 in controlling concrete by the water-ratio theory, the speaker feels that the chemical value of the author's work may be good and can neither be criticized nor discussed. This phase presents a viewpoint that is interesting to a contractor and may explain surface disintegration or weathering. Such weathering, however, is slight in properly made standard concrete. Therefore, the paper is perhaps more applicable to work showing lack of control during the process of making the concrete. Where good contractors do the work and proper supervision of the water-cement-ratio takes place, there need be no danger of disintegration. In fact, the author really affirms the importance and necessity of water-ratio control of concrete. There would seem to be no need of such a special test for void as he asks for if water-cement-ratio is controlled.

The speaker does not believe in specifying proportions of sand to gravel or in changing the size of the maximum grains in the sand. Both the water-ratio theory and the paper itself indicate that the size and grading of the sand grains have no effect on the strength or the density of plastic concrete. Size and grading affect only the economy of the mixture. If a change is made, it should be as the author recommends, to have sand of ¼ in. and less, and two other sizes to combine with this; this is almost the same as the speaker's practice—to buy commercial "ready-mix" and check the quantity of fine sand contained. The remainder of the aggregate, as Mr. Baylis points out, is of no material consequence except as a filler for the resulting concrete.

It would be better, if "ready-mix" is not used, to have three sizes of aggregate—from ¼ in. down, from ½ in. to ¼ in., and from 1½ in. up—with which a very economical concrete can be made. It is a question whether there is any advantage in cost to the contractor by using three sizes, having in mind

that the size and grading of the aggregate have no effect on the resulting strength.

Thus, there need be no fear of poor concrete providing a standard cement is used in a proper way, and providing the water-cement-ratio is controlled by some approved device. There are a thousand examples of good work to every instance of disintegrating or poor concrete.

Contractors as well as engineers know good clean aggregate and can obtain it, test it, and use it, if they are willing to pay the price. This quality of the aggregate affects the job constant and the job curve, but is a separate factor from that of grading, which is not important, as it affects only the yield or economy in volume of concrete made.

The effect of the paper is valuable in calling attention to the importance of control by the water-cement-ratio and in emphasizing the need of hiring contractors who are equipped and know how to handle the production of concrete under such control.

M. N. CLAIR,* JUN. AM. SOC. C. E.—In his method of "Measurement of Mortar and of Voids in It"† the author mentions that he heats a specimen to a temperature of about 212° Fahr. for about 24 hours. If the specimen is broken and heated, the moisture content will be very different from that obtained by the unbroken specimen in a drying oven.

In analyzing a number of samples of concrete, the specimens were dried in the manner described by Mr. Baylis. The specimens were 4-in. and 6-in. cylinders of various heights, and consistent results could only be obtained by first breaking them and then drying them.

R. H. BOGUE,‡ ESQ. (by letter).§—This paper is of interest because the author reports data on chemical experiments and interprets these data in terms of generalizations on the nature and behavior of Portland cement.

In the case of all scientific papers, and more especially those in which generalizations are formulated which may affect profoundly the treatment of a most important structural material, it seems imperative that the author exercise the utmost caution in his statements and make no assertion whatsoever for which he does not give either his authority or his experimental evidence. Even a conjecture on matters of such importance is impermissible unless the bases for an opinion are expressly stated. This is the more compulsory when such statements or conjectures appear to be at variance with established chemical or physical law.

The author has presented almost no references to published investigations, and confirming data on many statements are lacking. In many cases statements are made which leave the impression that they represent established truths while as a matter of fact they are the author's interpretations on observed phenomena. These interpretations may or may not be justified, but in every case it should be made clear that they represent only the author's views.

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* Testing and Designing Engr., The Thompson & Lichtner Co., Inc., Boston, Mass.

† *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 563.

‡ Research Director, Portland Cement Association Fellowship, U. S. Bureau of Standards, Washington, D. C.

§ Received by the Secretary, June 18, 1926.

Herewith are given a few typical instances that will serve to illustrate these points.

1.—It is stated:*

"When a solid has attraction for both acids and alkalies there is a certain hydrogen-ion concentration at which practically no acid ions are adsorbed, and another concentration at which practically no alkaline ions are adsorbed. Between these points both ions appear to be attracted to or near the solid surface in variable quantities, depending on the hydrogen-ion concentration and a number of other variables."

This statement seems not to be in accord with the usually accepted principles of electrolytic equilibrium and requires supporting data or authority.

2.—"In the hydration of Portland cement", it is reported,* "the dissolving and reprecipitation as the cement compounds unite with the water takes place with little change in volume or form from that in which the particles of cement are left after they have been mixed with water." It seems that the statement must be qualified, for it is hardly conceivable that a conversion of the several anhydrous crystalline compounds of clinker to the partly amorphous and partly crystalline and usually hydrous compounds resulting from reaction with water could be effected without change in form. Evidence supporting the author's opinion is necessary to establish these points.

3.—The statement is made that "stagnated layers of water or air close the pores against rapid communication with the surrounding water or air".† Since this statement constitutes the basis for the development of a method for testing concrete structures, it should be fully supported by authority or experimental data.

4.—The setting of Portland cement is referred to in various places in the paper as concerned with or brought about by a change in the oxides to the hydroxides and the presence in the product of both aluminum hydroxide and hydrated silica. No authority or data are offered for these statements, and since they are not in accord with recent published investigations, they require supporting evidence.

5.—It has not been explained what is meant by the "hydrogen-ion concentration of a solid mass". Such a conception is not recognized in physico-chemical science.

6.—The author states that diffusion is greatly retarded by the presence of gelatinous materials, and quotes an experiment in which 30 days were required for diffusion to proceed 2.5 in. Similar slow diffusion may be demonstrated in pure solutions with no gelatinous material present. For example, copper sulfate solution placed in a jar below water may take years to diffuse through a distance of a few inches to form a homogeneous solution.‡

In addition to these statements of the author, which lack supporting evidence, there are a few general points that require detailed attention.

Surface Adsorption.—A few experiments are described on the leaching out of the "calcium" from various cements. From the description presented it is

* *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 557.
† *Loc. cit.*, p. 558.
‡ Smith's "College Chemistry."

inferred that the solvent was distilled water and that after each interval of several days, when a part of the solution was removed for test, a similar quantity of water was added. The data are plotted with "alkalinity" in parts per million expressed as calcium carbonate as the ordinate against "percentage of total calcium dissolved" as the abscissa (Fig. 2*). The curves obtained show a more or less regular drop, indicating that as the total "calcium" dissolved becomes greater, the "alkalinity" of each fraction tested decreases. The author regards the smooth nature of the curves (the points on Curve *B* show a distinct break that is not explained) and the drop in "alkalinity" as evidence that "either there are a number of compounds formed in concrete, or that stable compounds of high calcium content do not exist".† He adds,

"The gradual change in equilibrium, as shown by the curves in Fig. 2, points strongly to the fact that if there are definite stable compounds of calcium and silica in concrete, they are probably compounds of lower calcium content than tri-calcium silicate, and that any additional calcium above these stable compounds may be merely in the nature of a surface adsorption, or, as has already been suggested, a solid solution."

These conclusions leave it optional whether there are a number of compounds formed in concrete, or surface adsorption complexes, or solid solutions. The author, however, makes it apparent in the following section on "Surface Adsorption"* that he purposes to champion the surface adsorption idea. The main point seems to have been missed entirely. The curves presented are typical curves for hydrolysis. Any salt which contains either a weak acid radical, or a weak basic radical, or both, will undergo in water a decomposition into its acidic and basic constituents.‡

The case in point involves the tri-calcium silicate, di-calcium silicate, and other aluminates, ferrites, etc. To deny the existence of such compounds in clinker is to lay aside the well-known work of the Geophysical Laboratory on the system, $CaO - Al_2O_3 - SiO_2$. Of course, errors may have been made in the attempt to carry over this work on pure components into the much more complicated polycomponent cement, but at least any denial of such work must be accompanied by convincing evidence. By the principle of hydrolysis, therefore, these silicates, aluminates, and ferrites of lime would be decomposed in water into calcium hydroxide and either the oxides of silica, alumina, and iron, or a silicate, aluminate, and ferrite of lower lime content. If the water were continuously or intermittently removed, the end products would probably be hydrated silica, alumina, or ferric oxide. If the water were not removed, the hydrolysis would be expected to continue until an equilibrium condition were reached at which the rate of recombination of acid and base just equalled the rate of hydrolysis. This condition, however, is determined by the dilution and the nature of the remaining solid material. This latter has changed continuously by becoming less basic, so that the equilibrium concentration of hydroxyl-ion to just counterbalance hydrolysis of the remain-

* *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 553.
† *Loc. cit.*, p. 552.
‡ Publications of A. A. Noyes, or any modern textbook on Physical Chemistry.

ing material becomes less at each test. That is, the "alkalinity" should become less at each test, which is the condition found by the author. This is the state of the reaction at equilibrium. If equilibrium is not attained, the departure from a constant hydroxyl-ion concentration would be even more pronounced.

Thus, in the data presented by the author, there is nothing that is not altogether comprehensible and completely explainable on the basis of the recognized laws and theories of chemistry. The injection of a theory of surface adsorption seems unnecessary and unwarranted.

Significance of the Void Test.—In the paper much significance is attached to the void test. It is imperative, however, that an adequate method for the measurement of voids be at hand before conclusions may legitimately be made on the significance of the voids. The method suggested consists in taking a special 24-hour old sample, drying at 210 to 250° Fahr., cooling, weighing, submerging in water for periods of 5 min., 2 hours, and 4 hours, and reweighing. Omitting details, the percentage of voids is then calculated as the ratio of the water adsorbed to the volume of the sample.

It must be realized that by drying to constant weight at temperatures up to 250° Fahr., a considerable quantity of water will undoubtedly be lost by dehydration and desorption, and that the water so lost will probably again be taken up on submergence. Furthermore, the wetter the original mix, the greater, presumably, will be the surface exposure, and consequently the greater will be the quantity of water taken up in a limited period of time by these processes, which is the condition observed by the author.

Thus, the void test as described appears to be a measure of a number of factors including the water of wetting, hydration, and hydrolysis, as well as that which actually enters the voids. Methods previously described* avoid these difficulties to a large extent, and Talbot has discussed at length the usefulness of the test, but none of these is referred to in the paper.

Calcium Solubility Index.—The author suggests a method† for determining whether any given concrete structure is continuing in a satisfactory state, or whether it is weakening and becoming unsafe or subject to rapid disintegration. The method consists of a determination of the "alkalinity" of a sample. In general, this becomes a calculation of the uncarbonated lime in the sample. High "alkalinity" means that the lime has not become carbonated. The author states that the "alkalinity" ¼ in. from the surface should not be less than 200; and ¼ to ½ in. from the surface, not less than 1 200. The reason given for this is that a high value indicates a concrete of such imperviousness that carbon dioxide cannot enter, while a low value indicates a porous structure.

This suggestion is interesting and should be properly tested. The experiments quoted are inadequate for basing any conclusions, but they seem to be sufficiently suggestive to warrant a carefully planned and comprehensive series. Such a series of investigations, however, should consider the testing

* By A. N. Talbot, Past-President, Am. Soc. C. E., and, also, F. E. Richart, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. for Testing Materials, Vol. 21 (1921), pp. 940 and 964.

† *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 573.

not only of structures at the present time, but structures subjected to exposure over a period of years, in order to observe any changes that may be taking place.

Conclusions.—The foregoing remarks are intended to bring out the necessity of presenting searching confirmatory data whenever new theories or ideas are expressed. Progress in science is best served by the announcement of such theories only after sufficient evidence has been gathered to justify the formulation of clearly defined conclusions. By following such a procedure, there will result a better understanding of Portland cement and a warranted confidence in the soundness of the theories presented. From this understanding and confidence will follow a more intelligent application of its use by engineers.

THE FINANCING OF IRRIGATION DEVELOPMENTS BY PRIVATE CAPITAL

Discussion*

By Messrs. Elwood Mead and F. H. Newell.

Elwood Mead,† M. Am. Soc. C. E. (by letter).‡—This is an interesting discussion of a feature of reclamation about which there is marked difference of opinion. The author lays down nine conditions§ which must be met to enable an irrigation project to be financed by private capital. Condition No. 8 requires the entire cost of the work, with a reasonable profit, to be paid and still have conditions compare favorably with the cost of a farm in humid sections. Condition No. 9 says the project must be so financed that it will attract the man of limited means to locate thereon.

Both these conditions must be largely decided by the judgment or bias of those in charge of the enterprise. No mathematical rules can be laid down. No doubt the promoters of nine out of ten of the private projects that have failed believed they would succeed; but their judgment was faulty. The paper is not clear as to how such mistakes are to be averted in the future.

What this irrigation planning lacks in the United States is attention to the costs which must be incurred in changing the raw land under an irrigation project into improved productive farms. Under the cheaper projects of twenty-five years ago this was not necessary, although it would have been valuable. When, however, construction costs of canals are more than $75 per acre, as practically all canals of any size are at present, and an equal or greater sum has to be spent leveling the land and putting up the necessary buildings and fences, the total investment runs so high that the settler of small means fails before he gets started. The future success of private and Government irrigation depends on the measures adopted to overcome this obstacle.

The plan proposed by Mr. Shepherd is a corporation, created under an Act of Congress, part of the members of the board of directors being appointed by the President, with the Government subscribing for part of the capital stock, this corporation to be connected with the Department of the Interior or the Department of Agriculture. This is not private enterprise because in the end the Government would be the sole moneyed member.

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* Discussion of the paper by R. E. Shepherd, Esq., continued from April, 1926, *Proceedings*.

† Commr. of Reclamation, Dept. of the Interior, Washington, D. C.

‡ Received by the Secretary, April 16, 1926.

§ *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 405.

It is not clear how such a complex organization would have any advantage over the present Reclamation Bureau; it would have a much greater overhead expense, the risk of loss would be equally great, and the responsibility of management less because the private directors would be operating largely with public funds. The main advantage urged for such an organization is that it would be an independent board, free from political control. Speaking from some considerable experience with both public and private development, the writer is doubtful whether this freedom would be secured. The Tariff Commission has not taken the tariff out of politics.

The only way that reclamation can be taken out of politics is to carry it on without Government money and that is precisely what cannot be done under present conditions. There are now pending in Congress appeals from about twenty private enterprises (that is, enterprises started with private capital and as private undertakings) that are not completed, are financially embarrassed, and are appealing to the Government to be a financial life-saver. There are many more that are not paying interest on bonds. The reasons are not the mistakes in design or construction, but solely that the profits of agriculture have not been sufficient to enable farmers to make a living and pay irrigation charges, or the opportunities of the projects have not been sufficiently attractive to enable the land to be settled and brought under cultivation. It is the farmer who determines whether or not an irrigation enterprise is a financial or economic success.

This paper, therefore, does not touch on the fundamental weakness of all past development, public or private, namely, the failure to include in the plans for development a study of the economic questions connected with land settlement and the improvement and equipment of farms. Until there is a definite land settlement policy, and unless the question of how raw land is to be changed into farms is thought out as clearly as the size of canals and the design of engineering structures, there will be delayed development and uncertainty as to the financial outcome.

About one-half the Government reclamation projects are meeting their payments. No equal number of private enterprises in the United States make a better showing. They have no financial problems, but they do have an economic problem in the increase in tenancy and non-resident ownership of land. The other half of the reclamation projects are in arrears to the Government in their payments, but this is not due to bad design of works or, in many cases, to poor land. On one-half the projects that are in financial difficulties, the whole trouble is lack of settlement—there is good land, there is plenty of water, but this land fell in the first place into the hands of people unable to develop it, or who acquired it for speculative purposes and have held it for that purpose ever since.

The adjustment now under way has as its fundamental principle the closer settlement of land, the selection of settlers who will be first of all good farmers, and then the establishment of a type of agriculture suited to the markets and the local climatic conditions. These are as important for private enterprise as for Government projects, but it looks as if the Government were destined to be a leader in the introduction of the new principles and new

methods needed to bring reclamation into accord with the economic conditions created by the World War.

F. H. NEWELL,* M. AM. SOC. C. E. (by letter).†—The author goes to the foundation of the vexing problems encountered by engineers and economists in the financing of irrigation development by private capital. Collateral with this he gives an answer to a question of National concern, as to the relation the Federal Government should have to irrigation enterprises. He suggests that Congress create a corporation, with continuity of policy, to take over all the duties and responsibilities of irrigation now imposed on the Secretary of the Interior and to conduct future operations through a board of directors, with full authority to carry on the business with reference to the larger interests of the public as well as of the people concerned. He also suggests‡ that:

"Such corporation should have the power to issue its own bonds which should enjoy the same privileges and relation to the Federal Government as the bonds of Federal Land Banks, and should take over, complete, and liquidate the existing Government projects."

To any one who has been intimately concerned with the initiation, construction, and operation of the Federal enterprises, this suggestion has a peculiar appeal. It is in accord with the experience of the past not only in irrigation enterprises dependent on private and public funds but is in line with other transactions involving the efficient investment and recovery of large sums of money. The need for a corporation separate from purely governmental functions or political control to handle Federal reclamation funds with efficiency and economy arises from the fact that departmental machinery with its many restraints growing up from ancient checks and balances has been evolved with reference to enforcing laws, mainly restrictive. It does not readily permit nor stimulate the exercise of sound judgment to fit present-day exigencies. It reflects past rather than future needs.

This is exemplified by the striking losses in amounts due the Reclamation Fund from the land owners (not necessarily farmers) whose lands have been increased in value from practically nothing to sales prices of more than $100 per acre. The inefficiency and losses in making collections from these land owners are in striking contrast to the activities of the Farm Loan Board. This is due to the fact that this Board, virtually a Government-owned or controlled corporation, has been able to pursue a persistent policy, one which recognized that collections must be made and has not tried to exempt groups of land owners but has proceeded with its work, not as a side issue or incident to some other more important operations, but as a business by itself, one worthy of exclusive attention.

It is now practically admitted that if, in place of the transient personality of the Secretary of the Interior, a permanent board or small continuing group of experts (chosen for special ability and experience) had determined on the policy of requiring prompt payment from land owners of the funds

* Pres., The Research Service, Inc., Washington, D. C.
† Received by the Secretary, May 13, 1926.
‡ *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 409.

due under the terms of the Reclamation Act, speculation in reclaimed lands would have been checked and the present unfortunate situation would never have developed. One of the best informed experts who has devoted years to the study of the situation on the ground asserts that the present situation would now be largely overcome if a business-like fiscal method of collection had been adopted and the persistent idea of repudiation of debts had been discouraged.

The greater part of irrigation development, with its benefits to individuals, communities, States, and less directly to the nation, as noted by Mr. Shepherd, has been made by private investment. Moreover, this condition is continuing; more land is being brought under cultivation by the use of private or corporate funds than can be developed by any conceivable appropriations of public funds. There is no doubt that as a whole the public funds were wisely and effectively expended in the earlier enterprises, and the public has gained thereby. The real reason public funds should not be more largely appropriated lies in the conditions recited by Mr. Shepherd; but these obstacles to such further wise investment of public funds may be removed by a National reclamation corporation.

To appreciate the situation fully it is necessary to emphasize the peculiar condition which has resulted with reference to Federal investments in irrigation enterprises. This work, involving the expenditure of many millions of dollars, was entrusted by Congress to a single official. No provision was made for any organization, bureau, or group of advisers. Upon one man alone, a Cabinet Officer, usually a lawyer chosen for political acumen, was imposed this responsibility, to fill a position to which he could devote only such fragments of time as might be taken from innumerable other more obvious or pressing demands.

It is true that, by inference, the Secretary of the Interior, the official named in the law, could employ whom he pleased, when he pleased, at any compensation, and without hindrance from civil service or any other restriction, except the will of the President, or such laws as might later be enacted by Congress. Such unlimited opportunity and discretion has hardly ever been given to a single official. He held in his hand the future of large communities.

At the same time this single official was charged with a multitude of other and most exacting duties in connection with public lands, National parks, Indian affairs, patents, pensions, education, geology, hospitals, insane asylums, a list of which would require a page even to enumerate the titles and a volume to describe the duties. It is true that for most of these certain responsible assistants were provided, but in infinite detail he is practically the court of last resort and must give some personal attention to each and every item.

The condition would not be so bad if this official and his principal assistants were educated or trained for the work or remained in office long enough to understand what it was all about. For two or more decades Secretaries of the Interior have held office on an average only a little more than two years. With few exceptions they have not been selected with reference to the work in hand; traditionally, they must come from the West; above all things

they must be effective party men. An enumeration of the names of those who have had personal charge of the Reclamation Fund—Hitchcock, Garfield, Ballinger, Fisher, Lane, Payne, Fall, etc.—is sufficient to indicate the wide variation in temperament and experience. With notable exceptions, these men were not skilled in economics or finance.

All these men, although differing widely, have agreed in one respect, namely, that immediately upon assumption of office each has taken a lively interest in irrigation, and has promptly reversed the policies of his predecessor so far, at least, as these pertain to finances. Each has entrusted the technical or engineering problems to the Reclamation Service, created by the Hon. James R. Garfield, but each has reserved to himself everything which had to do with finances and particularly the collection of the money, for the reason that, as administered by them, this involved many intricate political considerations.

With continually shifting personnel and policies regarding finance, it is indeed a wonder that so much money has been collected from the land owners to whom water was sold on installment. For analogy, imagine what would happen to the income tax if the Secretary of the Treasury announced that he did not believe that any considerable number of the taxpayers would pay and that, as far as he was concerned, he would not oppress any man!

A volume might be written upon the psychology or politics involved in this experiment in finance; it is sufficient to point out that Mr. Shepherd sees the difficulty and offers the solution, namely, that of taking the responsibility out of the hands of temporarily appointed politicians. He would entrust the enforcement of the policies, determined in a broad way by Congress, to a corporation in such manner as to permit the exercise of a sound discretion and a continuity of policy. This can be made possible by entrusting the powers to a small number of directors, preferably appointed for long terms, which overlap in such a way as to insure this necessary continuity.

Another point brought out by Mr. Shepherd is that success in all these enterprises, whether financed by private or public funds, must be a matter of evolution, that is, of slow growth based on experience, year after year. The greatest losses of public funds have been due to the anxiety of each successive Secretary of the Interior to push the maximum of construction during his term of office.

The original idea embodied in the Roosevelt Reclamation Act of June 17, 1902, was to provide a small revolving fund, utilize this in building large works, reservoirs, and main-line canals, the magnitude of which were such that they could not then be built by private capital; then to permit a natural evolution, re-investing the money, as it slowly came in, on the basis of the experience already acquired.

Preliminary to this investment authorized in 1902 the most careful studies possible were made (beginning in 1888) of the water supply, the topographical conditions, and the related physical facts; the best experts available were obtained to consider soils and economic conditions. The unknown factors which could not be determined in advance were to be ascertained by the steady development of the projects for which these main structures were built.

Unfortunately, this gradual progress was not acceptable to the short-term politicians. The otherwise orderly sequence of events was interrupted; a loan of $20 000 000 was urged to expedite work. It is significant to note that none of this money was expended for five years after the loan was authorized. This was because of the fact that the then conditions of securing rights of way, of establishing water rights, or economical construction, did not justify this expenditure. The same motives which dictated the securing of this loan were closely akin to those of indefinite extension of time of repayment and practical repudiation. Had the rule been rigidly adopted of limiting expenditures to the amounts which might be obtained from past investments much, if not most, of the criticism now rampant might have been avoided.

This relatively reckless attitude contributed to the inflation of land prices and to the spirit of speculation in governmentally reclaimed lands, so extensively commented upon. An analysis shows that the excessive speculation in lands, irrigated by the use of non-interest bearing public funds, lay in the fact that these speculative values resulted not so much because of the possible productive values of the land, but were based on the assumption that the owners of these lands would continue "to get something for nothing" from the Government. That is, they would get not merely exemption from interest payment on the cost of reclamation for a period of 10, 20, or more, years, but it was asserted that ultimately a considerable part of the debt, if not all of it, might be repudiated.

This avowed policy of repudiation was a plank in the platform of many a politician running for office; under this stimulus excessive land prices were agreed upon and in some cases paid; namely, on the claim that the Government would never dispossess any land owner. It was asserted that liberal Uncle Sam, who could afford to build and throw on the scrap heap a $10 000 000 battleship, or put millions of dollars in diking swamp lands, or building hard roads, would never stoop to collect the little driblets from a few thousand owners of irrigated lands.

With this lesson fresh in mind and with the advice of experienced men like Mr. Shepherd it would seem as if the people of the country and Congress would see to it that these intricate financial matters were entrusted to a small body of men especially chosen and that they would bring into line with practice the methods which experience everywhere is showing to be necessary in the investment and recovery of large sums of money.

PRESENT POLICY OF THE UNITED STATES BUREAU OF RECLAMATION REGARDING LAND SETTLEMENT

Discussion*

By Messrs. George C. Kreutzer and O. L. Waller.

George C. Kreutzer,† Esq. (by letter).‡—The author points out that the cost of construction of Federal irrigation projects has been increasing and now varies from $100 to $160 per acre for new projects approved by Congress. With this increased cost of construction a corresponding increase has taken place in the cost of developing and equipping irrigated farms. A 40 or 80-acre farm with suitable buildings and fences, including leveling and farm ditches, live stock, and farming equipment, will cost the farmer, exclusive of the land, from $5 000 to $10 000.

Experience has shown that settlers with such amounts of capital are scarce and when found are inclined to buy improved farms in established communities. The majority of settlers who take raw land on new irrigation enterprises have capital varying from $1 000 to $2 500.

The cost of water and farm development, high taxes, and labor costs have precluded the growing of low priced crops at a profit and have made large farms with few exceptions unprofitable. The small irrigation farmer, largely doing his own work, cultivating intensively and being free from burdensome debts bearing high rates of interest, has been able to pay fixed charges, including those for water, and to get ahead. His crop values per acre are relatively high, his labor bill small, and he has no idle land. He grows a large part of the food consumed by his family. These advantages over the larger operator gives him security and freedom from the other's difficulties. When the gross income from a 40 or 60-acre farm reaches $2 000 to $3 000 per year its owner is usually prosperous.

The selection of settlers is a step in the right direction. It is not expected that these settlers know how to do every farm task efficiently and well but they should have had experience enough to realize that to operate a farm calls for hard work and to know that they enjoy this means of making a living. In addition, they should have character and industry and a reasonable amount of capital. These qualifications when possessed by a settler indicate that he is adaptable for rural life and has a good chance of succeeding. Farming demands frugal expenditures and the practice of economies seldom experienced by the city dweller. The settler who has had previous experience knows these

* Discussion of the paper by Elwood Mead, M. Am. Soc. C. E., continued from April, 1926, *Proceedings*.

† Director of Reclamation Economics, U. S. Dept. of the Interior, Washington, D. C.

‡ Received by the Secretary, May 1, 1926.

principles and practices them from the beginning. Selecting settlers in accordance with desirable characteristics will not preclude failures but will greatly reduce them.

The only kind of irrigated farms that pay are those that are completely developed and well equipped. If settlers have not the capital nor the credit to do this it results in delayed development and the piling up of debts. The loans required to complete development must be repayable over a long period and with a low rate of interest. The returns from farming demand this.

The weakness of Federal reclamation and also of private irrigation enterprises is that the construction of the irrigation system is accomplished by the use of borrowed money and as such is a loan, yet the only security given is raw land of low value having no immediate earning power. The land itself usually is not worth more than a few dollars per acre. Earning power must be created. This can be done only by putting labor and capital into the raw land to convert it into profitable farms. The enterprise is not complete until then. When it is not completed there is a default in the construction payments. This is like building a factory and omitting the machinery. It leaves too much to work itself out. Certainly it has neither of the two conditions demanded by any other financial agencies in making a loan—security for protection and income to pay the debt. In irrigation developments these two conditions must be created. The safest way to do it is to put on the land a good settler who has some capital. His investment of capital and labor will create the security. Then money may be loaned him on terms he can meet to create income.

Settlers come from regions where the climatic and agricultural conditions differ from those of their new locations. To many, irrigation farming is new. They do not know how to lay out land for irrigation, which field to prepare first, or what kinds of crops to plant. Mistakes can be avoided if settlers are given friendly assistance and guidance from the beginning. They require counsel and help in the preparation of sound financial and agricultural programs. Co-operative institutions are brought into being where this service is extended by a competent leader. The services of such a leader is doubly required and his duties multiplied if the selection of settlers becomes a part of his duties and if credit which requires supervision is included in reclamation development.

The principles advocated by the author are entirely sound and if put into practice would alleviate many of the difficulties of reclamation.

O. L. WALLER,* M. AM. SOC. C. E. (by letter).†—Irrigation development rests not only on the cost of water delivered on the land, but also on the productive value of the water and on the price the crop will be likely to bring. The net value of the crop is a major element in determining the feasibility of a project. Before lands are developed, there should be an agricultural economic survey. Such a survey should assemble data showing the productivity of the soil, the adaptability of the climate to the production

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† Received by the Secretary, May 3, 1926.

of crops, the length of the growing season, the kind of crops that would be likely to be grown on the project, the transporting and marketing facilities, and the probable market value of the crop. Without these facts, the prospective settler cannot make a rational estimate of the income that he may reasonably expect to receive from his farm.

The history of irrigation shows that it has been the custom to make a serious study to determine whether water could be delivered to the farms and at a reasonable cost. The author states* that:

"It was the popular opinion that if water were provided, irrigated agriculture would follow, that settlement of land and farm development under canals could be left to take care of themselves."

The agricultural, economic, and social viewpoints have largely been omitted. Fortunately, the Federal Reclamation Service is now adding to its program a study of the economics of the irrigated farm and has extended its researches to include settlement and farm development, and is treating these as important parts of the reclamation program.

As in the past, studies must include not only the water supply, the necessary engineering treatment of the project, and the cost of delivering water on the land, but in addition the cost of settlement and farm development. Formerly, projects were financed, but now the settlers must be financed. The day when small irrigation projects could be developed by a single farmer or a group of farmers at a minor cost is past. Future development will cost much more; and ordinarily men of established financial credit are not buying such farms. Only settlers qualified by farming experience are to be permitted to undertake to farm irrigated land on Federal projects. Usually they will be men of moderate means who will need financial help to make it possible for them to improve their farms. Shall this new policy be accepted, these additional studies made, and credit arranged for the development of the farms? Or shall the statutes providing for Federal irrigation be repealed and arid lands allowed to remain barren, to raise sagebrush, and to breed horned toads and jack-rabbits; or shall the Federal Government be forehanded and develop its natural resources, irrigate its arid lands, and thereby provide homes and food for its rapidly increasing population?

In coming to a conclusion as to whether unproductive lands are to be developed or to remain barren, it must be kept in mind that the last great West has passed into history, as far at least as cultivable lands are concerned, and that future tillable soils must come from irrigating arid lands, from draining swamps, or from clearing logged-off areas. The world's population is constantly increasing, while the limit of tillable areas has nearly been reached. O. E. Baker, Agricultural Economist, U. S. Bureau of Agricultural Economics, says:† "The agricultural production of the United States is no longer keeping pace with our increasing population." In the rural districts of the United States, the yearly increase of population is about 600 000. These young people know farming, and farming ought to be made profitable enough to induce them to remain on the farms. The demand for agricultural

* *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 411.
† Year Book, U. S. Dept. of Agriculture, 1921, p. 408.

land has constantly been increasing, as is shown by the increase in land values. The Hon. Henry C. Wallace, former Secretary of Agriculture, says:*

"The average value of farm land (including buildings) in the United States increased between 1850 and 1920 at a rate equivalent to compound interest on the 1850 valuation of 2.65 per cent. The increase from 1900 to 1920 was at the rate of 6.47 per cent."

Since 1920, there has been a slight drop from war prices.

When arid lands can be made to carry the financial load for water with which to irrigate them; to pay for the lands, for their development and improvement; to pay a reasonable interest on the farmer's investment and pay wages for his labor and that of his wife and children who work on the farm, the Government should be interested in their development. The nation needs the homes thus made possible for its rapidly increasing population.

President Coolidge, in his letter to the Farmers' Conference of November 17, 1924, pointed out that the United States already imports foodstuffs that it should raise. At present, farmers are raising too much of some kinds of food and not enough of others. Probably the yearly fluctuations in yields will always produce temporary over-production in one crop or another. It is not a question of permanent over-production, but of raising the right kinds of crops and of proper selling organizations. The remedy is not to be found in importing food produced by under-paid foreign labor.

In determining whether new lands are needed, the really important matter is whether the new lands may be profitably worked by farmers living on them. In that case, the more farms, the better. Otherwise, the more people that fail in the business of farming, the worse off will the nation be. The real effort should be to get as much land as possible into continuous and profitable use.

As much farming land is annually being abandoned and going back to the jungle state as is reclaimed by irrigation and drainage. The poorer lands are constantly being exchanged for more fertile lands. Business men quit poor locations and move to better ones. Why should not the farmer leave his worn-out non-productive land that is not yielding a fair return for his investment and his labor and move to more fertile lands? To do this, the arid and swamp lands must be developed. They are the nation's most fertile land asset. People talk about abandoned farms; they ought to be abandoned—many of them ought never to have been deforested. Mr. Baker says:†

"The trend of land utilization in the United States is toward the more intensive use of the more fertile or more favorably situated land * * *; and toward the less extensive utilization of the less fertile or less favorably situated lands."

Practically all the land that is easily available for agricultural purposes is now under the plow. Only by draining and reclaiming the wet lands, by clearing the cut-over lands, and by irrigating the arid lands can the producing area be increased. Soon all the national land resources will be needed for domestic food production. Mr. Wallace estimated that the population

* Year Book, U. S. Dept. of Agriculture, 1921, p. 4.
† *Loc. cit.*, p. 409.

of the United States in 1950 would be 150 000 000. Dr. Raymond Pearl, Specialist in Vital Statistics at the Johns Hopkins University, estimated* a population of 150 000 000 in Continental United States in 1950. This is an increase of 45 000 000 over the 1920 Census. The same thought is ably discussed† by Dr. Edward M. East, of Harvard University.

To provide food for the normal increase in population, not counting immigration, Mr. Wallace estimated that it would be necessary to bring under cultivation 8 000 000 acres per year, or approximately 250 000 000 acres between 1920 and 1950.‡ Mr. Baker states that improved land increased less than 5% from 1910 to 1920, as compared with 15 to 50% for the previous decades. This 5% increase was practically confined to the precariously productive semi-arid lands of the Great Plains region.

"The land in the United States suitable for agricultural use without irrigation, drainage, or heavy fertilization is nearly all occupied. Consequently, one of the great questions before the American people is how to maintain the supply of foods and fibers for the increasing population at that high level to which we are accustomed."§

A study of population and food statistics would convince one that the Federal Government should continue the policy of developing its non-productive lands; that all such areas should be carefully studied and plans should be made for their improvement. These plans should contain a careful examination of all features pertaining to the soil, climate, crops, availability of water, and engineering treatment, and should provide for the selection and financing of home builders. A plan should be provided for the elimination of the land speculator. Otherwise, the cost to the settler will be much increased and the development of irrigation projects will be greatly delayed.

The development of the land resources of the United States should be advanced as rapidly as the new lands can be made to provide proper guaranties for the return to the Treasury of the costs of developing them. The writer most heartily concurs in the opinions of the author, but, in addition, suggests that a systematic study should be made of all available areas that may ultimately be irrigated, cleared, or drained. In other words, let a Federal inventory of the nation's land resources be obtained.

* In an article on "World Over-Crowding."
† "Mankind at the Cross Roads."
‡ See Year Book, U. S. Dept. of Agriculture, 1921.
§ *Loc. cit.*, p. 408.

LAND SETTLEMENT OF IRRIGATION PROJECTS

Discussion*

By Messrs. R. A. Hart, R. K. Tiffany, and F. H. Newell.

R. A. Hart,† M. Am. Soc. C. E. (by letter).‡—The author points out the almost universal need for artificial drainage on irrigation projects. The possibility—nay, probability—that such need will arise should be kept in mind from the beginning on every project and the settlers should be educated to the idea. It has been suggested even that the drainage works should be constructed at the same time as the original irrigation works. In few cases would such procedure be practical and in no case would it be economical, since the annual charge against the additional investment would run for years before the works actually were needed.

Drainage works should be installed, however, before the inhibitive condition becomes serious. Not only would the cost of installation be less, but loss of crops would be avoided and the danger of losing settlers would be lessened. No one is better fitted to redeem injured land than its original owner, and the cost of securing a new settler will be saved. It has been found expedient by drainage officials to go to considerable length to keep the present owners on the land after reclamation by drainage.

What is full colonization at one time or under one set of conditions may not be full colonization at another time or under a different set of conditions. Drainage reclamation is a case in point. With the drainage charges added to the original cost, it may be necessary to change the type of crops or, at least, operate on a more intensive scale in order to make the enterprise pay. This means that the average ownership must be reduced by a considerable margin. The new colonization represents a process of infiltration—the most successful and satisfactory kind of colonization. If this sort of colonization could have been applied to irrigation projects generally many of the difficulties encountered in the past would have been avoided. The seed of these difficulties lay in the fact that the supply of new lands greatly exceeded the real demand for them.

R. K. Tiffany,§ M. Am. Soc. C. E. (by letter).‖—This able and comprehensive paper shows a knowledge of the subject that can be gained only by direct and intimate contact with all its various phases. If the paper and the

* Discussion of the paper by Augustus Griffin, M. Am. Soc. C. E., continued from April, 1926, *Proceedings*.

† Secy.-Mgr., Western Clay Products Assoc. Salt Lake City, Utah.

‡ Received by the Secretary, March 26, 1926.

§ State Superv. of Hydraulics, Olympia, Wash.

‖ Received by the Secretary, April 29, 1926.

interesting discussion following it appear to wander from engineering into broader fields, it only emphasizes the fact that the engineer who would serve his clients and the public in planning development projects must make a comprehensive study of the social and economic features involved.

Had such a study been made of all the present irrigation projects, public and private, it is quite certain that many of them would not have been built. They did not at the time of construction and do not to-day meet the requirements outlined by Mr. Griffin for a successful project, or the requirements that Commissioner Mead of the U. S. Bureau of Reclamation has indicated to be essential to his approval of a new project.

It may be admitted that for such projects—some of them ill-conceived and without intrinsic merit, some inherently sound but built before their time—an artificial aid to land settlement will be necessary, and that in order to save present investments and assure the success of the settlers already on the ground, Federal subsidy of such aid on these Federal projects is necessary and justified by the conditions. To establish State or Federal subsidy of land settlement as a permanent policy for the future is, however, quite another question and opens up the much broader field of future State and Federal reclamation policy.

The country as a whole will eventually need the production of all its arid lands. Most of the remaining projects, with their high cost, large area, and consequently their long development period, will require a subsidy to the extent at least of freedom from interest charges during the construction and development period; but further than this it will not be necessary to go if the projects are built only as there is shown to be an economic need for their products.

It will be said, however, that the nation should plan for the future. This is granted; it should perhaps even build for the future within certain limits. Mr. Hoover and other distinguished engineers and economists have suggested that some great public works, such, for example, as those that will be required in the development of the Colorado River and Columbia Basin Projects, should be planned years in advance and their construction undertaken by the Government during the periods of industrial depression to neutralize the slackness of employment in private enterprise at such times.

No attempt should be made by Government subsidy to force into production new areas of land in competition with that of tax-paying farmers who are not now receiving fair returns on their labor and investment. Further irrigation development in the West should be steady and gradual to keep pace with the natural growth of its markets. The settlers to provide for such extension can be largely found in the natural increase of the present farm population. The experience of the past twenty-five years has shown a steady supply from this source for new units of projects that are economically sound.

As to artificial selection of settlers, the writer would question any further requirement than these—a love for the land and for growing things, demonstrated habits of thrift and industry, and a willing and able helpmeet. Rejection of the inexperienced for that cause alone would have eliminated many

of the outstanding successful farmers on the various projects with which the writer has been connected. In an experience of 25 years involving close personal contact with more than 5 000 water users, it is his observation that the percentage of success is greater among those without previous farming experience, but having these qualifications, than it is among those who had farm experience, but lacked one or more of the other requirements.

What of organization, guidance in selection of crops, stock, etc.? The value of such service has been amply demonstrated; but cannot this service be better rendered by existing agencies, such as the State College or University, the County Agent, and representatives of the State and Federal Departments of Agriculture, rather than by the agency that builds the works and must collect their cost from the settler?

Is it not obvious that such advice and assistance is far more effective when it comes from a disinterested agency? If furnished by the creditor agency, every failure will be blamed on the adviser and the very effort to be helpful becomes the basis of personal and political appeals for financial relief, postponement, and reduction or cancellation of the debt. If, in addition, the building agency becomes banker to the settler for a home-building loan, a chattel loan, and possibly a loan for seed and equipment, it would simply mean that the sources of friction and the difficulties of collection increase in geometrical ratio with the number of loans.

Admittedly the problem of making a success of many Federal irrigation projects is a most difficult one. The experiment of land settlement financed, aided, and directed by the Federal Government may be worth while. Undoubtedly it will be if it is in friendly and capable hands; but for future reclamation development is it not feasible to work out a policy and a program that will be more simple and more economical in administration; that will eliminate the necessity of the settler dealing in his most intimate business and financial affairs with an agency 3 000 miles away; that will make of the new irrigation projects not a specially favored and pampered community but a self-reliant unit of the State, served, developed, and supported by the same agencies as those on which the other sections of the State depend?

The experience of the past, the unfortunate condition of many projects at present, the confusion and utter lack of a definite program before the present (69th) Congress, all join to make the moment opportune for serious consideration of the whole reclamation problem and for an earnest effort to formulate a sound and permanent State and National reclamation policy.

F. H. Newell,* M. Am. Soc. C. E. (by letter).†—Mr. Griffin presents for discussion certain ideas which, if translated into action, might enable the "promoter" (Government, district, railroad, or colonization agency) to secure settlement on irrigation projects. These problems of land settlement arise primarily because the land to be settled is not worth what it costs; no really experienced farmer of his own motion will purchase it at the price asked. Thus, the promoter must resort to extraordinary tactics to attract and hold

* Pres., The Research Service, Inc., Washington, D. C.
† Received by the Secretary, May 6, 1926.

families, promising advice and financial aid such as may enable them ultimately to gain a foothold.

The existence of a settlement problem is *prima facie* evidence of poor land. Really good irrigated land, offered on reasonable terms, needs no special inducements to attract and hold good farmers. It is the marginal or doubtful tracts that call for this special treatment.

The danger, if it may so be called, of the exploitation of "high-pressure" methods of settlement lies in the fact that promoters are encouraged to urge development of other tracts of similar marginal land. To this extent they create or perpetuate unstable conditions. There is plenty of good farming land to be had throughout the United States, particularly in the arid West. There the quantity of water is limited and the available supply should be kept for and applied to this really good land and not be developed for land of such character that a settlement problem arises. When brought to the good land at a reasonable cost per acre there has not been, nor is there likely to be, any especial difficulty in the matter of settlement.

Under ordinary conditions, the margin of profit in farming is small; on the marginal or inferior lands there is frequent loss at least of the time or equivalent wages of the man who attempts to cultivate it. He is naturally attracted to the larger wages paid by "quantity production" industries. As a result thousands of acres of marginal lands are lying idle or are being abandoned, especially in the older settled parts of the United States. Efforts are being concentrated on the better lands. With improved systems of farming and of marketing, also with a larger use of cheap power and machinery, the more skillful farmer, concentrating his efforts on the best lands, is making a living. Even after the abandonment of these inferior lands, the farmers can increase the total crop production whenever there is a sufficient demand to assure them of a fair profit.

In face of these conditions it is questionable wisdom to pour more money and more effort into schemes to settle lands which in the open competitive market are not profitable, and especially to do so by the use of public or corporate funds taken directly or indirectly from the unwilling or uninformed taxpayers or stockholders. Well-informed farmers and their organizations throughout the country are beginning to view with distrust these attempts to interfere with the action of the laws of supply and demand.

There is a tradition or belief that any dry land that may be irrigated, or for that matter any wet land that may be drained, must necessarily have good productive soil. It has been argued that any such land that has remained idle up to the present time and now can be reclaimed must necessarily be good land because it has not been cropped or exhausted and that, having incurred large expenses directly or indirectly in reclamation, it is the duty of some one to see that it is settled.

Granting that the degree of moisture in the soil can be controlled by irrigation or by drainage, it does not follow that this soil under the new or changed conditions will continue to be tillable even if some crops may be had for several seasons in succession. It has been shown by Mr. Carl S. Scofield*

* *Journal of Agricultural Research*, Vol. 21, p. 266.

that "soft water makes land hard and hard water makes land soft". The soil after reclamation is not in a stable or fixed condition. The good farmer appreciates this and is becoming more and more cautious of risking his time and money on these newly reclaimed lands. The failure of three or more successive owners of such lands has been popularly attributed to some defect in the methods of selecting, directing, or financing the settlers. Too little attention has been given to the fact that fundamentally the cause has been in the inferiority of the land. Recognition of these defects has properly deterred the best farmers from attempting settlement, leaving the field open to those less skilled or competent. "Good land attracts good farmers—poor land remains for poor farmers."

Mr. Griffin notes* the saying "that the first two crops of settlers will fail and that the third and following crops may achieve success on the pioneer work of the first two"; he shows that it is not the failures but the early successes that insure a greater percentage of success among later comers. To put it in another way, may it not be that these few early successes were due rather to the fact that the better lands were selected by better settlers and that the superior qualities manifested by these settlers were associated with their ability to select the better lands?

When attention is given to the subject, it is comparatively easy to show that among the pioneers in Kansas, Nebraska, Minnesota, and on through the irrigated West, the good farmers who selected good lands remained to enjoy such success. On the irrigation projects, while there has been as a whole a notable shifting of population, it has been shown that this shifting or these successive waves of settlement have been confined almost wholly to those settlers who took up the inferior lands, some of which, it now appears as soil changes have proceeded, should not have been reclaimed since they are not worth the cost of reclamation and cultivation.

The obscuring of this fact and the special emphasis placed on the beneficial results that may flow from the solution of these "problems of settlement" has its baneful effect in stimulating promoters to urge on the Government the continued reclamation of lands of doubtful value such as would not be selected by good farmers. It is asserted that by careful selection, by aided and directed settlement, by loans, by continual supervision, it may be possible to hold settlers on certain areas. The consideration of the really productive value of these areas, the possibility of marketing the products, and the other fundamental conditions which should be given weight, are pushed into the background by the enthusiastic discussion of what is termed "the human element". "Ignorance is bliss" and as pointed out* by Mr. Griffin, "the art of getting settlers is well developed"; also "within certain limits the newer the project and the less development it has, the easier it is to get settlers".

Skillful practice of this art of getting settlers is being relied on by promoters to offset serious deficiencies in the quality of land. A precedent is sought in the fact that selection is exercised and largely depended upon

* *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 417.

by an employer, or by the Civil Service, or when enlisting men in the Army. If selection is then necessary or desirable, why not in selling irrigable or reclaimed lands?

A wide difference exists; in the case of the selection of employees or even of farm tenants there is in mind the fact that the persons thus selected will remain indefinitely under the direction of the employer or owner. Every action taken by these selected people will be guided for the benefit of the employer and only indirectly for the benefit of the person selected.

This idea is diametrically opposed to the basic conception of Federal reclamation, which seeks to provide homes on the land for Americans—independent voters and citizens—working first, last, and all the time for the immediate benefit of themselves and their families. People of that sort resent paternalism and do not willingly remain under the direction of an agent or employee of the promoter even if it be the Government.

Under a system of "aided and directed settlement" the assumption is made that the selecting agency must and will dominate and that the promoter or his agents will continue to direct and finance the settlers in their operations, practically assuming the responsibility for their acts. Such selection and direction may be necessary with renters but the theory of American land settlement is to discourage tenancy and to develop an independent land-owning community free from bureaucratic control. Experience has shown that where such aid and direction has been undertaken, it has been followed by the demoralization of the people aided, producing a condition dangerously approaching peonage.

This is touched on by Mr. Griffin, who comments* that, in his opinion, it is almost always unwise to forego payments when due as it increases the farmer's debt and dulls his conscience. Striking examples of this unfortunate condition are all too prevalent where, chiefly because poorly informed people have been located on sub-normal lands, there has been a resultant breaking down of the morale. This breakdown has been made more complete through well-meaning efforts to "aid and direct" the land owner by liberal extension of credit and then by permitting a partial repudiation of his debts. It is inevitable that these groups of dependents located on inferior land will call each year for more and more concessions. They are well aware that the promoter, having made a large investment in them, does not dare to let them go. He has already put up a bonus directly or indirectly† and they are firmly convinced that a further subsidy is due them as a partial recompense for their disappointments or for lack of success on these lands. In their minds the promoter has assumed a moral responsibility for their success.

While all this is going on, while the promoter is trying to select and hold his people, he is struggling against a general movement of agricultural population. Under the inexorable law of supply and demand the marginal or inferior lands are being permitted to go out of production, steadily reducing the area of cropped lands in many of the States. This is due to the fact that with improved methods, increased use of machinery and power, crops

* *Proceedings*, Am. Soc. C. E., March 1926, Papers and Discussions, p. 421.
† *Loc. cit.*, April, 1926, Papers and Discussions, p. 769.

can be produced on the better lands in excess of needs or markets. The officials of the Department of Agriculture have long since recognized this, and have not been disturbed by the abandonment of inferior acreage. They have steadily admonished against a rapid increase of tilled land until market conditions demand, and then only of the best lands. Economists show that there is already more than enough land under the plow and that the farmers' earnings should be increased not so much by bringing more land under cultivation as by wisely selecting the best areas and steadily improving these.

It is incumbent on the engineer and economist to view with a keenly critical eye the question as to whether in any particular case the probable productive value of the land to be reclaimed will justify the risk and cost of aided and directed settlement. Imagination and sympathy warm toward the man who "made two blades of grass grow where one grew before" but some one must consider whether the value of these two blades justifies their cost. The question properly arises as to whether the promoter, even if this happens to be the Government, is now justified in adopting high-pressure methods of settlement accompanied by loans. No one should be swept away by sentiment and popular appeal to undertake the salvage or settlement of lands regardless of ultimate costs and values simply because money has been spent in their reclamation.

INTERSTATE WATER PROBLEMS AND THEIR SOLUTION

Discussion*

By D. C. Henny, M. Am. Soc. C. E.

D. C. Henny,† M. Am. Soc. C. E. (by letter).‡—The argument made by the authors in favor of amicable settlement of water disputes between States on interstate streams as against Court proceedings carries a strong appeal. Engineers familiar with trials of water cases in the Courts realize fully the difficulty inherent in this method of reaching practicable and just conclusions and will be the first to admit the desirability of substituting for Court proceedings some less cumbersome, rigid, time-consuming, and expensive method of dealing with water questions.

The progress made in recent years with interstate compacts, as recorded by the authors, adds force to the argument, whereas on the other hand the obstacles which the compact method has encountered expose certain weaknesses which deserve careful consideration.

States at the head of an interstate stream system hold a strong strategical position. As one of such States, Colorado granted and still grants to its citizens the right of diversion and storage limited only by prior rights within the State. Prior rights in lower States were not only disregarded in practice but were alleged by reason of their position to be legally subsequent.

The effect of increased diversions in Colorado on lower States was not injurious until, through irrigation expansion, shortage resulted and thus the controversy started and is still waging. The State of Colorado made full use of its strategical position and of the doubt as to interstate water rights and at least in one case has for a long time worked serious damage on one of the lower States.

Two of the lower States suffering or anticipating injury resorted to the Courts for redress, which gave rise to the famous Kansas-Colorado and the Wyoming-Colorado cases. Technically, of course, Kansas and Wyoming were plaintiffs in these suits, but it can hardly be pleaded from this that Colorado was not the real aggressor, as was done by the authors.

The money spent by Colorado on these cases, reported by the authors as totaling about $1 000 000, was intended to make good its sweeping claims to all water arising within its borders, and similar expenditures in lower States were incurred in defense of their prior claims.

* Discussion of the paper by M. C. Hinderlider and R. I. Meeker, Members, Am. Soc. C. E., continued from May, 1926, *Proceedings*.

† Cons. Engr., Portland, Ore.

‡ Received by the Secretary, May 19, 1926.

The earlier Kansas-Colorado suit resulted in a negative decision by the U. S. Supreme Court, as the Court found no injury had been suffered up to that time by Kansas and failed to settle respective State rights. It was not until June 5, 1922, that the highest Court gave its verdict on this main point of controversy and decided in favor of the lower State, Wyoming.

This decision brought about a radical change in the attitude of Colorado. When it found that lower States could find effective redress in the Courts, it began to point out the objections to Court proceedings and to advocate amicable negotiations and settlement by treaty, otherwise called the compact idea.

A compact to be satisfactory and lasting should be based not on the somewhat haphazard result of a clash between opposing extreme claims but on the primary willing adoption by all sides of the views expressed by the U. S. Supreme Court, which virtually wipes out State lines in the consideration of respective water rights, at least as regards States with substantially similar water laws and water practice. It should be approached in a mutual desire for fairness and in a truly friendly and co-operative spirit. Pending compact negotiations, no effort should be made by either side, directly or indirectly, to radically change the *status quo* as any such step would tend to destroy mutual confidence, without which no real progress in reaching a compact agreement can be expected.

The idea of central bureau administration is as obnoxious to most Westerners, including the writer, as it is to the authors. The growing tendency to demand too much Federal assistance is distinctly injurious to the much lauded pioneer spirit of the Western farmers. It is believed, therefore, to be for the best interests of the West that local control shall prevail as much as possible.

It should not be inferred from this, however, that decisions of the U. S. Supreme Court do more than declare what the law is. They do not as such interfere with local self-government. Resort to the Courts cannot be logically objected to as setting up some central bureaucratic authority.

In regard to river control, no matter whether rights are settled by the Court or adjusted by negotiation, it is well to bear in mind that a mere verdict, by whomsoever rendered, cannot execute itself, and of necessity will call for some regional authority to carry it into effect, especially as against the States which have the geographical advantage of being at the head of a stream.

Actual conditions of river flow as affected by flow from the head, side inflow, and diversion, present an ever-changing problem for division of waters. This problem is of sufficient complexity when confined within the limits of a single State, but if extended to an interstate stream system its treatment requires the highest administrative talent.

Additional diversions will affect the stream in a manner which cannot be definitely foretold. The upper State may claim that benefit through return flow late in the season will offset injury during the earlier part, and this claim may be denied by lower States and can only be definitely judged after the

event, dependent as it is on the extent to which drainage is provided and on other factors only partly known.

Actual administrative control, therefore, must be provided in any case, and such control will probably be local in character, possibly by boards on which interested States are represented through an administrative officer. Whatever solution may be found of this phase of the question it is independent of whether the fundamental scheme of division of waters is the result of a verdict by the Court or by a compact commission.

A possible weakness in campact agreements is that their fairness depends on the wisdom of a number of men, generally appointed by the respective Governors of the States involved. In spite of the fact that appointments made in the past may have been of a high order, it is nevertheless conceivable that the exigencies of local politics may influence future appointments. It is, therefore, quite possible, if not probable, that representatives of States will be less trained to render impartial decisions than the judges of the Federal Courts, than whom as to previous training and impartiality no better men can be found to deal with difficult problems judiciously. Some lawyers also argue that the rigidity and circumscription of Court proceedings to which general objection is made and which are to be avoided in compact commission hearings, are nevertheless the result of vast experience and their irksomeness is offset by material advantages.

It may be inferred from the authors' arguments that the compact method eliminates Court proceedings. It is believed to be probable that such proceedings will be lessened. However, the States and even Congress have no power by legislative or other process to interfere with vested rights so long as the Fifth Amendment of the Constitution stands. Claimants in one State may have good reason to believe that their interests have not been properly safeguarded in the final compact, even if they have been given full opportunity to be heard before the Commission. They may be tempted, therefore, even against great odds, to obtain redress through the Courts.

The compact idea offers the greatest chance of success where only a few States are involved. It becomes more cumbersome as the number of interested States increases because any one State may fail to give the compact its legislative sanction and may make a strong effort also to prevent Congressional approval. This is understood to be the present status of the Colorado River Compact, Arizona being in this instance the objecting State.

There are cases in which conflict is not merely as between States but in which the United States itself is an interested party. In most Western States there are Indian tribes living on reservations for whom the United States acts as guardian, whose prosperity is in part dependent on water rights often secured by treaty. By the very fact of their legal incompetency they need to be represented by their guardian and not by State representatives. Similarly, there are in every arid State reclamation projects, claiming water through filings made on their behalf by the United States, on which, therefore, should rest in large part the obligations of defense against illegal encroachments such as may unwittingly be committed by compact commissions. In addition, there are in almost every State great possibilities for further reclamation of public lands in which the entire nation has an interest.

The compact idea, therefore, would gain in attractiveness and be less subject to criticism and active opposition if it included in its program a composite commission on which the Department of the Interior, by which most of these Federal interests are administered, is directly represented by a commissioner.

The authors devote a few paragraphs to international river problems and refer to various treaties, among others to the treaty of 1906 between the United States and Mexico dealing with the division between the two nations of the water in the Rio Grande above Fort Quitman, Texas.

This treaty provides for the delivery to Mexico of 60 000 acre-ft. per annum from the Elephant Butte Reservoir in New Mexico, built by the Reclamation Bureau in connection with its Rio Grande Project in New Mexico and Texas. The supply to the reservoir is seriously threatened by contemplated storage in Colorado and although the quantity assigned by the treaty to Mexico is small in relation to the full capacity of the reservoir, a series of dry years, such as has occurred in the past, may so deplete the reservoir as to render delivery of the stipulated quantity impossible. If this results merely from climatic conditions, the treaty itself holds the United States harmless, but if this condition can be proved to have been definitely aggravated by new diversions above, it is not unlikely that Mexico may demur and come forward with a large claim of damages as it did prior to the date of the treaty.

Thus, it would seem that in cases of this kind the United States is interested not merely through its Department of the Interior, but also through the Department of State, and that the latter also should be represented by a commissioner. Where this is not provided the compact idea will find opponents in land owners whose individual interests will be seriously jeopardized when at some time in the future the United States, through the Department of State, should be called on to take hold of the situation to make good its international obligations.

The objections to the compact idea as developed to date, carried out by State Commissioners under the chairmanship, at times, of some United States representative, are thus brought forward, not for the purpose of belittling its great possibilities, but to indicate its weak points and to explain why it has met with some well-founded opposition.

In order to be reasonably successful the compact idea should be conceived in a spirit of true fairness without any effort to take undue advantage; it should be carried through without permitting any change in the *status quo* as to main questions in dispute, after initiation of a proposed compact, so as not to destroy the feeling of mutual confidence which is essential to success; it should be handled by a commission on which all interests are fairly represented and the resultant decisions should be sufficiently elastic to permit the authority entrusted with their execution to deal with the ever-changing river conditions as they arise.

It is believed that as to some of these points the foundation for arriving at compact agreements has not at all times been satisfactory.

The authors deserve full credit for bringing up for discussion a matter of great and pressing importance and for their able presentation of the subject.

STREAM REGULATION WITH REFERENCE TO IRRIGATION AND POWER

Discussion*

By Messrs. J. W. Swaren and William B. Landreth.

J. W. Swaren,† M. Am. Soc. C. E. (by letter).‡—Commendation is due the author for bringing before the Society in concrete form one of the biggest problems in hydraulic engineering. It involves the economics of agriculture and industry and of finance from the point of view of the investor and the promoter; as well as the purely technical problems of design to meet operating conditions with numerous and changeable variants.

The enthusiastic irrigationist proclaims that all water falling on a watershed draining through, or contiguous to, a sufficient acreage, should be conserved by storage and utilized on the land, leaving only dry stream beds at the entrance to the sea. At the most he would concede only canalized streams to water transportation. The operating power official, on the other hand, would fit the stream flow to his daily and seasonal load curve, with irrigation and navigation surviving as they may.

An engineering undertaking comprises three phases, named in the order of their importance as follows:

(*a*) The accumulation of pertinent data, which is crystallized into a decision or plan of action;

(*b*) The formulation of designs; and

(*c*) The construction of the works.

It is not often that the practicing engineer is equally familiar with the three major interests of power, irrigation, and transportation, so that he can, or does, take an entirely disinterested view. More often such a point of view bears the imprint of a regulatory body, and on this body rests the economic decision as to the maximum beneficial use. These bodies have no, or at the most, a meager, staff for the collection of data. The sole reliance for this important phase must rest entirely on the applicant. Only one who has had experience in the review of such submissions of data can appreciate the wide variance of the material offered. In spite of carefully drawn regulations, specifying the points to be covered so as to conserve the time of both applicant and examining officer, such submissions vary from a mass of heterogeneous irrelevance to avowed briefs of special pleading. It is through these that the

* Discussion of the paper by J. C. Stevens, M. Am. Soc. C. E., continued from May, 1926, *Proceedings*.

† Cons. Engr., Washington, D. C.

‡ Received by the Secretary, April 17, 1926.

examining officer must wade in an endeavor to formulate a sound conclusion of the respective phases of interest.

Regulatory bodies are political (not partisan), and as such must take the political point of view, even though they are headed in many cases by engineers. As a result, the digestion and assimilation of these masses of data must be done by subordinates who formulate technical decisions for consideration by politically minded superiors. These, in turn, look to three parties in interest:

(1) The consuming public—irrigator, power customer, or shipper;
(2) The investing public—security holders and taxpayers; and
(3) The applicant, with its attendants, engineers, promoters, and security vendors.

State appointed bodies have approached the problem almost entirely from the point of view of the first class, having in mind the continued maintenance of rates. Where consideration has been given to earnings as affecting the investing public, the possible absorption of wreckage from failure, as a rate influence, usually has predominated.

A study of decisions handed down by the Federal Power Commission indicates that it takes a much broader view of the problem. It has rarely touched rates. Naturally the immediate interests of the Federal Government have been given first consideration; but where its decisions have departed from the immediate Government interest, its attention has been directed to the wider economic phases of the project under consideration. This is entirely proper, as rates are a local matter but economic utilization is of National import.

The popular conception of the financial phase is that the man who puts up the money is well able to take care of himself. This is true in the first instance, as the securities of large undertakings, the sale of which provides funds for construction, are purchased by interests whose chief concern is re-sale, and who anticipate that a certain percentage will turn "sour". This is not true, however, of the final purchasers. The only way to appreciate how widespread the final distribution of a group of securities extends, is to examine the mailing list of a dividend, or the coupon verification sheet of an interest date.

Needless to say the vendors of securities are not slow to point out that even tacit approval by a commission gives added strength to the offering. Only too frequently this is accepted by the final investor as tantamount to a guaranty, particularly when such approval is by a Federal body; and the vitalness of this matter to engineers is doubly apparent from the fact that it is possible for a security issue to be prepared so that it will pass the accounting analysis of any "blue sky" commission in the United States, when at the same time a brief analysis of its engineering economics will forecast certain failure. Many such instances have come before the writer, and he presumes that other engineers have had similar experiences in the conduct of examinations.

Thus far, securities of hydro-electric companies have a good record. John E. Field, M. Am. Soc. C. E., does not present a happy case for irrigation projects.* Such debacles as he instances are preventable. Their lasting effects are shown in the difficulty of privately financing even an exceptionally meritorious irrigation project.

Any study of a specific stream to determine methods of control for power or irrigation is not complete without a study of the economic phases. It is not safe to assume that any one phase is paramount; only competent analysis can determine the particular phase. Usually the ultimate decision will rest with some extraneous regulatory body, whose judgment should be free from purely local influence, having in mind not only the needs of the community, but also the National welfare. A stream study that merely provides data for the design of structures to harmonize an imposed group of physical restrictions is not complete. It must also provide the data from which may be evolved a plan that will result in the maximum economic benefit.

WILLIAM B. LANDRETH,† M. AM. SOC. C. E. (by letter).‡—The writer has been much interested in Mr. Stevens' statement§ in regard to forestation as a means of water storage and stream regulation. Study of the flow of Eastern streams leads him to agree that forestation has little if any economic value in water storage or supply.

In discussing the paper, E. H. Sargent, M. Am. Soc. C. E., describes‖ studies made in forming an operating chart for the proposed Sacandaga Reservoir. This calls to mind an operating diagram made by the late G. Edward Gibson and the writer in 1919, for an existing reservoir at Hinckley, N. Y.

Part of the water supply for the Rome summit level of the New York State Barge Canal is secured by the use of the Hinckley Reservoir on West Canada Creek. This reservoir has a crest capacity of 77 000 acre-ft. and a usable capacity of 3.85 in. on its water-shed.

The Hinckley Reservoir has been in use since January, 1915, and furnishes water for the Barge Canal during May to November, inclusive.

Several water powers are situated on West Canada Creek both above and below the point of diversion of the canal supply. Any plan of regulation that would distribute water not needed for the Barge Canal would tend to reduce the damage that might result from the diversion of water. It was realized that the plan adopted should be a practical one and rely as little as possible on individual judgment or guesswork. Gaugings had been made of the flow of the creek at various points since 1907 and were continuous from January, 1907, to January, 1918.

A study of the surface elevations of the water in the reservoir showed that during 1915, 1916, 1917, and 1918, the general plan of operation was to keep the reservoir full or nearly so during the canal months and then draw

* *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 776.
† Cons. Engr., Schenectady, N. Y.
‡ Received by the Secretary, May 15, 1926.
§ *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 624.
‖ *Loc. cit.*, May, 1926, Papers and Discussions, p. 1040.
¶ Report of New York State Engineer, 1920, p. 7.

the water down, ready for the next spring flood, which resulted in inefficient operation. The reservoir is not large enough for a complete regulation of the stream. The most beneficial use to which it can be put is to smooth out the flood peaks and to increase the low-water flow.

As the records of stream gaugings were very complete after January, 1907, it was decided to use them in the preparation of an operating diagram. The records of stream flow from 1907 to 1914, inclusive, were used in the study to determine the best plan of operation and those of the four years, 1915 to 1918, inclusive, as a test period. It was believed that the best results would be obtained by dividing each month into three nearly equal periods, beginning, say, on the 1st, 11th, and 21st, of the month. By such a division of the time the necessary equalizing storage would be reduced to a small amount and, in fact, could be disregarded. The rate of inflow on corresponding dates varied from year to year. The rate of draft at any particular time would be dependent on the amount of storage and the rate of inflow at that time. In actual use the quantity of the draft is determined from an operating diagram based on the quantities referred to. The net inflow, the draft, and the storage were expressed in terms of the equivalent depth, in inches, on the water-shed. A mass curve was constructed using the mean net inflow for each "one-third-of-a-month" period for the years 1907 to 1914, inclusive. From this mass curve the maximum uniform drafts possible, with varying amounts of storage from an empty to a full reservoir, were determined for the date beginning each "one-third-of-a-month" period. From the data for each date the drafts for various amounts of storage were determined, and by plotting those drafts for the various amounts of storage for each "one-third-of-a-month" period in chronological order, an operating diagram was obtained. This diagram showed varying allowable rates of draft for varying amounts of storage through the year.

For storage above the crest of the dam the draft is uniform, the additional storage being disregarded—as was true also for the storage in the lower 3.5 ft. of the reservoir. As a test, the first operating diagram was applied to the 4-year period, 1915 to 1918, inclusive, as those years had not entered into the formation of the diagram. As the reservoir was in use during those years the flow below the reservoir was corrected for the influence of the storage, so as to show the inflow into the reservoir. The operating diagram was then applied to the entire 12-year period, 1907 to 1918, inclusive, and such corrections made in it as seemed warranted and advisable.

During 1919, the diagram was in use and the results of its operation were carefully noted. A final operating diagram was drafted in December, 1919, based on the operating diagram in use. This final diagram has been in use since January, 1920, and differs only from the first one by a slight increase in the amount of draft and a decrease in the reserve storage. The diagram in use calls for a nearly empty reservoir on April 1, in order to obtain the full effect of it during the spring floods that generally come in April or May, and a full reservoir about June 1.

Use is made of the operating diagram as follows: At the beginning of any "one-third-of-a-month period" the gate operator would obtain the surface-

water elevation of the reservoir from a gauge. In the operating diagram at the intersection of the vertical date-line and the horizontal reservoir-elevation-line he would read the draft, in second-feet, for the ensuing "one-third-of-a-month" period and set the reservoir gates to feed that quantity. The operator would check the gate setting by a gauge about 1 mile down stream from the reservoir dam and, if necessary, reset the gates. Observation of the surface elevation in the reservoir and a study of the operating diagram during the "one-third-of-a-month" period would show whether the draft was too great or too small and it could be corrected at the beginning of the next period. For several years the regulation of the draft from the reservoir by the operating diagram has been under the control of the State Engineer.

TABLE 1.—MONTHLY MEAN DRAFT FROM HINCKLEY, N. Y., RESERVOIR WHEN IT WAS NOT OPERATED BY THE AID OF A DIAGRAM.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1915	1119	677	663	1525	731	275	298	871	459	1222	663	723
1916	1505	1423	965	1983	1775	643	475	340	334	370	983	1230
1917	1136	469	460	1693	1086	1485	445	436	304	1034	1487	494
1918	418	472	957	1505	903	434	358	326	451	1133	1333	1359

TABLE 2.—MONTHLY MEAN DRAFT FROM HINCKLEY, N. Y., RESERVOIR WHEN IT WAS OPERATED BY THE AID OF A DIAGRAM.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1920	958	688	762	2660	1120	473	434	579	630	695	750	1390
1921	1140	920	2330	1400	870	524	607	695	588	640	1000	1250
1922	730	676	1110	3390	1330	2100	1060	688	624	643	619	523
1923	607	709	728	2690	2060	570	492	382	396	469	647	916
1924	1070	1080	784	1980	2580	636	455	366	498	969	767	646
1925	570	741	1640	2340	1200	666	996	754	684	...	...	....

TABLE 3.—YEARLY RAINFALL, IN INCHES, FROM STATIONS AT OR NEAR HINCKLEY, N. Y., RESERVOIR.

Year.	Inches.
1915	47.41
1916	44.27
1917	45.73
1918	44.57
1920	43.67
1921	43.56
1922	54.91
1923	47.05
1924	44.83
1925	61.41

In Tables 1, 2, and 3 are given the mean monthly drafts from the Hinckley Reservoir since January, 1915, except that for 1919 when the record was badly broken, and the yearly rainfall at or near it. These records are taken from the Annual Reports of the New York State Engineer.

In 1917 the total run-off on the water-shed was 31.68 in.; in 1923, it was 32.49 in. In 1923 when the diagram was used, the draft was much more uniform than in 1917, when no diagram was used, although the total run-off was nearly the same.

The use of an operating diagram in determining the draft from the reservoir has shown that such regulation is a very practical question and can be solved within reasonable limits by a study of the gaugings of the stream. The past performance of a stream, as shown by its gaugings, forms a good measure of what it will do, its value depending on the length of the gauging period.

Regulation for canal supply from May 1 to December 1 in each year does not differ greatly from that for irrigation purposes, during those months.

DEVELOPMENT OF INDUSTRIAL SITES AND TRACKAGE LAYOUTS AT KANSAS CITY, KANSAS, AND LOS ANGELES, CALIFORNIA, BY THE UNION PACIFIC SYSTEM

Discussion*

By Messrs. A. D. Butler, Lester Bernstein, and Augustus Hunt.

A. D. Butler,† M. Am. Soc. C. E.—Cities, like people, have their individuality. It is a strange thing, and one often commented upon, that of all the people in the world no two persons are exactly alike; and this characteristic of individuality is even more pronounced in cities than in persons. The problems that confront each city are those peculiar to its particular needs; and a solution that meets the requirements of one will only apply to another after modification.

When a railroad terminal is to be established in a city built in a flat, level country, it is often a serious problem to select a route that will meet the needs of the road, and yet not seriously invade some residential district, greatly to its detriment.

Nature has been kind to Spokane, Wash., in many ways, and has provided against the problem of a railroad company invading to any considerable extent any of its residential districts. The City of Spokane was started along the banks of the Spokane River, and has developed about equally both north and south of this dividing line; the ground rises on the south to a plateau about 400 ft. above the river and on the north side, more gradually, to an elevation of approximately 100 ft. As is the case with most of the Western cities, the valley through which the Spokane River flows is the only practical place for the railroads to build; so, by nature, all the railroad terminals and industrial sections of the city are in the flat valley on each side of the river and interfere little with any residential section.

The down-town section of the city is badly cut up, however, with three main-line transcontinental railways. This has been very expensive, both to the municipal government and to the railway companies. If, twenty years ago, the railway companies had been able, as they do now, to have seen this problem in the broad sense of transportation and service to the public, rather than in an impassioned desire to get business away from their competitors, such things as this could have been avoided.

* This discussion (of the paper by Walter R. Armstrong, M. Am. Soc. C. E., presented before the City Planning Division, Salt Lake City, Utah, July 9, 1925, and published in May, 1926, *Proceedings*) is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

† City Engr., Spokane, Wash.

The existence of railroads is only justified because they render service to the public; and this fact should be kept uppermost in mind by both railroad and city officials, to the end that the terminals may be designed so that the public will be best served, and not merely that one company may take away some of the business that its competitor now receives.

The present trend of thought in city planning, however, seeking through experience in the past to provide for the future growth of cities, is certainly an encouraging sign. Although mistakes will be made in future development, yet this method should go a great way toward eliminating some of the unfortunate situations in which some cities now find themselves.

LESTER BERNSTEIN,* ASSOC. M. AM. SOC. C. E.—As many industries can bear only a limited charge for doing business (including taxes), some of the older cities of the United States will soon have to consider the establishment of industrial districts to take proper care of their industries. Taxes have become quite a serious problem with some plants situated in the central parts of towns. In a number of instances taxes, with high real estate values that prohibit plant additions, have compelled long-established industries to move outside the city limits.

AUGUSTUS HUNT,† ASSOC. M. AM. SOC. C. E.—The arrangements made for the terminal facilities at the Port of Los Angeles are particularly impressive. Of special interest is the kind and amount of aid offered by the City of Los Angeles to further this development. The acquiring of the long strip of territory to be developed into railroad yards, unloading docks, store house, and terminal structures, is an example of what a progressive city can do to assist and procure for its industrial program aids that in the years to come will be of inestimable value to the growth of the city and will provide for more efficient and cheaper freight handling for the railroad. Both the City of Los Angeles and the Union Pacific Railroad are to be congratulated on their spirit of co-operation.

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MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

HARRY AUSTIN BELDEN, M. Am. Soc. C. E.*

DIED OCTOBER 30, 1925.

Harry Austin Belden, the son of James Belden, was born on March 28, 1865, at Wellington, Ohio. He was graduated from the High School in his native town and spent one year at the Case School of Applied Science at Cleveland, Ohio.

In 1887, he served as Rodman and Draftsman on cable railway construction for the Grand Avenue Railroad Company, Kansas City, Mo. In 1888, he was employed as Engineer at the Kansas City Stock Yards and, in 1889, he again accepted a position with the Grand Avenue Railroad Company as Draftsman on cable railway construction. Later, in the same year, he was similarly engaged by the Washington and Georgetown Railroad Company, at Washington, D. C.

Returning to Kansas City in 1890, Mr. Belden served as Assistant Chief Draftsman in the City Engineer's Office, but, in 1891, he returned to Washington to serve again as Draftsman on cable railway construction for the Washington and Georgetown Railroad Company. In 1892 he entered the service of the Baltimore City Passenger Railway Company, at Baltimore, Md., as Draftsman on cable railway construction and, in 1893, he was appointed Principal Assistant Engineer in charge of the Drafting Office and the construction of trolley lines of the City and Suburban Railway Company, at Baltimore.

In 1894, he became Assistant Engineer in the Frog, Switch, and Signal Department of the Pennsylvania Steel Company and, in 1895, was appointed Principal Assistant Engineer in charge of the Drafting Office and of the construction of conduit electric railway lines for the Metropolitan Railway Company of Washington, D. C. During 1896 and 1897, he served as Principal Assistant Engineer in charge of the extension of the cable line and of the erection of the Union Passenger Station for the Capitol Traction Company of Washington.

Mr. Belden was connected with the National Contracting Company of New York, N. Y., during 1898 and 1899, as Engineer in charge of construction of conduit electric railway lines for the Metropolitan Street Railway Company on the Eighth Avenue Line above 59th Street and on the Fifth Avenue Line below 50th Street.

From 1900 until 1902, he held the position of Engineer for Shewan, Tomes, and Company, an English firm of Hong Kong, China, and was in charge of the erection of electric plants and tramways in Canton and Hong Kong. He made many friends among the Chinese who called him the "Heavenly Light

* Memoir compiled from information on file at the Headquarters of the Society.

Man". After spending three years in China, Mr. Belden resigned his position to accept another in the Philippine Islands, as Superintendent of Construction for the J. G. White and Company, of New York, N. Y., where his work consisted of the construction of a street railway system, including 40 miles of track, an electric lighting system for the City of Manila, and harbor improvement contracts at Cebu and Iloilo.

During the World War Mr. Belden served as Chief Engineer on the construction of Camp Hancock, Augusta, Ga. After the completion of this work his health became impaired and on the advice of his physician he moved to Margaretville, N. Y. The change proved so beneficial that he was able in a short time to take charge of work in Porto Rico. For the greater part of three years prior to his death, Mr. Belden was engaged as Special Expert in the valuation of the properties of the Colorado Electric Light and Power Company, and spent most of his time in Denver, Colo.

In 1904 he was married to Susie C. McCay, of Covington, Ga., who, with a daughter, Jane, survives him.

He was a member of the English Club, of Hong Kong, the English and University Clubs, of Manila, and of the Engineers' Club, of New York.

Mr. Belden had the gift for making true and lasting friendships, and, with a mind enriched by the study of peoples and customs of the various countries visited during his extensive travels, was a most delightful companion. He was passionately devoted to the Orient, took a keen interest in Oriental art, and had made an interesting collection of Chinese paintings and Japanese color prints. He was an engineer whose professional integrity and devotion to any undertaking in his line of work gave him a high standing in the estimation of his business associates, and many of his most enthusiastic friends and admirers were among the members of his profession.

Mr. Belden was elected a Member of the American Society of Civil Engineers on October 4, 1905.

PHELPS JOHNSON, M. Am. Soc. C. E.*

DIED FEBRUARY 20, 1926.

Phelps Johnson was born at Warwick, N. Y., on October 23, 1849. His school and academy training was received at Springfield, Mass., and when seventeen years of age he joined the R. F. Hawkins Iron Works as Draftsman, later becoming Assistant Engineer. In January, 1879, he accepted a position with the Wrought Iron Bridge Company as Assistant Engineer and remained with that Company until he went to Canada in the beginning of 1882 to join the Toronto Bridge Company as Assistant Engineer.

The Dominion Bridge Company, with its principal works at Lachine, Que., Canada, was formed in 1883 by the interests owning the Toronto Bridge Company, and Mr. Johnson then became the Manager and Engineer of the Toronto Works, a position he held until June, 1888, when these works were closed and

* Memoir prepared by G. H. Duggan, M. Am. Soc. C. E.

he moved to the main office as Chief Engineer of the Company. While Manager of the Toronto Works he was also in charge of the erection of the cantilever bridge over the Reversible Falls at St. John, N. B., Canada, and of the St. Lawrence River Bridge at Lachine, both important structures in their time.

Mr. Johnson continued as Chief Engineer of the Dominion Bridge Company until October, 1891, when he was appointed General Manager. In February, 1910, desiring to be relieved from some of the details of management, he was appointed Managing Director, and, in October, 1913, President, a position which he held until January, 1919, when he retired from active work. He was elected a Director of the Company in 1903, and remained a valued Director and Member of the Executive Committee until his death.

It will be seen from his record that Mr. Johnson occupied a leading position in the Dominion Bridge Company from the time of its inception until he retired from active business in 1919. This was the period of Canada's growth in the construction of railways and highways, during which notable bridges were constructed, many of them under his direction. These structures, the success of the Company, and the originality displayed by Mr. Johnson in solving difficult steel construction problems and in devising new erection methods, many of which are now common practice, are witness to the genius of the man.

For many years after the formation of the Dominion Bridge Company there were few consulting engineers in Canada, and Mr. Johnson was called upon frequently for advice in engineering matters both without and within the field of bridge superstructures. He occupied a position of peculiar authority and trust among those who came into professional or business contact with him, and his judgment was highly valued.

The erection of the 1 800-ft. span at Quebec placed the unprecedented problem before engineers of finding a practicable design that could be safely constructed with reasonable economy of plant and preparation. That engineering monument, the Forth Bridge, in Scotland, of practically the same span, had been built with consummate skill, plate by plate at the site, but climatic and industrial conditions in Canada prevented this method from being followed and it was necessary to construct the Quebec Bridge of members completely finished in the shops so that they could be quickly assembled in the field.

Modern traffic conditions also required the Quebec Bridge to be more than twice as heavy as the Forth Bridge, so that members of very large size were needed which would have been exceedingly difficult to handle if the bridge were constructed with any of the ordinary arrangements of web-bracing. Secondary stresses and distortions due to cantilever erection also became formidable considerations.

It was natural that Mr. Johnson should have been consulted and in close touch with the several projects for bridging the St. Lawrence River at Quebec and, although he was not successful in obtaining for his Company the first contract for the bridge, it may be said fairly that his suggestions enabled the Company to obtain the order eventually and successfully construct the bridge.

During the preparation of the competitive designs for tendering on the final construction of the bridge, Mr. Johnson suggested that many of the most serious obstacles would disappear by adopting a new form of web-bracing, making what is now known as the K-truss, but designated by him at the time as "half shear bracing" because of its function of dividing the shears between two members in each panel. It took little investigation to confirm his judgment that this offered the most desirable solution of the problem with decided advantages in economy of material and plant, uniformity in deflections, and safety and ease of erection.

To carry out this great work the St. Lawrence Bridge Company was formed and Mr. Johnson became its President and General Manager, at all times taking a deep interest in the engineering side of the work, particularly in its erection. It was an application on a large scale of the qualities which distinguished his work throughout his career, his practical turn of mind and a wonderful insight into difficult problems of handling material, supplemented by the skill to devise methods for overcoming the difficulties ahead.

Although called to executive positions with his Company he never lost his interest in the engineering side of the work and kept closely in touch with it even after his retirement from the actual direction of affairs.

Mr. Johnson was a bachelor, and led a very simple life, without any home ties. He had few recreations and did not mingle extensively in the social activities of his associates; there was thus nothing to interfere with his dominant interests, that of his profession and of his Company. He was, however, a public-spirited citizen, generously supporting the interests of others. No man in his community has been more revered among his associates and those who came in contact with him in business.

A little more than forty years ago, the writer became associated as a Junior with Mr. Johnson, and the respect which he early formed for his character and ability was never shaken but only deepened in that time. He feels that he owes much to his example and counsel.

Mr. Johnson became a Member of the Canadian Society of Civil Engineers (now the Engineering Institute of Canada) in 1883. He served on its Council for six years, was Vice-President for one year, and President in 1913.

McGill University, Montreal, Que., Canada, conferred on him the honorary degree of LL.D. in 1921, in recognition of his eminence in the Engineering Profession and particularly for the part he played in the construction of the Quebec Bridge.

Mr. Johnson was elected a Member of the American Society of Civil Engineers on July 1, 1891.

WILLIAM PIERSON JUDSON, M. Am. Soc. C. E.*

DIED FEBRUARY 12, 1925.

William Pierson Judson, the son of Col. John Work and Emily (Pierson) Judson, was born at Oswego, N. Y., on May 20, 1849. Both his father and his

* Memoir compiled from information on file at the Headquarters of the Society.

mother were of American Colonial descent, his mother having been a direct descendant of Abraham Pierson, the first President of Yale College.

He was educated in the public schools of Oswego, having been graduated from the High School in 1866. His education was continued by study and active practice under his father and Capt. Jared A. Smith, Corps of Engineers, U. S. Army, both of whom were graduates of the United States Military Academy, West Point, N. Y.

From 1866 to 1898, Mr. Judson was in active service as a Civilian Engineer with the United States Corps of Engineers on fort, river, and harbor surveys with headquarters at the U. S. Engineer Office at Oswego as follows:

From 1866 to 1868, he served as Draftsman at Fort Ontario, Oswego, under Captain Smith and Maj. Charles E. Blunt, Corps of Engineers, U. S. A. For the next four years he was engaged as Draftsman in connection with fort and harbor construction on Lake Ontario, and from 1873 to 1874, he served as Surveyor and Draftsman for forts, rivers, and harbors. In 1874, Mr. Judson was appointed U. S. Assistant Engineer in executive charge of the surveys, design, and construction of river and harbor works, lighthouses, and forts on Lake Ontario and the St. Lawrence River, including Lake Champlain; the harbors of Buffalo, N. Y., and Erie, Pa.; the rebuilding and repair of Fort Wayne, Detroit, Mich.; Fort Montgomery, Rouse's Point, N. Y.; Fort Niagara, New York; and many surveys of projected works, including 10 miles of the Narrows of Lake Champlain.

He also made canal surveys and presented reports, as follows: In 1890 he reported on the Niagara Ship Canal, advising a barge canal from Oswego to the Hudson River rather than a ship canal.* In 1896, he made a survey and report for the United States Deep Waterway Commission on "The Oswego-Oneida-Mohawk Valley Route from Oswego to Troy", advising the canalizing of the channels of these rivers.† In 1897, he was detailed as Assistant Engineer to Maj. Thomas W. Symons, Corps of Engineers, U. S. A., at Buffalo, and there wrote the body of the "Report on Ship Canal from the Great Lakes to the Hudson River", the construction advised being for barges rather than for ships.‡ The report was the basis for the plans for the Barge Canal as it was built by the State of New York (1905-20).

In 1898, Mr. Judson entered private practice and was engaged as Engineer for the Contractor on the construction of three miles of the Erie Canal, at Buffalo.

From 1899 to 1905, he served as Deputy State Engineer of New York for three terms, having been in executive charge of the organization and direction of the New York State Highway System during its first six years. He also edited the publications of the Department, directed the surveying and monumenting of county lines through the Adirondack Mountains, and co-operated with the U. S. Geological Survey in the survey of the State and the gauging of its streams.

* H. R. No. 283, 52d Cong., 1st Session, 1892.
† Report of the Commission, H. R. No. 149, 56th Cong., 1st Session, 1900.
‡ H. R. No. 86, 55th Cong., 1st Session, 1897.

From 1905 until his death, Mr. Judson was engaged in practice as a Consulting Civil and Electrical Engineer. During this time he was also President of the Broadalbin, N. Y., Electric Light and Power Company and the Broadalbin Supply and Construction Company, Incorporated, as well as a Director of the Broadalbin Knitting Company, Limited, and the Broadalbin Bank. He had also served on the Commission on the Varick Water Power Canal, at Oswego, since 1876.

He was married on October 9, 1888, to Mrs. Anna Littlejohn Thompson McWhorter, of Oswego, who survives him.

Mr. Judson was a man upon whom one could rely through all the vicissitudes and experiences of life, true to the end. Whether among those whom he knew well or others, he was recognized as one who possessed the highest conception of integrity and honor. He was devoted to his profession and maintained a deep interest in it as long as he lived. His death is a great loss to those who were privileged to enjoy his friendship.

He was the author of the following books: "From the West and Northwest to the Sea by Way of the Niagara Ship Canal" (1890); "An Enlarged Waterway from the Great Lakes to the Atlantic" (1893); "Lake Ontario to the Hudson River Through the Oswego-Oneida-Mohawk Valley" (1896); "History of Projects from Great Lakes to Tidewater, 1768-1901"; "City Roads and Pavements", editions of which were issued in 1894, 1902, 1906, and 1909; and "Road Preservation and Dust Prevention" (1908). He also wrote many engineering reports and papers.

Mr. Judson was a member of the Institution of Civil Engineers of Great Britain; American Society of Municipal Improvements; Massachusetts Highway Association; International Navigation Congress; American Association for the Advancement of Science; Genealogical and Biographical Society of New York; New York State Historical Association; American Defense Society; Sons of the American Revolution; and the Society of the War of 1812. He was also a member of the following clubs: Fortnightly, City, Country, and Yacht Clubs of Oswego; Antlers Club, of Amsterdam, N. Y.; Fort Orange and Institute of History and Art of Albany, N. Y.; Lake Placid Club, Lake Placid, N. Y.; and the Authors' Club, of London, England. He was also a Mason, and a member of the Protestant Episcopal Church. In politics he was a Republican.

Mr. Judson was elected a Member of the American Society of Civil Engineers on September 7, 1881.

WILLARD DATUS LOCKWOOD, M. Am. Soc. C. E.*

DIED FEBRUARY 22, 1926.

Willard Datus Lockwood was born at Mt. Kisco, N. Y., on November 29, 1870. He was the son of the late Ezekiel Sturgis and Caroline M. Ensign Lockwood. The Lockwood family were early settlers in Connecticut, being mentioned in the history of the State as far back as 1652.

* Memoir prepared by E. S. Skillin, Assoc. M. Am. Soc. C. E

His early life was spent in Rochester, N. Y., where he was educated. He was graduated from the Rochester High School and spent four years in the Rochester Atheneum Mechanics Institute. While in the Institute he acted as Instructor and Lecturer in descriptive geometry and took special courses in structural design, hydraulics, and strength of materials.

From 1888 to 1892, Mr. Lockwood was with the Rochester Bridge and Iron Works as Draftsman and Designer; the following year he became Assistant to the City Engineer of Rochester on the trunk sewer, a rock tunnel 10 miles long, and in 1893 and 1895 served as Engineer in connection with the Rochester Water Supply on the location and construction of 26 miles of steel water pipe.

In 1895 and 1896, he held the position of Assistant Engineer for the State of New York in charge of a division of surveys of State lands, and the following year was Engineer for contracting firms in charge of tunnel work and difficult foundations. During 1898 and 1899, he served as Principal Assistant Engineer with the United States Board of Engineers in the survey and investigation of water supply and power and the location of dams.

Following this work in 1899 and 1900, Mr. Lockwood was with the Isthmian Canal Commission in charge of surveys for the location of the Boca San Carlos Dam and the survey for Graytown Harbor. He was also in charge of estimates of cost for the proposed canal and the design of the harbor terminals.

For a year, in 1901, he was in Ecuador as Engineer on 50 miles of mountain railroad for the Guayaquil and Quito Railway. He again entered the Government service with the United States Corps of Engineers, in charge of fortifications and river and harbor work during 1901 and 1902.

Going to England in 1903, Mr. Lockwood was Engineer and Superintendent for the construction of a large blast furnace at Frodingham, and built a complete American plant for a British Steel Company.

Returning to Rochester in 1904 he became Assistant Engineer and Engineer in Charge of the construction of the Cobbs Hill Reservoir.

Leaving the professional engineering field, he was connected with several large contracting firms in the construction of dams, tunnels, dry docks, and submarine pipe lines in various parts of the United States and also in Santo Domingo, where he was in charge of the construction of concrete piers for the Dominican Government.

In 1913, Mr. Lockwood was made Manager for the Frederick Snare Corporation, Engineers and Contractors, at Philadelphia, Pa., and until 1925 had charge of all construction in and around Philadelphia for that Company. His work included the construction of several large steamship piers, both substructures and superstructures, and the supervision of a large part of the track installation on the elevated railway structure in Philadelphia. During the World War the construction of the first shipways at the Hog Island Yards and the large Quartermaster's Terminal at Philadelphia was carried out, as well as other important works.

Due to ill health Mr. Lockwood resigned his position with the Frederick Snare Corporation and in 1925, on the advice of his physician, moved to Miami, Fla. During the few months of his residence there he established himself in

business, also taking great interest in the various engineering problems in the large developments of Florida, and was elected Vice-President of the Engineers' Club of Miami.

His active life was spent in the various fields of engineering and construction in many places, in the United States, South America, the West Indies, and England. He had a very large acquaintance and many friends among engineers and those connected with engineering matters. He was an untiring worker and never saved himself when his time or energies were needed by his friends or business associates. He was always intensely active and interested in any problems to be solved and had a keen sense of ways and means to overcome unusual and difficult conditions.

By nature extremely sociable, Mr. Lockwood was interested and took an active part in the several societies and organizations of which he was a member, such as the Engineers' Club of Philadelphia, The Philadelphia Board of Trade, and the Manufacturers' Club of Philadelphia. He is survived by his wife and two daughters.

Mr. Lockwood was elected a Junior of the American Society of Civil Engineers on April 3, 1894, an Associate Member on October 7, 1896, and a Member on October 2, 1906.

ROBERT HUGH McCORMICK, M. Am. Soc. C. E.*

DIED FEBRUARY 23, 1925.

Robert Hugh McCormick, a son of David and Ann (McNaughton) McCormick, was born at Detroit, Mich., on May 1, 1853.

Mr. McCormick was educated in the public schools of Detroit and at the Bryant and Stratton Business College. In 1869 he found employment in his father's crockery and glass store. Upon the death of his father in 1872 and the closing of the business, young McCormick entered the Pingree and Smith Shoe Factory as a machine operator.

In the fall of 1878 Mr. McCormick became a Recorder on a Government survey of the Mississippi River. During the summer and fall of 1879 he was in charge of a soundings cutter on the St. Mary's River in connection with a survey for a new Hay Lake channel, made under the direction of the late Alfred Noble, Past-President, Am. Soc. C. E. From the fall of 1879 to the fall of 1880 Mr. McCormick was engaged as Rodman, Leveler, and Assistant Engineer on railroad location and construction in Eastern Michigan, and during the winter of 1880 he located and built twenty-five miles of narrow-gauge railroad for the late Gen. Russell A. Alger. From January, 1882, to July, 1883, Mr. McCormick was a Division Engineer for the Grand Trunk Railway Company on construction between Pontiac and Jackson, Mich.

In July, 1883, Mr. McCormick entered the office of the City Engineer at Detroit, as Assistant Engineer in charge of the Street Department. He became City Engineer in 1898 and continued in that position, except for

* Memoir prepared by John W. Reid, M. Am. Soc. C. E.

a period of about one year, until January 1, 1917, when he resigned and retired from active work. The increase in population of Detroit during the time of Mr. McCormick's contact with the office of the City Engineer was from about 150 000 in 1883 to about 800 000 in 1917, or more than 400 per cent.

He was a painstaking public official, possessed of a high sense of his responsibility to the public and to his profession. He was by nature careful and conservative and although quiet of disposition was keenly appreciative of the humorous. Mr. McCormick never married.

Mr. McCormick was elected a Member of the American Society of Civil Engineers on September 7, 1898.

MILTON FREDERICK STEIN, M. Am. Soc. C. E.*

DIED JANUARY 2, 1926.

Milton Frederick Stein, the son of Max and Elvira Stein, was born in Chicago, Ill., on December 17, 1886. He received his preliminary education in the Englewood Schools of Chicago and was graduated from the University of Illinois in 1909 with the degree of Bachelor of Science in Municipal and Sanitary Engineering. In 1916, Mr. Stein received the degree of Civil Engineer. His outstanding scholarship and unusual thesis merited special graduating honors and election to Sigma Xi.

After his graduation, Mr. Stein entered the employ of Chester and Fleming, Consulting Engineers (now The J. N. Chester Engineers), of Pittsburgh, Pa. Under Mr. Chester, he designed a number of reservoirs, pumping stations, water filtration plants, and other structures, at various places, in some instances acting also as Supervisor of construction and operation. Among these municipalities may be mentioned Charleroi, New Kensington, Phœnixville, and Erie, Pa.

In 1913 Mr. Stein was employed by the Water Department of the City of Cleveland, Ohio, where he supervised the preliminary tests for filtration of the water supply of that city. He was in immediate charge of the design of certain parts of the concrete work in the Division Avenue Filtration Plant, and assumed a major responsibility for the design of the filter details, chemical treatment equipment, laboratories, and the clear water reservoir. He also made some special investigations on the recovery of water sludge and its utilization.

In 1917, Mr. Stein left the employ of the City of Cleveland and joined the Engineering Staff of Hering and Gregory, Consulting Engineers, of New York, N. Y., where he remained for about a year, working particularly on the design of the sewage works for Albany, N. Y. He then returned to Cleveland and was employed by the Division of Sewage Disposal on special studies relating to the Westerly Sewage Treatment Works. In 1917, in order to be at home, Mr. Stein returned to Chicago and became connected with the work of the Sanitary District of Chicago, especially in the design of the Des Plaines

* Memoir prepared by George B. Gascoigne, M. Am. Soc. C. E.

activated sludge sewage works, one of the first large plants of this type. For a short time afterward, he was employed by Pearse, Greeley and Hansen, Hydraulic and Sanitary Engineers, of Chicago, and subsequently, became associated with the International Filter Company as Research Chemist and Engineer. It is understood that in this position he perfected a number of water-works improvements, particularly in rate controllers.

About 1921, owing to failing health, Mr. Stein gave up regular employment and spent his time in special investigation and study. In this connection, he developed and patented with Mr. C. A. Jennings, of Chicago, a fine screen for the removal of suspended solids from sewage. From 1923 until his death, most of his professional work was done as an associate of Mr. George B. Gascoigne, Consulting Sanitary Engineer, of Cleveland, as a special designing engineer, particularly on water-works problems.

Mr. Stein was a man of marked ability, very painstaking, and a great student. His contributions to the technical press were frequent and of high caliber. Aside from the magazine articles that appeared from time to time, his best known work is his book entitled "Water Purification Plants and Their Operation", which was first published in 1915, the second edition coming out in 1919, while a third edition, completed just before his death, has since been published. This book, which is of very high order, is considered a standard textbook in its field. Mr. Stein not only had a brilliant and analytical mind, but was extremely versatile. Not only was he at home in the field of sanitary engineering but he liked to delve into unusual and abstruse problems, requiring the most intricate mathematical solutions. Furthermore, he was an artist of no mean ability and an accomplished musician. By nature he was quiet and reserved and there were few who knew him intimately. These recognized in him a true friend as well as a capable engineer. His passing removes from the field of water purification engineering one who, perhaps more than any other, understood and was thoroughly conversant with the design, construction, and operation of such works.

Mr. Stein was elected an Associate Member of the American Society of Civil Engineers on December 3, 1912, and a Member on September 9, 1919.

JOSEPH STRACHAN, M. Am. Soc. C. E.*

DIED MARCH 20, 1926.

Joseph Strachan was born at Newburgh, N. Y., on December 31, 1859. He was a son of James Strachan and Susan (McCullough) Strachan, both of Ulster-Scotch stock.

His earlier education was obtained in the public schools of Brooklyn, N. Y., to which city his family moved when he was a child. In 1879, he entered New York University and was graduated in 1883 with Phi Beta Kappa honors and with the degrees of Bachelor of Science and Civil Engineer; in 1886, the degree of Master of Science was conferred on him by his Alma Mater.

* Memoir prepared by Willis R. Tenney, M. Am. Soc. C. E.

After graduation Mr. Strachan began his professional career by accepting a position on construction work with the South Pennsylvania Railroad Company, where he passed from the grade of Rodman to Chief of a construction party. On this work he spent about two years and then returned to his home in Brooklyn and began work with the Kings County Elevated Railroad Company in December, 1885, on its new elevated structure. For nearly four years he was engaged on elevated and surface railroad construction in Brooklyn, serving as Draftsman, Inspector, and Construction Superintendent. His work during this period embraced the design and construction of trestles, track layouts, coal pockets, conveyors, etc., and included work on the old New York and Bath Beach Railroad, the New York and Sea Beach Railroad, and the Fulton Elevated Railway.

In July, 1889, Mr. Strachan entered the employ of the then City of Brooklyn, N. Y., as an Assistant Engineer in the Department of City Works, Bureau of Water-Works Extension. In this capacity he prepared detail plans of masonry for the extension of the water mains east of Rockville Center, Long Island, designed changes of structures on the conduit line, made surveys, detail plans, and estimates for the relief of Baiseley's Pond from Jamaica sewage, and surveys and estimates for proposed changes in the grades of streets in Brooklyn.

In 1894 and 1895, he conducted the work of strengthening a part of the brick conduit near Liberty Avenue, Brooklyn, designed the connections of a proposed 66-in. steel pipe conduit with an existing conduit and the Milburn Pumping Station, and had charge of land purchases along the line of the conduit. From 1896 to 1900, he was engaged on the design of various pieces of work along the aqueduct, including the location survey for a proposed 66-in. steel pipe conduit from Milburn to Spring Creek; he superintended the layout and construction of the Oconee and Shetucket deep-well stations and of the change of the Springfield Pond Pumping Station to a deep-well station.

In 1901 and 1902, he had charge of the construction of a double pipe line of 48-in. cast-iron pipe from the Milburn Engine House to the Milburn Efflux Chambers.

From November, 1902, until his retirement from municipal service on January 31, 1926, Mr. Strachan was a Division Engineer in the Bureau of Highways, Borough of Brooklyn, New York, N. Y. For a considerable part of this time he was in charge of construction work, supervising the grading, paving, and repaving of many miles of streets. During his connection with street work he saw modern pavements of asphalt and granite on concrete foundation increase from about 95 miles to more than 870 miles in the Borough of Brooklyn, and the disappearance of the old cobble-stone and Belgian block pavements to the extent of 260 miles. In this work he had a large part, not only in the supervision of construction but in the preparation of the plans and specifications, in the study of local traffic conditions, and in the collection of data relating to improved methods in highway construction in various other cities.

Mr. Strachan was a Charter Member and Past-President of the Brooklyn Engineers' Club and had served as its Secretary for more than twenty-three

years, from January, 1903, until his death. During his incumbency of this office the Club grew rapidly, increasing in membership from about 200 to nearly 500, a result due in a very large measure to his efforts and his active interest in everything pertaining to it. He was also a Charter Member of the Brooklyn University Club, a member of the Zeta Psi Fraternity and of the Zeta Psi Club of New York.

As Secretary of the Brooklyn Engineers' Club he arranged, in 1914, the first Industrial Exposition held in the Borough of Brooklyn. This has now become a regular feature of the life of the city.

In 1888, Mr. Strachan was married to Mary Elizabeth Griswold, of Brooklyn, who died in 1921. A daughter, Louise, and two sons, Kenneth and Malcolm, survive him.

In August, 1925, he was compelled to give up the active practice of his profession, owing to heart trouble. From that date he gradually became weaker until his death on March 20, 1926.

Mr. Strachan was a man of exemplary habits, of a patient and kindly disposition, and one who had the respect of his associates and subordinates. His courteous manner, his willingness to assist others, and his consideration for the feelings of those about him made of him a companion whose loss will be a personal one to hosts of those who were fortunate in having had the opportunity of associating with him.

Mr. Strachan was elected an Associate Member of the American Society of Civil Engineers on May 4, 1892, and a Member on May 4, 1898.

WILLIAM ANDREW THOMPSON, M. Am. Soc. C. E.*

DIED DECEMBER 15, 1925.

William Andrew Thompson was born at Greenwich, N. Y., on December 16, 1853. He was the third son of Andrew A. and Eliza (Stevens) Thompson. His great-grandparents emigrated to this country from the north of Ireland in early Colonial days and both parents had several ancestors who served in the War of the Revolution.

After pursuing his studies in the local schools, he entered the University of Vermont and was graduated as Civil Engineer in the Class of 1877.

Immediately after his graduation Mr. Thompson sought employment with the United States Engineer Corps then being organized for improving the Upper Mississippi River from St. Paul, Minn., to the mouth of the Missouri River. From July, 1878, to March, 1879, he was in charge of surveys, and from March to December, 1879, served as Inspector in charge of contract work on the improvement of the river.

From August, 1880, to 1925, he was employed continuously on various engineering works for improving the Upper Mississippi, among which were the following: In 1887 he was in responsible charge of the construction of a breakwater in Lake Pepin, near Lake City, Minn., and in 1888 rebuilt the

* Memoir prepared by M. Meigs, M. Am. Soc. C. E.

breakwater in Lake Pepin near Stockholm, Wis. From May, 1896, to 1925, he was in charge, as Assistant Engineer, of the river improvements from Winona, Minn., to the mouth of the Wisconsin River, a distance of 96 miles. In this period he built another breakwater at Kings Coulee in Lake Pepin and planned and executed the river improvement works between Winona and the Wisconsin River, the first section to attain the full projected depth of 6 ft. at extreme low water. For this work he was greatly complimented by his superior officers.

Mr. Thompson was the first to advocate and use large quantities of brush in constructing the training works used to confine the channel. This method resulted in great economies in construction and in cost to the Government.

His Headquarters' Office at La Crosse, Wis., established in 1895, was relinquished on December 1, 1925, as the improvement work in that Section was completed, and the office force was transferred to St. Paul, Minn. Mr. Thompson died at his home in La Crosse, on December 15, 1925.

It is a curious coincidence that the date of the completion of his immediate work, of his retirement from service under the Civil Service Retirement Act, and of his death, all occurred on December 15, and that the anniversary of his birth was December 16. He was a man who spent a long life under one employer—the United States Government—and in one class of work, who attained the highest position to which he could aspire in that service and who was contented with the modest emolument allowed his position, but who gave his whole heart to his work and died respected and loved by his friends and associates.

Mr. Thompson was of a highly sociable nature and enjoyed the company of his many friends. He took a great interest in the civic affairs of his home town. He was a member of the La Crosse Club and the La Crosse Country Club.

His age prevented him from participating actively in the World War, but he gave a son to the service, Capt. Alexander M. Thompson, who went overseas and is now head of the La Crosse Dredging Company, at Minneapolis, Minn. Mr. Thompson took an intense and patriotic interest in the war and was active in promoting the sale of Liberty Bonds, serving on committees for that purpose.

On March 10, 1885, he was married to Myra Arntzen, of Quincy, Ill., the daughter of the Hon. Bernard Arntzen. His widow and son survive him.

Mr. Thompson was elected a Member of the American Society of Civil Engineers on October 7, 1896.

CARL DONALD BOSSERT, Assoc. M. Am. Soc. C. E.*

DIED SEPTEMBER 12, 1925.

Carl Donald Bossert was born in Washingtonville, Ohio, on May 3, 1884. He received his early education in the local grade and high schools and the

* Memoir prepared by C. R. Oberst, Esq., Columbus, Ohio.

Leetonia High School. He then entered Ohio State University and was graduated with the Class of 1908 with the degree of Civil Engineer.

During the summer vacations and immediately after his graduation from college, Mr. Bossert was with the Pennsylvania Railroad Lines West of Pittsburgh, as Instrumentman and Draftsman on location, working in Eastern Ohio. In 1909, he resigned to accept a position with the United States Government, laying out Government lands in Minnesota. He was also stationed at San Francisco, Calif., and from there went to the Navy Yard at Charleston, S. C., as Instrumentman.

Upon the completion of his work at the Charleston Navy Yard in 1910, he again engaged in railway work as Instrumentman on the location of the Sandy Valley and Elkhorn Railroad, in Eastern Kentucky.

In May, 1911, Mr. Bossert returned to Eastern Ohio to become Deputy County Surveyor of Columbiana County, where he had charge of the location and construction of several important roads, with bridges and drainage. He held this position until April, 1915, when he transferred to Wayne and Holmes Counties, Ohio, as Deputy Surveyor, which position he held until the World War. While in this position, he worked on the design of the 8 000-acre drainage project for these counties.

In October, 1918, he was commissioned First Lieutenant of Engineers, U. S. Army, and was about to go over seas when the Armistice was signed. He was discharged from the Army in January, 1919.

From March, 1919, to January, 1923, Mr. Bossert served as Bridge Engineer in Franklin County, Ohio, working on preliminary surveys and plans for the Greenlawn Avenue Bridge and the changes in design of the Mound Street Bridge, at Columbus. He had complete charge during the construction of the latter bridge.

In January, 1923, he entered the employ of Carl B. Harrop, Engineers and Constructors, as Construction Engineer and, later, became Superintendent of Construction. This position took him into the ceramic field in different parts of the United States, in which, as in all former enterprises, he was highly successful.

Mr. Bossert ranked high as a student, possessed a keen mind, and had the faculty of combining theoretical and practical engineering knowledge to a high degree. He was well liked among his fellow workers and, although a hard worker, always had time to help a younger engineer. He was an engineer in every sense of the word and was always very active in technical societies. Courteous, unassuming, and self-retiring, he held the esteem of all who knew him.

He was a member of the American Association of Engineers, an Associate Member of the American Ceramic Society, and a member of the Engineers' Club of Columbus. He was a member of the Masonic Fraternity and had taken all the degrees possible, having been a Knight Templar, Scottish Rite Mason, and Shriner. He was also a member of Acacia, National Collegiate Fraternity.

Mr. Bossert was elected a Junior of the American Society of Civil Engineers on October 31, 1911, and an Associate Member on December 6, 1915.